CLASS 10th

CBSE 2025

padhleakshay

33 DAYS™ CHALLENGE

DAY 1 TO DAY 33 CHAPTERWISE

DAILY TARGETS

MATHEMATICS (STANDARD)

PYQs from Last 10 Years

SAMPLE PAPERS Included

555+ Competency Questions

Based on Latest CBSE Sample Papers dated Sep 5th, 2024

"100% Swaha"

with padhleakshay

Based on CBSE Latest Sample Papers dated 05/09/24

HOW TO USE THIS BOOK?

1

Step 1:
Trust the book as you don't need to solve anything else than this.

2

Step 2:
Download our Android App 'PadhleAkshay' from the Google Playstore so that you get access to all the notes for quick revision before solving the sums.

3

Step 3:
Before starting with Day-1, have a look on the first page which gives an idea of typology of questions asked last year and the flowcharts page for brief intro to the chapter.

4

Step 4:
Follow it day-wise and try not to miss/skip any day in your journey.

5

Step 5:
Swaha! You've solved 100% of important questions after these 33 Days. Now, just solve the given sample papers to get the grip of the latest pattern.

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C-8, Sector 6, Noida, Uttar Pradesh-201 301
Phone: +91-99100 55604, 78400 40400
E-mail: info@shivdas.in • www.shivdas.in
Trade contact: sales@shivdas.in

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CONTENTS

# # #

PREFACE

Padhleakshay has become a trusted source of belief for lakhs of students since the 2020 Boards Examinations after he provided the best possible notes and a set of all important questions designed by himself and his team to all the CBSE students via his YouTube channel **Padhle (@Padhleakshay)**.

This book stands out from all other books present in the market, **but Why?**

The reason is once again, the structure of this book, if we count the most necessary resources for CBSE Boards Exams on fingers, those would be Concise notes, Previous Years' Questions, Competency Based Questions and Sample Papers. All of these resources are packed into this book with the best possible structure of 33 Days so that your mind is very clear about taking one day at a time, and that too chapterwise, **but Why?**

The reason this book has been organised chapterwise is to make sure that you've covered all the topics with all possible typologies of questions from that particular chapter starting from Objectives, Assertion Reasons, Subjective questions and the Case based questions. Not just the questions but the answers make this book special, **but Why?**

Each and every question in this book has been designed by experts keeping in mind the latest CBSE pattern of Competency Based Questions and it's quite evident on every page with the 'COMPETENCY' label on such questions with the answers containing 'Explanation' and 'Free Advice' boxes wherever required to enhance your clarity. After all these efforts, the book has been reviewed by the 'Toppers Bench', making it the best product the market will ever witness.

Akshay has been working on this book for the last 4 months with his team. We would like to convey big thanks to all of them, especially **Ayan, Aditya Kumar, Kaunain Ahmad** and **Anurag Yadav**.

That said, we believe that there is always scope for doing things in a better manner and hence we invite you to provide us with your candid feedback and suggestions on how we can make this series even better.

# # #

1 Surface Areas And Volumes

What did CBSE ask last year?

MCQs & A/R	1 Question (1 × 1 = 1 Mark)
Subjective	**No Very Short Question Asked**
	1 Short Question (1 × 3 = 3 Marks)
	1 Long Question (1 × 5 = 5 Marks)
Case Based	No Case Based Questions Asked

Note: *All the above typology of questions include 'Competency based Questions' labelled as* COMPETENCY.

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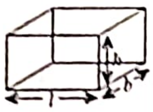
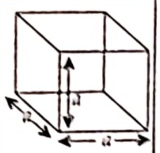
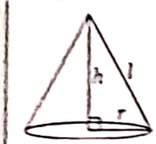
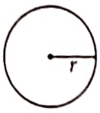
Introduction & Surface Area of a Combination of Solids

Table for Surface Area and Volume

Solid	Figures	Curved surface area (1)	Plane area (2)	Total area [1 + 2]	Volume	Remarks
Cuboid		Also known as lateral surface area = $2(lh + bh)$	Area of: Top face = lb Bottom face = lb $2lb$	$2(lb + bh + hl)$	$l.b.h$	l : length b : breadth h : height
Cube		Lateral surface area = $4a^2$	Top face = a^2 Bottom face = a^2 $2a^2$	$4a^2 + 2a^2 = 6a^2$	a^3	a : Side of cube
Right circular cylinder closed at top and bottom		Curved surface area = $2\pi rh$	Area of: Top face = πr^2 Bottom face = πr^2 $2\pi r^2$	$2\pi r^2 + 2\pi rh$ or, $2\pi r(r + h)$	$\pi r^2 h$	r : radius h : height of cylinder
Right circular cylinder open at top		Curved surface area = $2\pi rh$	Area of: Top face = 0 Bottom face = πr^2 πr^2	$2\pi rh + \pi r^2$ or, $\pi r(2h + r)$	$\pi r^2 h$	r : radius h : height of cylinder
Hollow cylinder (Pipe)		External surface area = $2\pi Rh$ → Internal surface area = $2\pi rh$	Area of: Top face = $\pi(R^2 - r^2)$ Area of: Bottom face = $\pi(R^2 - r^2)$	$2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$	$\pi R^2 h - \pi r^2 h$ (External Volume – Internal-Volume)	R : Radius of Outer Base r : radius of Inner Base h : height
Cone		πrl	Area of: Bottom Face = πr^2	$\pi r^2 + \pi rl$ or $\pi r(r + l)$	$\frac{1}{3}\pi r^2 h$	h : height of cone r = radius of cone l = slant height $= \sqrt{h^2 + r^2}$

SURFACE AREAS

Frustum		$\pi l(R + r)$	*Area of: Top Face* $= \pi r^2$ *Area of: Bottom Face* $= \pi R^2$	$\pi r^2 + \pi R^2 + \pi l(R + r)$	$\frac{1}{3}\pi h (R^2 + r^2 + Rr)$	h = *height of frustum* r = *radius of top face* R = *Radius of base* l = *slant height* $= \sqrt{h^2 + (R-r)^2}$
Sphere		$4\pi r^2$	*None*	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r : *radius of sphere*
Hemisphere		$2\pi r^2$	πr^2	$3\pi r^2$	$\frac{2}{3}\pi r^3$	r : *radius of hemisphere*
Spherical shell		$4\pi R^2$ *(Outer)* $4\pi r^2$ *(Inner)*	*None*	$4\pi R^2 + 4\pi r^2$	$\frac{4}{3}\pi (R^3 - r^3)$	R : *Radius of outer shell* r : *Radius of inner shell*

<u>Volume of a Combination of Solids</u>

Note: *Cubes, cuboids, spheres, hemispheres, right circular cylinders and cones form combinations together and ask questions to find surface areas and volumes.*

TIME SAVING HACKS

Area × Rate = Cost

1L = 1000 cm^3

1000L = $1m^3$

Density × Volume = Mass

Speed = $\dfrac{\text{Distance}}{\text{Time}}$

AND VOLUMES

OBJECTIVE QUESTIONS

(DAY 1)

Multiple Choice Questions

Q.1. The radius of a sphere (in cm) whose volume is $12\pi cm^3$, is *[CBSE 2020*

(*a*) 3 (*b*) $3\sqrt{3}$

(*c*) $3^{\frac{2}{3}}$ (*d*) $3^{1/3}$

Q.2. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is COMPETENCY

(*a*) 2 : 1 (*b*) 1 : 2 (*c*) 1 : 3 (*d*) 3 : 1

Q.3. The circumferences of two circles are in the ratio 4 : 5. What is the ratio of their radii ? *[CBSE 2023*

(*a*) 16 : 25 (*b*) 25 : 16

(*c*) 2 : $\sqrt{5}$ (*d*) 4 : 5

FREE ADVICE: The ratio of the circumferences of two circles is equal to the ratio of their radii. This is because the circumference of a circle is directly proportional to its radius.

Q.4. The table below shows the measurements of 3 right circular cones. COMPETENCY

Cone	Radius (in cm)	Slant Height (in cm)
P	3	5
Q	5	7
R	3.5	10

Which of these have the same curved surface area?

(*a*) only P and Q (*b*) only Q and R

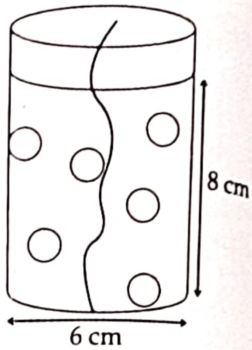
(*c*) only R and P

(*d*) P, Q and R have different curved surface areas because they have different radii and slant heights

Q.5. 6 spherical glitter balls with diameter 1 cm are present in a cylindrical candle made with transparent wax as shown in the figure.

Find the volume of wax used to make the candle.

[CBSE 2024



(*a*) 70 π cm^3

(*b*) 71 π cm^3

(*c*) 72 π cm^3

(*d*) 73 π cm^3

Q.6. If two solid-hemispheres of same base radius are joined together along their bases, then curved surface area of this new solid is

(*a*) $4\pi r^2$ (*b*) $6\pi r^2$

(*c*) $3\pi r^2$ (*d*) $8\pi r^2$

FREE ADVICE: Curved surface area of a hemisphere is $2\pi r^2$ and here, we've joined two solid hemispheres along their bases of radius r, from which we get solid sphere. Hence, the curved surface area of a new solid = $2\pi r^2 + 2\pi r^2 = 4\pi r^2$.

Q.7. A metallic sphere of radius 10.5 cm is melted and then recasted into small cones, each of radius 3.5 cm and height 3 cm. The number of such cones is COMPETENCY

(*a*) 63 (*b*) 126 (*c*) 21 (*d*) 130

FREE ADVICE: The number of small cones that can be made from the metallic sphere, we need to equate the volume of the sphere with the volume of one of the small cones.

Q.8. Bipin is making iced tea in 2.2 litre jar. He adds some ice spherical balls of diameter 2 cm into the jar, followed by 1.32 litre of tea until it's full. How many ice spheres does he add to the cup? COMPETENCY

(Note: 1 ml = 1 cm^3 and take π as $\frac{22}{7}$.)

(*a*) 126.25 (*b*) 210

(*c*) 315 (*d*) 1050

Q.9. The diameter of hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be outside the cone? **COMPETENCY**

(a) 50% (b) less than 50%

(c) 100% (d) more than 50%

FREE ADVICE: Aise question kabhi bhi aye tho bus dono solids k volume ka formula apas me divide kr do.

Q.10. A solid iron cylinder is melted to form rods of the same height. The radius of the iron rods is $\frac{1}{4}$ of the radius of the cylinder. How many rods were made? **COMPETENCY**

(a) 4 (b) 16

(c) 64 (d) depends on the volume of the cylinder

Q.11. A rectangular sheet of paper 40 cm × 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is [CBSE 2014

(a) 3.5 (b) 7 (c) $\frac{80}{7}$ (d) 5

FREE ADVICE: We need to use the formula for the lateral surface area of a cylinder, which is equal to the area of the rectangular sheet of paper i.e. $l \times b$

Q.12. A right circular cylinder of radius r and height $h(h > 2r)$ just encloses a sphere of diameter. **COMPETENCY**

(a) r (b) $2r$ (c) h (d) $2h$

Q.13. A container with a Grey hemispherical lid has radius R cm. In figure 1, it contains water upto a height of R cm. It is then inverted as shown in figure 2. What is the height of water in figure 2? **COMPETENCY**

(a) R

(b) $\frac{5R}{3}$

(c) 2R

(d) $\frac{7R}{3}$

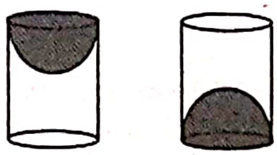


Fig. 1 Fig. 2

Q.14. During conversion of a solid from one shape to another, the volume of the new shape will **COMPETENCY**

(a) increase

(b) decrease

(c) remain unaltered

(d) be doubled

Q.15. The volume of a right circular cone whose area of the base is 156 cm^2 and the vertical height is 8 cm, is [CBSE 2023

(a) 2496 cm^3 (b) 1248 cm^3

(c) 1664 cm^3 (d) 416 cm^3

Q.16. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recasted into the form of a cone of base diameter 8 cm. The height of the cone is [NCERT EXEMPLAR

(a) 12 cm (b) 14 cm

(c) 15 cm (d) 18 cm

FREE ADVICE: Agar koi solid melt kar ke dusre solid me convert hota hai to dono ka volume equal hota hai.

Q.17. Two identical solid cubes are joined by a side to form a cuboid.
What fraction of the surface area of the two cubes is the surface area of the cuboid? **COMPETENCY**

(a) $\frac{6}{5}$ (b) $\frac{11}{12}$

(c) 1

(d) cannot be determined without the exact dimensions

Q.18. The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is: [NCERT

(a) 10 : 17 (b) 20 : 27

(c) 17 : 27 (d) 20 : 37

Q.19. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by:

(a) 3 (b) 4 (c) 5 (d) 6

Q.20. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is increased by **COMPETENCY**

(a) 300% (b) 700%
(c) 400% (d) 600%

FREE ADVICE: When the radius of a sphere is increased by 100%, it means the new radius is double the original radius.

Q.21. A solid hemisphere with radius 20 cm is melted to form 8 cones of the height 20 cm.
Which of these is the radius of the cones? **COMPETENCY**

(a) 5 cm (b) $2\sqrt{10}$ cm
(c) 10 cm (d) $10\sqrt{2}$ cm

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).
(c) (A) is true and (R) is false.
(d) (A) is false, but (R) is true.

Q.1. Assertion: The radius of the top and bottom of a bucket of slant height 35 cm are 25 cm and 8 cm. The curved surface of the bucket is 3630 sq.cm.
Reason: Curved surface of bucket $= \pi(R_1 + R_2) \times$ *slant height* (l)

Q.2. Assertion: Total surface area of the cylinder having radius of the base 12 cm and height 30 cm is 3318 cm².
Reason: If r be the radius and h be the height of the cylinder, then total surface area $= (2\pi rh + 2\pi r^2)$. **COMPETENCY**

Q.3. Assertion: Number of metal balls that can be made out of a solid cube of lead whose edge is 44 cm, each ball being 4 cm in diameter, is 2541.
Reason: *No. of balls* $= \dfrac{\textit{Volume of one ball}}{\textit{Volume of lead}}$

Q.4. Assertion: If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.
Reason: If r be the radius and h be the slant height of the cone, then slant height, $l = \sqrt{(h^2 + r^2}$.

Q.5. Assertion: The number of solid sphere of diameter 6 cm can be made by melting a solid metallic cylinder of height 45 cm and diameter 4 cm is 5. **COMPETENCY**
Reason: If three solid metallic spherical balls of radius 3,4,5 cm are melted into single spherical ball then its radius is 6 cm.

SUBJECTIVE QUESTIONS

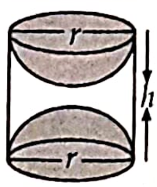
Very Short Answer Questions

Q.1. Two cubes each of volume 27 cm³ are joined end to end to form a solid. Find the surface area of the resulting cuboid.

Q.2. The volume of a right circular cylinder with the height equal to the radius is $25\frac{1}{7}$ cm³. Find the height of the cylinder. *[CBSE 2020*

Q.3. A solid is in the shape of a cone mounted on a hemisphere of same radius. If the curved surface area of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part. **COMPETENCY**

Q.4. A wooden article was made by scooping out a hemisphere of radius 7 cm, from each end of a solid cylinder of height 10 cm and diameter 14 cm. Find the total surface area of the article.



COMPETENCY

Q.5. A toy is in the shape of a solid cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 21 cm and 40 cm respectively and the height of cone is 15 cm, then find the total surface area of the toy.

COMPETENCY

Q.6. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes.

[CBSE 2020

Q.7. A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes. COMPETENCY [CBSE 2020

(DAY 2)

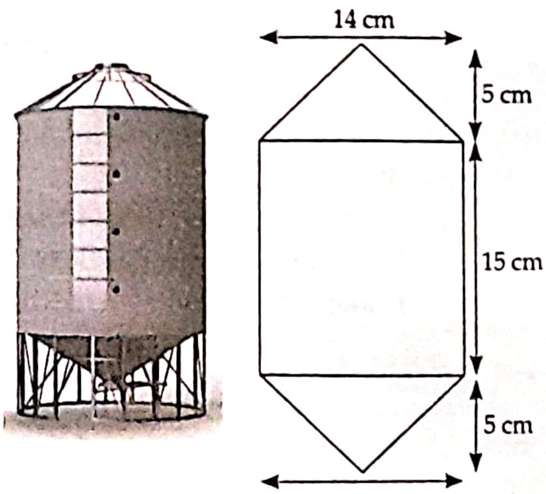
Short Answer Questions

Q.1. 500 persons are taking dip into a cuboid pond which is 80 m long and 50 m broad. What is the rise of water level in the pond if the average displacement of the water by a person is 0.04 m^3?

COMPETENCY

Q.2. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation? [CBSE 2017

Q.3. A silo is used to store grains. It can be observed as a cylinder with 2 cones on its circular bases as shown in the figure below. [CBSE 2024



(Note: The figure is not to scale.)

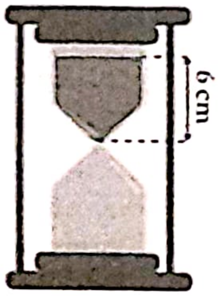
If the height of the grains in the silo is 20 m, what fraction of the silo's volume is filled with grains? Show your work.

Q.4. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the 22 solids. [CBSE 2019

Q.5. A juice seller was serving his customers using glasses as shown in fig. The inner diameter of the cylindrical glass was 5 cm but bottom of the glass has a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent and actual capacity of the glass. (Use π = 3.14) COMPETENCY

Q.6. A sphere of diameter 12 cm dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the vessel, raises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel. [CBSE 2018

Q.7. Shown here is a 5-minute sand timer. At the beginning, the sand has a height of 6 cm. After 5 minutes, the sand only occupies space in the cylindrical portion.



If the height of each conical section is h cm, then find the height of the sand in the bottom portion, after 5 minutes, in terms of h. Show your work. COMPETENCY

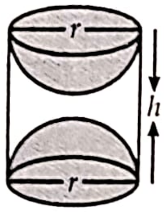
Q.8. Water flows at the rate of 10 minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm?

COMPETENCY

Long Answer Questions

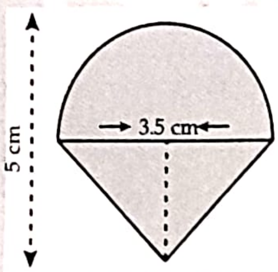
(DAY 3)

Q.1. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article. *[CBSE 2023*

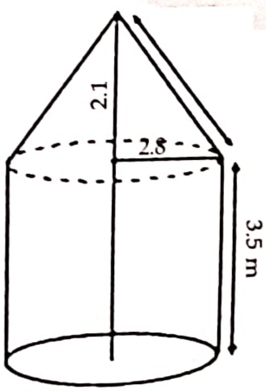


Q.2. A double cone is formed by a revolving right triangle having sides 5 cm, 12 cm and 13 cm about its hypotenuse. Find T.S.A. and volume of double cones so formed. **COMPETENCY**

Q.3. Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere (see Fig). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take $\pi = {}^{22}/_{7}$) *[NCERT Exemplar*



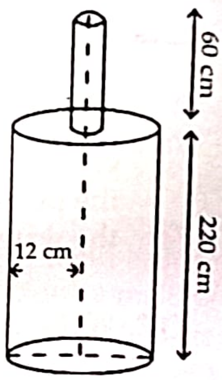
Q.4. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally.
The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹120 per sq. m, find the amount shared by each school to set up the tents. **(Use $\pi = {}^{22}/_{7}$)** **COMPETENCY**



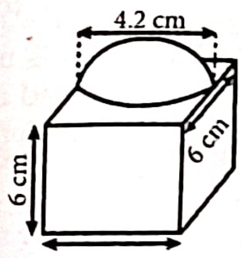
Q.5. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 minutes, if 8 cm standing water is required? **COMPETENCY**

Q.6. Water is being pumped out through a circular pipe whose internal diameter is 8 cm. If the rate of flow of water is 80 cm/sec, then how many litres of water is being pumped out through this pipe in one hour? *[CBSE 2021*

Q.7. A solid iron pole consists of a cylinder of height 220 m and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 gm mass. (Use $\pi = 3.14$) *[CBSE 2024*



Q.8. In Fig., a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 6 cm and the hemisphere fixed on the top has a diameter of 4.2 cm.
Find:
(*a*) the total surface area of the block.
(*b*) the volume of the block formed.
(Take $\pi = {}^{22}/_{7}$) *[CBSE 2019*



FREE ADVICE: Hemisphere ka base area neglect kr dena hai kyu ki vo total surface area ka part nahi hai.

CASE BASED QUESTIONS

Q.1. On a Sunday,your parents took you to a fair. You could see lot of toys displayed, and you wanted them to buy a RUBIK's cube and strawberry ice-cream for you.

Observe the figure and answer the questions:

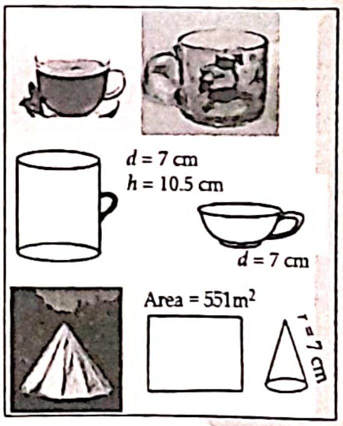
(*a*) What is the length of the diagonal, if each edge measures 6 cm?

(*b*) (*i*) Volume of the RUBIK's cube if the length of the edge is 7 cm, is ____.

Or, (*ii*) What is the curved surface area of hemisphere (ice cream) if the base radius is 7 cm? **COMPETENCY**

(*c*) If the radius is 7 cm and the height is 24 cm. What is the slant height of an ice-cream.

Q.2. Adventure camps are the perfect place for children to practice decision making or themselves without parents and teachers guiding their every move.



Some students of a school reached for adventure at Saleshpur.

At the camp, the waiters served some students with a welcome drink in a cylindrical glass and some students in a hemispherical cup whose dimensions are shown below. After that they went for jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter.

Each group of four students was given a canvas of area 551 m^2. Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting, would amount to 1 m^2, the students put the tents. The radius of the tent is 7 m.

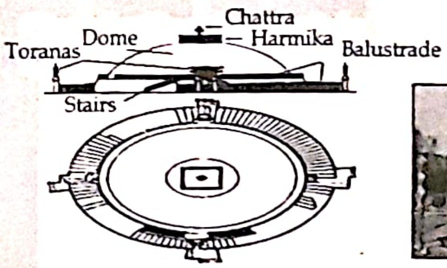
(*a*) What is the volume of cylindrical cup?

(*b*) (*i*) What is the volume of hemispherical cup?

Or,(*ii*) Which container had more juice and by how much? **COMPETENCY**

(*c*) What is the height of conical tent prepared to accommodate four students ?

Q.3. The Great Stupa at Sanchi is one of the oldest stone structures in India and an important monument of Indian Architecture. It was originally commissioned by the emperor Ashoka in the 3rd century BC. Its nucleus was a simple hemispherical brick structure built over the relics of the Buddha. It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it.



(*a*) The formula to find the Volume of Sphere is ________ .

(*b*) (*i*) Calculate the volume of the hemispherical dome if the height of the dome is 21 m.

Or (*ii*) The cloth required to cover the hemispherical dome if the radius of its base is 14 m is ________ .

(*c*) The total surface area of the combined figure *i.e.* hemispherical dome with radius 14 m and cuboidal shaped top with dimensions 8 m, 6 m and 4 m is ________ . **COMPETENCY**

* * * * *

ANSWERS

Multiple Choice Answers

1. (c) Volume of Sphere = $\frac{4}{3}\pi r^3$
Here volume is 12π cm^3
$\Rightarrow \frac{4}{3}\pi r^3 = 12\pi \text{ cm}^3 \Rightarrow 4r^3 = 36$
$\Rightarrow r^3 = 9 \quad \therefore \quad r = 3^{2/3}$

2. (a) Let the radius of the cone and cylinder be r
The base radii of cone and cylinder are equal.
Curved surface areas are also equal
$\therefore \pi rl = 2\pi rh \quad \therefore \quad \frac{l}{h} = \frac{2}{1}$

3. (d) Circumference of a circle = $2\pi r$
$\frac{2\pi R_1}{2\pi R_2} = \frac{4}{5} \quad \Rightarrow \quad \frac{R_1}{R_2} = \frac{4}{5}$
$\therefore$ **$R_1 : R_2 = 4 : 5$**

4. (b) As we know, C.S.A. of cone = πrl
$\therefore P = \pi(3)(5) = 15\pi \text{ cm}^2$
$Q = \pi(5)(7) = 35\pi \text{ cm}^2$
$R = \pi(3.5)(10) = 35\pi \text{ cm}^2$
Hence, Q & R have the same C.S.A.

5. (b) **Given:** radius, $r = \frac{D}{2} = \frac{1}{2}$ cm
Volume of 6 spherical balls
$= \frac{4}{3}\pi r^3 \times 6$
$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{1}{2}\right)^3 \times 6$
$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{8} \times 6 = \frac{22}{7}$
$= \pi \text{ cm}^3$
Volume of wax used to make candle
$= \pi r^2 h$ – Volume of 6 spheres
$= \pi(3)^2 \times 8 - \pi = \pi \times 9 \times 8 - \pi$
$= 72\pi - \pi =$ **71π cm^3**

6. (a) We know that,
Curved surface area of a hemisphere $= 2\pi r^2$
So, when two solids hemispheres of same base radius are joined together along their bases, then Curved surface area of newly formed solid sphere
$= 2 \times 2\pi r^2 =$ **$4\pi r^2$**

7. (b) Volume of sphere $= \left(\frac{4}{3}\right)\pi r^3$
$= \left(\frac{4}{3}\right)\pi(10.5)^3 =$ **1543.5π cm^3**
Volume of the cone $= \left(\frac{1}{3}\right)\pi r^2 h$
$= \left(\frac{1}{3}\right)\pi (3.5)^2(3) = 12.25\pi \text{ cm}^3$
Number of small cones = $\frac{\textit{Volume of sphere}}{\textit{Volume of cone}}$
$= \frac{1543.5\pi}{12.25\pi} =$ **126**

8. (b) **Given.** Volume of Jar = $2.2l$
$= 2200$ ml
...[$\therefore$ 1 ml = 1 cm^3
Volume of tea = $1.32l$ = 1320 ml
Remaining volume for ice
$= 2200 - 1320 = 880 \text{ cm}^3$
Volume of one ice sphere $= \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi (1)^3 = \frac{4}{3} \times \frac{22}{7} \times 1$ ...[d = 1 cm
= 4.19 cm^3
$\therefore$ **No. of Ice spheres** $= \frac{880}{4.19} =$ **210**

9. (d) If the hollow cylinder could cover exactly 50% of the spherical ball.
The base of the cone would lie on the ball's equator. The spherical ball is at its widest around its equator. If the cone could come up to the ball's equator, it could cover whole ball since after the equator the spherical ball only keeps tapering. This is why, cone can cover only less than half the ball.
Hence, more than 50% of the ball would lie outside the cone.

10. (b) Radius of cylindrical rod = R
Radius of iron rod $= \frac{1}{4}$R

Height of cylindrical rod

= Height of iron rod = h

Volume of cylinder = $\pi R^2 h$ ...(i)

Volume of iron rod = $\pi\left(\frac{1}{16}\right)R^2h$ ...(ii)

$\therefore$ **No. of Rods** = $\frac{\pi R^2 h}{(1/16)\pi R^2 h}$ = **16**

11. **(a)** We'll equate the area of the rectangle to the lateral surface area of the cylinder:

$2\pi rh = l \times R$

$\Rightarrow 2\pi rh = 40 \times 22 \Rightarrow 2\pi r \times 40 = 40 \times 22$

$\Rightarrow 2\pi r = 22 \quad \therefore r = \frac{22}{2\pi} = \frac{11}{3.14}$ = **3.5 cm**

12. **(b)** Sphere of diameter

= Diameter of base of cylinder

$\therefore$ **$d = 2r$**

13. **(b)** $\pi R^2 h - \frac{2}{3}\pi R^3 = \pi R^3$

$\Rightarrow \pi R^2 h = \pi R^3 + \frac{2}{3}\pi R^3$

$\Rightarrow \pi R^2 h = \pi R^2\left(R + \frac{2}{3}R\right) \quad \therefore h = \frac{5R}{3}$

14. **(c)** During conversion of a solid from one place to another, the volume of the new shape will ***remain unaltered.*** It remains constant.

15. **(d)** Height of cone = 8 cm

Area of base = 156 cm^2.

Putting values we get,

$\therefore$ **V** = $\frac{1}{3} \times 156 \times 8 = \frac{1248}{3}$ = **416 cm^3**

16. **(b)** We know that,

Volume of spherical shell = $\frac{4}{3}\pi(r_2^3 - r_1^3)$

$= \frac{4}{3}\pi(4^3 - 2^3) = \frac{4}{3}\pi(56) = \left(\frac{224}{3}\right)\pi$

We know that,

Volume of cone = $\left(\frac{1}{3}\right)\pi r^2 h,$

$= \left(\frac{1}{3}\right)\pi(4)^2 h = \frac{16\pi h}{3}$

Here,

Volume of spherical shell = Volume of cone recast by melting

$\Rightarrow \frac{224\pi}{3} = \frac{16\pi h}{3} \quad \Rightarrow \quad 16h = 224$

$\therefore$ **h = 14 cm**

17. **(a)** Let length of cube be l cm

Surface area of cube = $6a^2$

Surface area of two cubes = $2 \times 6 \times l^2$

$= 2 \times 6l^2 = 12l^2$ ...(i)

and length of cuboid, $l = 2l$,

breadth = l, height = l

Surface area of cuboid

$= 2(lb + bh + hl)$

$= 2[2l^2 + l^2 + 2l^2] = 10l^2$ ...(ii)

$\therefore$ **Fraction S.A. of two cubes to the area of cuboid** = $\frac{12l^2}{10l^2} = \frac{6}{5}$

...[From (i) & (ii)]

18. **(b)** Let the radii of two cylinders be r_1 and r_2 and height of the cylinders be h_1 and h_2.

Given, $r_1 : r_2 = 2 : 3$ and $h_1 : h_2 = 5 : 3$

$\therefore$ Ratio of volumes = $\frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$

$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3} = \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$

Hence, the ratio of their volumes is 20 : 27.

19. **(a)** Let the rise in water level in the cylindrical vessel be 'h' cm.

Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of liquid displaced in the cylindrical vessel = $\pi R^2 h$

If the sphere is completely submerged in the vessel, then volume of liquid displaced in the cylindrical vessel = Volume of the sphere

$\therefore \pi R^2 h = \frac{4}{3}\pi r^3$

$\Rightarrow 18^2 \times h = \frac{4}{3} \times (9)^3$

$\Rightarrow h = \frac{4}{3} \times \frac{1}{(18)^2} \times (9)^3$ = **3 cm**

Thus, the rise in water level in the cylindrical vessel is 3 cm.

20. **(b)** Initial Volume (V) of sphere = $\frac{4}{3}\pi r^3$

If the radius of a sphere increases by 100%, it means the radius is doubled.

New volume of sphere is V_1.

Now $V_1 = \frac{4}{3}\pi(2r)^3$

$V_1 = 8V$

So, volume will be 8 times the original volume.

% change in volume $= \frac{(8V - V)}{V} \times 100$

$= 700\%$

21. (*c*) Volume of hemisphere

= Volume of 8 cones

$$\frac{2}{3}\pi R^3 = 8\left(\frac{1}{3}\pi r^2 h\right)$$

$$\frac{2}{3}\pi(20)^3 = \frac{8}{3}\pi r^2(20)$$

$\Rightarrow (20)^3 = 4r^2 \times 20 \Rightarrow r^2 = \frac{20 \times 20}{2 \times 2}$

$\Rightarrow r^2 = 100$ $\qquad \therefore r = 10$ cm

Assertion Reason Answers

1. (*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation. *Here 'l' is slant height, r_1 and r_2 are top and bottom radius respectively.*

Curved surface area $= \pi l(r_1 + r_2)$

$= \frac{22}{7} \times 35(25 + 8)$

$= 22 \times 5 \times 33 =$ **3630 cm²**

2. (*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation. *The total curved surface area for a cylinder is $2\pi r(r+h)$.*

3. (*c*) *(A) is true and (R) is false.*

Explanation. *We have to find the number of spherical lead shots of diameter 4 cm that can be made out of the solid cube.*

Given, edge = 44 cm

Vol. of solid cube $= (a)^3 = (44)^3$

$= 85184$ *cm³*

Diameter of lead shot = 4 cm

Radius $= \frac{4}{2} = 2$ *cm*

Volume of leadshots $= \left(\frac{4}{3}\right)\pi r^3$

$= \left(\frac{4}{3}\right)\left(\frac{22}{7}\right)(2)^3$

$= \left(\frac{4}{3}\right)\left(\frac{22}{7}\right)(8) = 33.52$ *cm³*

$\therefore$ *Number of spherical lead shots that can be formed* $= \frac{\text{Volume of solid cube}}{\text{Volume of lead shot}}$

$= \frac{85184}{33.52} = 2541$

4. (*d*) *(A) is false, but (R) is true.*

*If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is **25 cm.***

5. (*b*) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*

Volume of single solid sphere

= Volume of all three sphere

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3)$$

$R^3 = 27 + 64 + 125$

$\therefore R = \sqrt{216} = 6$ *cm*

Very Short Answers

1. As volume of cube = 27 cm³

$\Rightarrow (\text{side})^3 = 27$

side $= \sqrt[3]{27}$

$\therefore$ side = 3 cm

Length of the resulting cuboid

$= 3 + 3 = 6$ cm

its breadth, $b = 3$ cm and

its height, $h = 3$ cm

Now, Surface area of the resulting cuboid $= 2(lb + bh + hl)$

$= 2(6 \times 3 + 3 \times 3 + 3 \times 6)$

$= 2(18 + 9 + 18) = 2 \times 45 =$ **90 cm²**

2. Given: $h = r$

Vol. of right circular cylinder $= 25\frac{1}{7}$ cm³

We know that volume of the right circular cylinder $= \pi r^2 h$

$\Rightarrow \pi r^3 = 25\frac{1}{7}$ $\qquad [\because r = h$

$\Rightarrow r^3 = \frac{176}{7} \times \frac{7}{22}$ $\qquad \Rightarrow r = 2$

$\therefore$ $h = r = 2$ **cm**

3. We have, Surface area of Hemisphere

= Surface Area of Cone

Let the radius of hemisphere $= r$

and slant length of Cone $= l$

But according to given question, $2\pi r^2 = \pi r l$

$2r = l$

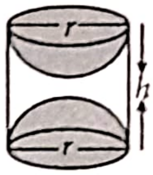
We know that in cone,

$$h = \sqrt{(l^2 - r^2)} = \sqrt{(4r^2 - r^2)}$$

$$\Rightarrow h = \sqrt{3}r \qquad \Rightarrow \frac{h}{r} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow h : r = \sqrt{3} : 1$$

4. Curved surface area of cylinder = $2\pi rh$



$r = 7$ cm, $h = 10$ cm

$$2 \times \frac{22}{7} \times 7 \times 10 = 440 \text{ cm}^2$$

Surface area of hemisphere = $2\pi r^2$

$$\Rightarrow 2 \times \frac{22}{7} \times 7 \times 7 = 308 \text{ cm}^2$$

Surface area of 2 hemisphere

$= 2 \times 308 \text{ cm}^2 = 616 \text{ cm}^2$

Total surface area = *Curved surface area of cylinder + Surface area of hemispheres*

$= 440 + 616 = $ **1056 cm²**

5. **Given.** Radius, $r = \frac{D}{2} = \frac{40}{2} = 20$ cm;

Height of cylinder, H = 21 cm

Height of cone, h = 15 cm

Slant height of cone, $l = \sqrt{(h^2 + r^2)}$

$$l = \sqrt{[(15)^2 + (20)^2]}$$

$$l = \sqrt{225 + 400} = \sqrt{625} = 25 \text{ cm}$$

Total surface area of toy = *Curved surface area of cone + Curved Surface area of cylinder + Area of bottom part*

$= \pi rl + 2\pi rH + \pi r^2$

$= 3.14[20 \times 25 + 2 \times 20 \times 21 + 20 \times 20]$

$= 3.14[1740] = $ **5463.6 cm²**

6. The ratio of heights of cones is 1 : 3.

The ratio of radius of cones is 3 : 1.

We know that, volume of the cone,

$$V = \frac{1}{3}\pi r^2 h$$

Assume the heights to be h_1 and h_2, and radius to be R and r

$$\frac{V_1}{V_2} = \frac{\left(\frac{1}{3}\pi R^2 h_1\right)}{\left(\frac{1}{3}\pi r^2 h_2\right)} = \frac{R^2 h_1}{r^2 h_2} = \frac{9}{3} = \frac{3}{1}$$

Hence, $V_1 : V_2 = 3 : 1$

7. **Given.** radius of cylinder

= radius of cone = r

Let the height of cylinder = h

then height of cone = $3h$

Volume of cone, $V_1 = \frac{1}{3}\pi r^2 h_1$

Volume of cylinder, $V_2 = \pi r^2 h_2$

Now, ratio of volume of cone to the

$$\text{Volume of cylinder, } \frac{V_1}{V_2} = \frac{\left(\frac{1}{3}\pi r^2 3h\right)}{\pi r^2 h}$$

$$= \frac{\left(\frac{1}{3}\pi r^2 3h\right)}{\pi r^2 h}$$

$$\frac{V_1}{V_2} = \frac{1}{1}$$

$\Rightarrow V_1 : V_2 = 1 : 1$

Hence , the required ratio is 1 : 1

Short Answers

1. Let the rise of water level in the pond be h meter, when 500 persons are taking a dip into a cuboidal pond.

Given that,

Length of the cuboidal pond = 80 m

Breadth of the cuboidal pond = 50 m

Now, volume for the rise of water level in the pond

= Length × Breadth × Height

$= 80 \times 50 \times h = 4000\,h$ m³

And the average displacement of the water by one person = 0.04 m³

So, the average displacement of the water by 500 persons = 500 × 0.04 m³

Now, by given condition,

Volume for the rise of water level in the pond = Average displacement of the water by 500 persons

$\Rightarrow 4000h = 500 \times 0.04$

$$\therefore h = \frac{(500 \times 0.04)}{4000} = \frac{20}{4000} = \frac{1}{200} \text{ m}$$

= 0.005m

= 0.005 × 100 cm = **0.5 cm**

($\because$ 1m = 100 cm)

Hence, the required rise of water level in the pond is 0.5 cm.

2. Given. Width of the canal = 5.4 m
Depth of the canal = 1.8 m
Speed of the flowing water = 25 km/h

$= \frac{25000}{60} = \frac{1250}{3} = \text{m/min}$

Volume of water flowing out of the canal in 1 min

$= \text{Area of opening of canal} \times \frac{1250}{3}$

$= 5.4 \times 1.8 \times \frac{1250}{3} = 4050 \text{ m}^3$

Volume of the water flowing in 40 min
$= 4050 \times 40 = 162000 \text{ m}^3$
Height of the standing water = 10 cm
= 0.10 m

$\therefore$ *Area to be irrigated* $= \frac{162000}{0.10}$

$= \mathbf{1620000 \text{ m}^2}$

3. Diameter of cone = 14 cm

$\therefore$ Radius of cone $= \frac{14}{2} = 7$ cm

Height of cone = 5 cm
Volume of cylinder $= \pi r^2 h$

$= \pi \times (7)^2 \times 15$

$= 735\pi \text{m}^3$

Volume of 2 cones

$= 2 \times \left(\frac{1}{3}\pi r^2 h\right)$

$= 2 \times \left(\frac{1}{3} \times \pi \times 49 \times 5\right)$

$= \frac{490\pi}{3} \text{ m}^3$

$\therefore$ Total volume of silo $= \frac{735\pi}{1} + \frac{490\pi}{3}$

$= \frac{2695\pi}{3} \text{ m}^3$

4. We know,
The radius of the cylinder equals to the radius of the hemisphere.

Radius of the cylinder $= \frac{7}{2} = 3.5$ cm

The total height of the solid = Height of the cylinder - two radius of the hemispheres

Height of the cylinder = 20 - (3.5 × 2)
= 13 cm

Volume of a cylinder $= \pi \times r^2 \times h$

$= \frac{22}{7} \times 3.5^2 \times 13 = 500.5 \text{ cm}^3$

We know that the two hemispheres at the ends of the cylinder form a sphere.

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{22}{7} \times 3.5^3 \times \frac{4}{3} = 179.67 \text{ cm}^3$

$\therefore$ **Total volume of the solid**
$= 179.67 + 500.5 = \mathbf{680.17 \text{ cm}^3}$

5. **Actual capacity of Glass**
= *Vol. of cylinder – Vol. of hemisphere*
= *Apparent Capacity – Vol. of Hemisphere*

$= \pi r^2 h - \frac{2}{3}\pi r^3$

$= 3.14 \times \frac{5}{2} \times \frac{5}{2} \times 10 - \frac{2}{3}$

$\times 3.14 \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$

$= 196.25 - 32.71 = \mathbf{163.54 \text{ cm}^3}$

6. Increase in the height of the cylinder due to the sphere, $h = 3\frac{5}{9} = \frac{32}{9}$

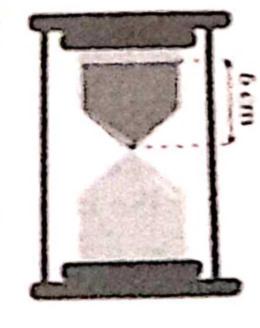
Diameter of sphere = 12 cm
Radius of the sphere, R = 6 cm
$\therefore$ Rise in the volume of water in cylinder = Volume of sphere

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi R^3$

$\Rightarrow r = \sqrt{\frac{4 \times 6 \times 6 \times 6 \times 9}{3 \times 32}} \quad \therefore r = 9 \text{ cm}$

Hence, Diameter of the cylindrical vessel = 2 × 9 = **18 cm**

7. Radius of base = r cm
Height of sand in cylinderical bottom after 5 minutes
$= h_1$ cm
Height of sand in cylinderical top portion = $(6 - h)$ cm
Volume of sand



$= \pi r^2 h_1 = \frac{1}{3}\pi r^2 h + \pi r^2 (6 - h)$

$\pi r^2 h_1 = \pi r^2 \left[\frac{1}{3}h + 6 - h\right]$

$\Rightarrow h_1 = \frac{1}{3}h - \frac{h}{1} + \frac{6}{1} \quad \therefore h_1 = \left(6 - \frac{2}{3}h\right) \text{cm}$

8. We have radius of conical vessel,

$$r = \frac{40}{2} = 20 \text{ cm}$$

and height, $h = 24$ cm

Volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(20)^2 \times 24$$

$$= \frac{1}{3} \times \pi \times 400 \times 24 = \mathbf{3200\pi \text{ cm}^3}$$

Now, let us find the volume of water that flows through the cylindrical pipe in 1 minute.

We get, The volume of water that flows through a cylindrical pipe

$$= \pi r^2 \times \text{water flowing in 1 minute}$$

Substituting the value of $r = \frac{5}{2}$ mm

$$= 2.5 \text{ mm} = \frac{2.5}{10} \text{ cm}$$

Water flowing in 1 min = 10 m = 1000 cm

The volume of water that flows through the cylindrical pipe in 1 minute

$$= \pi\left(\frac{2.5}{10}\right)^2 \times 1000 \text{ cm}^3/\text{min} = \mathbf{62.5\pi \text{ cm}^3}$$

So, we get the total time taken to fill the

Conical vessel $= \dfrac{\textit{Volume of conical vessel}}{\textit{Volume of water that flows through the pipe in 1 minute}}$

$$= \frac{3200\pi}{62.5\pi} = \mathbf{51.2 \text{ minutes.}}$$

Hence, we get the total time taken to fill the conical vessel as 51.2 minutes.

Long Answers

1. Height of the cylinder, $h = 10$ cm

Radius of the cylinder = radius of the hemisphere, $r = 3.5$ cm

TSA of the article = 2 × CSA of the hemispherical part + CSA of the cylindrical part

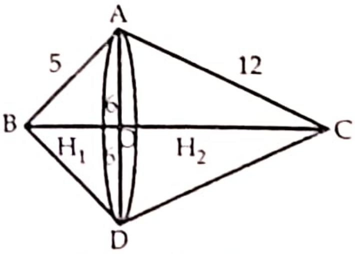
$= 2 \times 2\pi r^2 + 2\pi rh = 2\pi r(2r + h)$

$= 2 \times \frac{22}{7} \times 3.5 \times (2 \times 3.5 + 10)$

$= 22 \text{ cm} \times 17 \text{ cm} = \mathbf{374 \text{ cm}^2}$

Hence the total surface area of the article is 374 cm².

2.



Here, a right triangle is revolved about its hypotenuse and thus the above diagram of double cone is formed.

Now in ΔABC,

Let, AB = 5 cm, AC = 12 cm & BC = 13 cm. Also, let $OB = h_1$, $OC = h_2$ and $OA = OD = r$

Thus, OB + OC = 13

$$h_1 + h_2 = 13 \quad \text{...(1)}$$

Also, in ΔABO, $AB^2 = AO^2 + OB^2$

$$\Rightarrow (5)^2 = h_1^2 + r^2 \quad \text{...(2)}$$

And, in ΔAOC, $AC^2 = AO^2 + OC^2$

$$\Rightarrow 12^2 = h_2^2 + r \quad \text{...(3)}$$

Now, subtracting equation (2) from equation (3), we get

$$\Rightarrow 12^2 - 5^2 = h_2^2 + r^2 - h_1^2 - r^2$$

$$\Rightarrow 144 - 25 = h_2^2 - h_1^2$$

$$\Rightarrow 119 = (h_2 - h_1)(h_2 + h_1)$$

Substituting Eq. (1) in above equation

$$\Rightarrow 119 = (h_2 - h_1)(13)$$

$$\Rightarrow h_2 - h_1 = \frac{119}{13} \quad \text{...(4)}$$

Adding equation (4) and equation (1) we get

$$h_2 - h_1 + h_2 + h_1 = \frac{119}{13} + 13$$

$$\Rightarrow 2h_2 = \frac{119 + 169}{13}$$

$$\Rightarrow 2h_2 = \frac{288}{13} \qquad \Rightarrow h_2 = \frac{144}{13}$$

Putting $h_2 = \frac{144}{13}$ in equation (4) we get

$$\frac{144}{13} - h_1 = \frac{119}{13}$$

$$\Rightarrow \frac{144}{13} - \frac{119}{13} = h_1 \qquad \Rightarrow h_1 = \frac{25}{13}$$

Thus, we get $h_1 = \frac{25}{13}$ and $h_2 = \frac{144}{13}$

Now, substituting the value $h_1 = \frac{25}{13}$ in equation (2), we get

$$5^2 = \left(\frac{25}{13}\right)^2 + r^2$$

$$\Rightarrow r^2 = 25 - \frac{625}{169}$$

$$\Rightarrow r^2 = \frac{4226 - 625}{169}$$

$$\Rightarrow r^2 = \frac{3600}{169}$$

$$\Rightarrow r = \sqrt{\left(\frac{3600}{169}\right)}$$

$$\Rightarrow r = \frac{60}{13}$$

Thus, $r = \frac{60}{13}$

Now, T.S.A. of the double cone can be given by the sum of C.S.A. of both cones.

$\Rightarrow$ T.S.A. of the double cone

= Sum of C.S.A. of both cones

$$= (\pi \times r \times AB) + (\pi \times r \times AC)$$

$$= \left(\pi \times \frac{60}{13} \times 5\right) + \left(\pi \times \frac{60}{13} \times 12\right)$$

$$= \pi \times \frac{60}{13}(5 + 12)$$

$$= \frac{22}{7} \times \frac{60}{13} \times 17 = \mathbf{246.59\ cm^2}$$

Then, volume of double cone is given by the sum of volumes of both cones.

$\therefore$ Volume of double cone = Sum of volumes of both cones

$$= \left(\frac{1}{3} \times \pi \times r^2 \times h_1\right) + \left(\frac{1}{3} \times \pi \times r^2 \times h_2\right)$$

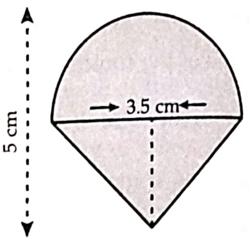
$$= \left[\frac{1}{3} \times \pi \times \left(\frac{60}{13}\right)^2 \times \frac{25}{13}\right] + \left[\frac{1}{3} \times \pi \times \left(\frac{60}{13}\right)^2 \times \frac{144}{13}\right]$$

$$= \frac{1}{3} \times \pi \times \left(\frac{60}{13}\right)^2 \left(\frac{25}{13} + \frac{144}{13}\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{3600}{169} \times \frac{16}{13} = 290\ cm^3$$

Thus, we get the value of T.S.A. of the double cone as 246.59 cm² and the volume of double cone as 290 cm³.

3. Radius of hemispherical portion of the lattu, $r = \frac{3.5}{2} = \frac{7}{4}$ cm



Radius of the conical portion $r = \frac{3.5}{2} = \frac{7}{4}$

Height of the conical portion,

$h = \left(\frac{5-3.5}{20} = \frac{13}{4}\right)$

Slant height of the conical part,

$l = \sqrt{(r^2+h^2)}$

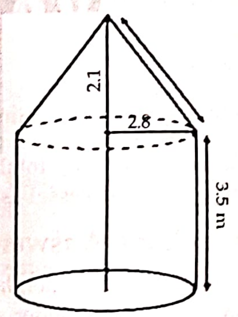
$= \sqrt{\left(\frac{7}{4}\right)^2 + \left(\frac{13}{4}\right)^2} = \sqrt{\left(\frac{218}{16}\right)} = 3.69$ cm

Total surface area of the top $= 2\pi r^2 + \pi rl$

$= \pi r(2r + l) = \frac{22}{7} \times \frac{7}{4} \times \left(2 \times \frac{7}{4} + 3.7\right)$

$= \frac{22}{4} \times 7.2 = 39.6$ **cm²**

4.



Radius of the base of cylinder (r) = 2.8 m
Radius of the base of the cone (r) = 2.8 m
Height of the cylinder (h) = 3.5 m
Height of the cone (H) = 2.1 m.
Slant height of conical part (l)

$= \sqrt{(r^2 + H^2)}$

$= \sqrt{(2.8^2 + 2.1^2)} = \sqrt{(7.84 + 4.41)}$

$= \sqrt{(12.25)} = 3.5$

Area of canvas used to make tent = CSA of cylinder + CSA of cone

$= 2 \times \pi \times 2.8 \times 3.5 + \pi \times 2.8 \times 3.5$

$= 61.6 + 30.8 = 92.4$ m²

Cost of 1500 tents at ₹120 per sq.m

$= 1500 \times 120 \times 92.4 =$ ₹16,632,000

$\therefore$ **Share of each school** $= \frac{16,632,000}{50}$

= ₹3,32,640

5. Canal is in the shape of cuboid where,
Breadth = 6 m; Height = 1.5 m
Speed of canal = 10 km/hr
Length of canal in 1 hour = 10 km
or, Length of canal in 60 minutes = 10 km

or, Length of canal in 1 min $= \frac{1}{60} \times 10$ km

Length of canal in 30 min $= \frac{30}{60} \times 10$ km

= 5 km

= 5000 m

Now, Volume of the canal
= Length × Breadth × Height
= 5000 × 6 × 1.5

$\therefore$ Volume of water in canal $= \frac{\textit{Area irrigated}}{\textit{Height}}$

$\Rightarrow 5000 \times 6 \times 1.5 =$ Area irrigated $\times \frac{8}{100}$

$\Rightarrow$ Area irrigated $= \frac{(5000 \times 6 \times 1.5 \times 100)}{8}$

= 562500 m²

Now, **Area irrigated** $= \frac{562500}{10000}$ hectares

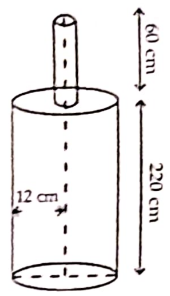
= 56.25 hectares

6. Diameter = 8 cm, Radius = 4 cm
Length of flow of water in 1 sec = 80 cm
In 1 hour = 80 × 60 × 60 = 288000 cm
Volume of cylindrical pipe in 1 hr. $= \pi r^2 h$

$= \frac{22}{7} \times 4 \times 4 \times 288000 = \frac{101376000}{7}$

$= \frac{101376000}{7} \times \frac{1}{1000} = \frac{101376}{7}\ l$

7. **Given.** Radius of larger cylinder,



$R = \frac{24 \text{ cm}}{2} = 12$ cm

Height of larger cylinder, H = 220 cm
Radius of smaller cylinder, r = 8 cm
Height of smaller cylinder, h = 60 cm

Volume of the solid iron pole = *Volume of larger cylinder + Volume of smaller cylinder*

$= \pi R^2 H + \pi r^2 h$

$= \pi$ (12 cm × 12 cm × 220 cm + 8 cm × 8 cm × 60 cm)

= 3.14 × (31680 cm³ + 3840 cm³)

= 3.14 × 35520 cm³ = 111532.8 cm³

Mass of 1 cm³ iron = 8g

Mass of iron in the pole

= 8g × volume of the solid iron pole

= 8 g × 111532.8 = 892262.4 g

$= \frac{892262.4}{1000}$ kg = 892.2624 kg

Thus, the mass of iron in the pole is 892.26 kg.

8. We have Edge of cube = a = 6 cm
Diameter of hemisphere= 4.2 cm
radius = 2.1 cm

(*a*) **Surface area of decorative block** = *Total surface area of the cube + curved surface area of hemisphere – base area of hemisphere*

$= 6a^2 + 2\pi r^2 - \pi r^2 = 6a^2 + \pi r^2$

$= \left[6 \times 6 \times 6 + \frac{22}{7} \times 2.1 \times 2.1\right]$ cm²

= (216 + 13.86) cm² = **229.86 cm²**

(*b*) Volume of the block = *Volume of the cube + volume of the hemisphere*

$= a^3 + \left(\frac{2}{3}\right)\pi r^3$

$= 6^3 + \frac{2}{3} \times \frac{22}{7} \times (2.1)^3$

= (216 + 19.404) cm³ = **235.404 cm³**

Case Based Answers

1. (a) Given, Edge of cube = 6 cm
We know that, Diagonal of cube
$= \sqrt{3} \times$ edge of cube $= 6\sqrt{3}$ cm

(b) (i) Given, Side of cube = 7 cm
We know that,
Volume of cube $= a^3$
$= 7 \times 7 \times 7 = $ **343 cm³**

Or, (ii) Given, Radius (r) = 7 cm
Curved surface area of hemisphere
$= 2\pi r^2$
$= 2 \times \frac{22}{7} \times 7 \times 7 = 44 \times 7$
$=$ **308 cm²**

(c) Given, Radius (r) = 7 cm
height (h) = 24 cm
Here, $l^2 = r^2 + h^2$
So, $l^2 = 7^2 + 24^2 = 49 + 576 = \sqrt{625}$
$\therefore$ **l = 25 cm**

2. (a) We know that the formula of the volume of a cylinder, $V = \pi r^2 h$
$= \frac{22}{7} \times \frac{49}{4} \times 10.5 = $ **404.25 cm³**

(b) (i) The volume of a hemisphere with radius, $R = \left(\frac{2}{3}\right)\pi R^3$
$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3 = 11 \times \frac{49}{6} = $ **89.83 cm³**

Or

(ii) Vol. of cylindrical cup = 404.25 cm³
& Vol. of hemispherical cup
= 89.83 cm³
So, volume of cylindrical cup has more soup = 404.25 – 89.83
= **314.42 cm³**

(c) Given that, Area of the canvas = 551 m²
Area left after wastage = 551 – 1 = 550 m²
So, the area of the conical tent $= \pi r l$
$\Rightarrow 550 = \frac{22}{7} \times 7 \times l$
$\therefore l = 25$
Now, $h = \sqrt{(l^2 - r^2)} = \sqrt{(625 - 49)}$
$= \sqrt{576} = $ **24 m**

3. (a) We know the formula of volume of sphere i.e. $= \frac{4}{3}\pi r^3$

(b) (i) Radius of the hemisphere = 21 m
So, the required volume $= \frac{2}{3}\pi R^3$
$= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = $ **19404 m³**

Or

(ii) Cloth required to cover the hemispherical dome $= 2\pi R^2$
$= 2\left(\frac{22}{7}\right)14^2 = $ **1232 sq.m**

(c) The total surface area of the combined figure i.e hemispherical dome with radius 14 m and cuboid shaped top with dimensions 8 m × 6 m × 4 m
= 1232 + 2 × 8 (6 + 4) = **1392 sq.m**

(DAY 3 SWAHA)

* * * * *

2 Statistics

What did CBSE ask last year?

MCQs & A/R	2 Questions ($1 \times 2 = 2$ Marks)
Subjective	No Very Short Question Asked
	No Short Question Asked
	1 Long Question ($1 \times 5 = 5$ Marks)
Case Based	No Case Base Question Asked

Note: *All the above typology of questions include 'Competency based Questions' labelled as* COMPETENCY.

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Mean of Grouped Data

- Direct Method = $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

- Assumed Mean = $\bar{x} = a + \dfrac{\sum f_i d_i}{\sum f_i}$ (Where 'a' is assumed mean)

- Step Deviation Method = $\bar{X} = A + \left[\dfrac{\sum f_i u_i}{\sum f_i}\right] \times h$

...[Where $u_i = \dfrac{x-a}{h}$, and h is a common divisor

Mode of grouped Data

- $l + \left[\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Where,

- l = lower limit of the modal class
- h = size of class internal
- f_1 = Frequency of modal class
- f_0 = Frequency preceding the modal class frequency
- f_2 = Frequency succeeding the modal class frequency

- **Mode of Ungrouped Data:** The value of the observation having maximum frequency is the mode.

- **Relationship between Mean, Median and Mode**

1. Mode = 3 Median − 2 Mean

2. Median = $\dfrac{Mode + 2Mean}{3}$

3. Mean = $\dfrac{3Median - Mode}{2}$

Remember!

Class with maximum frequency is the modal class.

Median of Grouped Data

- Representing a cumulative frequency distribution graphically as a cumulative frequency curve, or an ogive of the less than type and of the more than type.

- The median of grouped data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives for this data.

- Median $= l + \left[\dfrac{\frac{n}{2} - cf}{f}\right]h$

  Where,
  - l = Lower limit of the median class
  - n = total frequency
  - c = Cumulative frequency of class before the median class
  - f = Frequency of the median class
  - h = Class width (Upper limit – Lower limit)

- **Median (Ungrouped Data)**

If n is odd $\rightarrow$ Median $= \left(\dfrac{n+1}{2}\right)^{th}$ observation

If n is even $\rightarrow$ Median $= \dfrac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$

Remember!
For ungrouped data, first arrange the values of observation in ascending order or descending order.

OBJECTIVE QUESTIONS

(DAY 4)

Multiple Choice Questions

Q.1. The median and mode respectively of a frequency distribution are 26 and 29, Then its mean is [CBSE 2020

(a) 27.5 (b) 24.5

(c) 28.4 (d) 25.8

Q.2. While computing mean of grouped data, we assume that the frequencies are COMPETENCY

(a) evenly distributed over all the classes.

(b) centered at the class marks of the classes.

(c) centered at the upper limits of the classes.

(d) centered at the lower limits of the classes.

Q.3. The mean of 5 numbers is 27. If one number is excluded, their mean is 25. The excluded number is COMPETENCY

(a) 30 (b) 35

(c) 32 (d) 36

FREE ADVICE: We can set up a system of equations with these two Mean results.

Q.4. The table below shows the time taken by a group of students to complete 100 m race. COMPETENCY

Time taken (in sec)	*Number of students*
18 -20	3
20 - 22	18
22 - 24	26
24 - 26	19
26 - 28	9
28 - 30	5

Which of these is the mean time taken, in sec, by the group of students to complete the 100 m race when calculated using direct method?

(a) 18.16 (b) 18.96

(c) 22.7 (d) 23.7

Q.5. The table below shows the results of a survey conducted on 40 gamers on how many games did they play on a particular day. Which of the following is the model class? COMPETENCY

Number of games	*Number of gamers*
1 - 2	10
2 - 3	12
3 - 4	5
4 - 5	6
5 - 6	4
6 - 7	2
7 - 8	1

(a) 1 - 2 (b) 2 - 3

(c) 4 - 5 (d) 7 - 8

Q.6. In the formula $x = a + h(f_iu_i / f_i)$, for finding the mean of grouped frequency distribution, u_i = [NCERT EXEMPLAR

(a) $(x_i + a)/h$ (b) $h(x_i - a)$

(c) $(x_i - a)/h$ (d) $(a - x_i)/h$

Q.7. If the mean of first n natural number is 15, then find n. [CBSE 202

(a) 28 (b) 27 (c) 29 (d) 30

Q.8. What is the arithmetic mean of first n natural numbers? [CBSE 202

(a) $\frac{(n+1)}{2}$ (b) $\frac{n}{2}$

(c) n^2 (d) $\frac{(n-1)}{2}$

Q.9. Shreya collects the following data on the number of movies watched by her friends in the month of June.

Names	*No. of Movies watched*
Shailja	3
Nikita	8
Arima	9
Meena	4
dune	1

What is the average number of movies watched by Shreya's friends in that particular month? [CBSE 2024

(a) 4.16 (b) 4.20 (c) 5 (d) 9

Q.10. The approximate relationship between the mean, mode and median can be expressed using an empirical formula. Shown below are the measures of central tendency of the marks obtained by Class 8 students in a test.

Mean: 5 marks

Mode: 5.3 marks

Which of the following could be the approximate median of the marks? **COMPETENCY**

(*a*) 5.10 marks (*b*) 5.15 marks

(*c*) 5.20 marks (*d*) 10.15 marks

Q.11. Using the empirical formula, find the mode of a distribution whose mean is 8.32 and the median is 8.05. [CBSE 2020

(*a*) 31 (*b*) 7.51

(*c*) 7.22 (*d*) 6.51

Q.12. For finding the popular size of ready made garments, which central tendency is used? **COMPETENCY**

(*a*) Mean (*b*) Median

(*c*) Mode (*d*) Both Mean and Mode

Q.13. A survey was conducted on 80 gamers on how many games did they play in a day. The data is given below.

No. of games	*No. of gamers*
1 - 2	20
2 - 3	24
3 - 4	10
4 - 5	12
5 - 6	8
6 - 7	4
7 - 8	2

Which one of the following is the modal class? **COMPETENCY**

(*a*) 1 - 2 (*b*) 2 - 3

(*c*) 4 - 5 (*d*) 7 - 8

Q.14. Which one of the following is the value of the observation having the maximum frequency. **COMPETENCY**

(*a*) Mean

(*b*) median

(*c*) mode

(*d*) both (*a*) and (*b*)

Q.15. From the following frequency distribution, find the median class. [CBSE 2017

Cost of living index	1400-1550	1550-1700	1700-1850	1850-2000
No. of weeks	8	15	21	8

(*a*) 1400-1550 (*b*) 1550-1700

(*c*) 1700-1850 (*d*) 1850-2000

Q.16. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The sum of lower limits of median class and modal class is: [CBSE 2023

(*a*) 15 (*b*) 25 (*c*) 30 (*d*) 35

Q.17. The mean of the first four data points in a data set is 10, while the mean of the remaining sixteen data points is 20. What is the mean of the entire dataset? **COMPETENCY**

(*a*) 1.5 (*b*) 12 (*c*) 15 (*d*) 18

Q.18. Consider the following frequency distribution of the heights of 60 students of a class.

Height (in cm)	*Number of students*
150-155	15
155-160	13
160-165	10
165-170	8
170-175	9
175-180	5

The upper limit of the median class in the given data is **COMPETENCY**

(*a*) 165 (*b*) 155 (*c*) 160 (*d*) 170

Q.19. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below. **COMPETENCY**

Class	*Frequency*
13.8 – 14	2
14 – 14.2	4
14.2 – 14.4	5
14.4 – 14.6	71
14.6 – 14.8	48
14.8 – 15	20

The number of athletes who completed the race in less than 14.6 seconds is:

(*a*) 11 (*b*) 71
(*c*) 82 (*d*) 130

Q.20. **In statistics, an outlier is a data point that differs significantly from other observations of a data set. If an outlier is included in the following data set, which measure(s) of central tendency would change?** *[CBSE 2024*

12, 15, 22, 44, 44, 48, 50, 51

(*a*) only mean
(*b*) only mean and median
(*c*) all mean, median, mode
(*d*) cannot be said without knowing the outlier

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*
(*b*) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*
(*c*) *(A) is true and (R) is false.*
(*d*) *(A) is false, but (R) is true.*

Q.1. **Assertion:** Class width = Upper class limit – Lower class limit

Reason: Class mark = $\dfrac{\textit{Upper Class Limit} + \textit{Lower Class Limit}}{2}$

Q.2. **Assertion:** The runs scored by a batsman in 5 ODIs are 31,97, 112, 63, and 12. The standard deviation is 25.79.

Reason:

Mean = $\dfrac{\textit{total sum of number in data sets}}{\textit{total number in data sets}}$

Q.3. **Assertion:** The arithmetic mean of the following frequency distribution table is 13.81. COMPETENCY

x	4	7	10	13	16	19
f	7	10	15	20	25	30

Reason: x = value of $(n + 1)$th/2 observation.

Q.4. **The table below shows the marks obtained by students of Sections A and B of grade 10. The results are recorded in groups as follows:** COMPETENCY

Marks	*Number of students*	
	Section A	*Section B*
20 - 29	1	2
30 - 39	1	1
40 - 49	10	9
50 - 59	11	11
60 - 69	5	4
70 - 79	2	3

Assertion (A): The median mark of Section A is equal to the median mark of Section B.

Reason (R): The cumulative frequency of the median class and the preceding class are the same for both the sections.

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. If the mean of the first n natural number is 15, then find n. *[CBSE 2020*

Q.2. Find the class-marks of the classes 10-25 and 35-55. *[CBSE 2020*

Q.3. The mean and median of 100 observations are 50 and 52. The value of the largest observation is 100. It was later found that it is 110 and not 100. Find true mean and median. *[CBSE 2017*

Q.4. A cooking oil manufacturing company sells oil in three different bottle sizes. Now, it wants to sell only one size in the market. It has data on how the three sizes perform in the market.
Based on which measure of central tendency should the company fix the size of the oil bottle? Justify your answer. COMPETENCY

Q.5. Find the mode of the following frequency distribution. *[CBSE 2019*

Class Interval	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Q.6. Find the value of λ if the mode of the following data is 20 :

15, 20, 25, 18, 13, 15, 25, 15, 18, 17, 20, 25, 20, λ, 18 **COMPETENCY**

Q.7. Following is the data on the number of maths question attempted by a student in a week. **COMPETENCY**

10, 15, 25, 10, 25, 15, 25

What is the mode of given data?

Q.8. The numbers of runs scored by a player in 11 cricket matches are: 5, 19, 42, 11, 50, 30, 21, 0, 52, 36 and 27. Find the median. **COMPETENCY**

(DAY 5)

Short Answer Questions

Q.1. The mean of the following frequency distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f. *[CBSE 2020*

Class Interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Q.2. The arithmetic mean of the following frequency distribution is 53. Find value of k. *[CBSE 2019*

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	k	13

Q.3. The mean of the following distribution is 31.4. Determine the missing frequency x. *[CBSE 2016*

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	10	12	7	8

FREE ADVICE: *Get the frequency of the class by subtracting it from the proceeding one.*

Q.4. Following frequency distribution shows the daily expenditure of milk of 30 households in a locality. *[NCERT*

Daily Expenditure on milk (₹)	0-30	30-60	60-90	90-120	120-150
No. of households	12	15	32	k	13

Find the mode of the above data.

FREE ADVICE: *f_0, f_1, f_2 ki value formula put karte time dhyaan rakhna.*

Q.5. **If the median of the following frequency distribution is 32.5. Find values of f_1 and f_2.** **COMPETENCY**

Class	Frequency
0-10	f_1
10-20	5
20-30	9
30-40	12
40-50	f_2
50-60	3
60-70	2
Total	**40**

Q.6. Find the mode of the following frequency distribution. *[CBSE 2020*

Class	Frequency
0-20	6
20-40	8
40-60	10
60-80	12
80-100	6
100-120	5
120-140	3

Q.7. The mean temperature of a certain city for 30 consecutive days was found to be 34° C.

Further, the mean temperature of the first 10 days was 30° C. The mean temperature of the next 10 days was 35° C.

Find the mean temperature of the rest of the days. Show your work.

Q.8. The maximum bowling speeds, in km per hour, of 33 players at a cricket coaching centre are given as follows.

Speed (in km/h)	85-100	100-115	115-130	130-145
No. of Players	11	9	8	5

Calculate the median bowling speed.

COMPETENCY

Q.9. The frequency distribution of daily rainfall in a town during a certain period is shown below.

Rainfal (in mm)	Number of Days
0 – 10	2
10 – 20	6
20 – 30	x
30 – 40	7
40 – 50	4

Unfortunately, due to manual errors, the information in the 20-30 mm range got deleted from the data.

If the mean daily rainfall for the period was 27 mm, find the number of days when the rainfall ranged between 20-30 mm. Show your work. **COMPETENCY**

Long Answer Questions

Q.1. A sports teacher records the given data about the heights (in cm) of all the students of classes 6, 7, and 8.

Height (in cm)	Class		
	6	7	8
120 - 130	15	13	10
130 - 140	13	15	12
140 - 150	12	18	16
150 - 160	10	5	8
160 - 170	7	8	10
170 - 180	3	2	3

Find the mean height of all the students in all three classes together, using any suitable method. Show your work and round your answer upto to two decimal places. *[CBSE 2024*

Q.2. In the class test, marks obtained by 120 students are given in the following frequency distribution. If it is given that mean is 59, find the missing frequencies x and y. **COMPETENCY**

Marks	No. of Students
0-10	1
10-20	3
20-30	7
30-40	10
40-50	15
50-60	x
60-70	9
70-80	27
80-90	18
90-100	y

Q.3. A medical camp was held in a school to impart health education and the importance of exercise to children. During this camp, a medical check-up of 35 students was done and their weights were recorded as follows:

Weight in kg.	No. of Students
Below 40	3
Below 42	5
Below 44	9
Below 46	14
Below 48	28
Below 50	31
Below 52	35

Compute the modal weight.

[CBSE 2016

Q.4. The following data was collected on the number of potted plants in each of the 20 houses in a locality.

Number of Plants	Number of Houses
0 – 2	4
2 – 4	3
4 – 6	2
6 – 8	5
8 – 10	6

Ram and Deepak calculate the mean number of potted plants in the locality using assumed mean as 5 plants and 6 plants, respectively.

Will their results be same or different? Show your work and justify your answer. **COMPETENCY**

CASE BASED QUESTIONS

Q.1. The NRSJPS Sangathan or 'Private School Organisation' oversees the functioning of the schools with its headquarters in Prayagraj. The chairman of NRSJPS Sangathan is providing a common programme of education.

Chairman of Regional office Prayagraj prepare a table of the marks obtained of 100 students which is given below.

Marks	0-20	20-40	40-60	60-80	80-100
No. of Students	15	18	21	29	p

He was told that mean marks of a student is 53.

(*a*) What is the lower limit of model class?

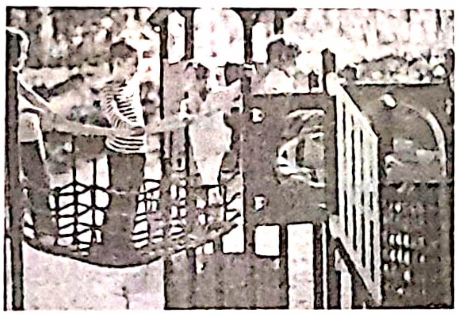
(*b*) (*i*) How many students got marks between 80-100?

***Or,* (*ii*) What is the value of the modal marks?** COMPETENCY

(*c*) What is the value of median marks?

Q.2. Anurag decided to make a playground in various colony parks. He decided to study the age-group of children playing in a park of the particular colony. The classification of children according to their ages, playing in a park is shown in the following table.

Age group (in years)	6 - 8	8 - 10	10 - 12	12 -14	14 - 16
Number of children	43	58	70	42	27



Based on the above information, answer the following questions.

(*a*) Which age group has the maximum number of children?

(*b*) (*i*) Find the lower limit of the modal class.

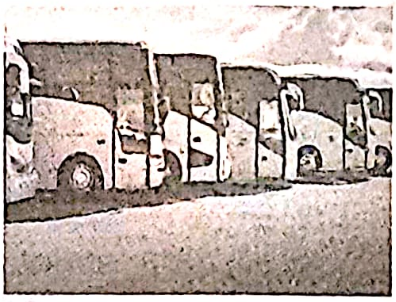
***Or,* (*ii*) Find frequency of the class succeeding the modal class.**

(*c*) Find the mode of the ages of children playing in the park.

COMPETENCY

Q.3. Transport department of a city wants to buy some Electric buses for the city, for which they want to analyse the distance travelled by existing public transport buses in a day.

Daily distance travelled (in km)	200-209	210-219	220-229	230-239	240-249
Number of buses	4	14	26	10	6



The following data shows the distance travelled by 60 existing public transport buses in a day.

(*a*) The upper limit of a class and lower limit of its succeeding class differ by ________ .

(*b*) (*i*) The cumulative frequency of the class preceding the median class is ________ .

***Or,* (*ii*) The median of the distance travelled is ________ .**

(*c*) If the mode of the distance travelled is 223.78 km, then mean of the distance travelled by the bus is ________ .

*****

ANSW

Multiple Choice Answers

1. (*b*) Median is 26; Mode is 29.

By using the empirical relation of mean, median and mode formula,

Mode = 3 Median – 2 Mean

$\Rightarrow$ Mean $= \frac{1}{2}$ [3Median – Mode]

$= \frac{1}{2}[3(26) - 29]$

$= \frac{1}{2}[49] = 24.5$

Hence, the mean value is 24.5.

2. (*b*) While computing mean of grouped data, we assume that the frequencies are centred at the classmarks of the classes.

Therefore, the frequencies are centred at the classmarks of the classes.

3. (*b*) The mean of 5 numbers = 27

$$\text{Mean} = \frac{\textit{sum of given values}}{\textit{total number of given value}}$$

Hence, sum of numbers

$= 5 \times 27 = 135$

Given one number is excluded.

So, mean of 4 numbers = 25

Sum of 4 numbers = 4 × 25 = 100

Therefore, the excluded number

$= 135 - 100 = 35$

4. (*d*) 23.7

5. (*b*) The highest number of gamers is 12.

$\because$ The model class is **2 – 3**.

6. (*c*) From the question,

$x = a + h\,(f_i u_i / f_i)$

It is the step deviation formula

...*where* $\left[\begin{array}{l} x_i = \text{data values} \\ a = \text{assumed mean} \\ h = \text{class size} \end{array}\right.$

VERS

When the class size is similar, ... calculation of mean can be simplif... by using the coded mean of $u_1, u_2, ...$ .....

Where $u_i = \frac{(x_i - a)}{h}$

7. (c) It is given that mean of first natu... number is 15.

$\therefore \frac{(1+2+3+4+....+n)}{n} = 15$

we know, sum of first 'n' natu... numbers is given by

$\frac{n(n+1)}{2}$

$\therefore \frac{\frac{n(n+1)}{2}}{n} = 15 \Rightarrow \frac{(n+1)}{2} = 15$

$\Rightarrow n + 1 = 30 \quad \therefore n = 29$

8. (a) First n natural numbers are, 1, 2, 3, ... n

Sum of these numbers = 1 + 2 + 3 + ...+ $n = \frac{n(n+1)}{2}$

Hence arithmetic mean

$= \frac{\textit{sum of numbers}}{\textit{Total numbers}}$

$= \frac{n(n+1)}{2} \times \frac{1}{n}$

$\therefore \bar{x} = \frac{(n+1)}{2}$

9. (c) Average number of movies watched

$= \frac{3 + 8 + 9 + 4 + 1}{5} = \frac{25}{5} = 5$

10. (a) 3 median = mode + 2 mean

3 median = 5.3 + 2(5)

$\Rightarrow$ 3 median = 5.3 + 10

$\therefore$ **Median** $= \frac{15.3}{3}$ = **5.10 marks**

11. (b) From empirical formula, we know that;

3Median = Mode + 2Mean

Putting given values in above equation we get:

3 × 8.05 = Mode + 2 × 8.32

$\therefore$ Mode = 24.15 - 16.64 = **7.51**

12. (c) The mode is the most appropriate measure of central tendency to use. This is because it is the value that appears most frequently in the data set and is therefore a good indicator of the popular size.

13. (b) The highest frequency among given data = 24

Hence, modal class of the given data is 2-3.

14. (c) A mode is that value among the observations which occurs most often, that is, the value of the observation having the maximum frequency.

15. (c) Here, $n = 52 \quad \Rightarrow \frac{n}{2} = \frac{52}{2} = 26$

The cumulative frequency greater than or equal to 26 is 44 *or* 26th term lies in the class interval 1700 – 1850.

$\therefore$ **Median class is 1700 – 1850.**

16. (b) Here, N = 66.

$\Rightarrow \frac{N}{2}$ = 33, which lies in the interval 10-15.

So, the lower limit of the median class is 10.

The highest frequency is 20, which lies in the interval 15-20.

Therefore, the lower limit of modal class is 15.

So, the required sum is 10 + 15 = 25.

17. (d) Mean of first four data set = 10

Sum of observations = 10 × 4 = 40

Mean of remaining 16 data set = 20

Sum of remaining 16 observations = 20 × 16 = 320

Entire data set:

Total sum = 40 + 320 = 360

Total data points = 4 + 16 = 20

$\therefore$ **Mean of entire data set** $= \frac{360}{20}$ = **18**

18. (a) We have, N = 60

$\frac{N}{2} = \frac{60}{2} = 30$

Cumulative frequency greater than or equal to 30 lies in class 160–165.

$\therefore$ **The upper limit of the median class is 165.**

19. (c) The number of athletes who completed the race in less than 14.6 seconds is the sum of all frequencies upto the class interval 14.4–14.6

= 2 + 4 + 5 + 71 = **82**

20. (c) all mean, median, mode

Assertion Reason Answers

1. (b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

Explanation: *The class mark, also known as the class midpoint, is calculated by taking the average of the upper class limit and the lower class limit. It represents the central value or mid-point of a class interval.*

2. (b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

3. (c) (A) is true and (R) is false.

4. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Very Short Answers

1. The first n natural numbers are 1, 2, 3, ..., n.

Given that mean of n natural numbers is 15

$\Rightarrow \frac{(1+2+3+...+n)}{n} = 15$ ...(i)

We know that sum of first n natural numbers is

$1 + 2 + 3 + ...+ n = \frac{n(n+1)}{2}$ ...(ii)

Substituting (ii) in (i) we get

$\Rightarrow \frac{n(n+1)}{2} = 15n$

$\Rightarrow n + 1 = 30 \quad \therefore$ **n = 30 – 1 = 29**

2. We know that,

Class mark $= \frac{\textit{(Lower class limit + Upper class limit)}}{2}$

$\therefore$ **Class mark of 10 - 25 :**

$$\text{Class mark} = \frac{(10+25)}{2} = \frac{35}{2} = \mathbf{17.5}$$

Class mark of 35 - 55 :

$$\therefore \text{Class mark} = \frac{(35+55)}{2} = \frac{90}{2} = \mathbf{45}$$

3. Given. Mean = 50; N = 100

$$i.e.,\ 50 = \frac{\Sigma X}{100}$$

$$\Rightarrow \Sigma X = 5000$$

$$\text{Correct Mean} = \frac{[5000+110-100]}{100}$$

$$= \frac{[5110-100]}{100}$$

$$= \frac{5010}{100} = \mathbf{50.10}$$

$\therefore$ **The real or true mean value is 50.10 Median will remain same *i.e.*, Median value is 52.**

4. The company should fix the size of the oil bottle based on the mode as mode gives the information about the size that is sold most often.

5. Given. Max Frequency = 50,

$\Rightarrow$ class (modal class) = 35 – 40

$$\therefore \textbf{Mode} = 35 + \frac{(50-34)}{(100-34-42)} \times 5$$

$$= 38.33$$

6.

x_i	13	15	17	18	20	z	25
f_i	1	3	1	3	3	1	3

As we know, mode is the value that occurs most in the data set. From above table, it is clear that for observations, 15, 18, 20 and 25 are repeated 3 times. Since the mode is 20 and Modal class will have the maximum frequency. Therefore $z = 20$.

7. We know that 25 comes the maximum number of times hence, it is the mode of the data.

8. Arranging the terms in ascending order, 0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

$$\Rightarrow \text{Median value} = \frac{(11+1)}{2}\text{th}$$

$$= 6\text{th value} = 27$$

Short Answers

1. We prepare following table to find f,

Class	Class Mark (x_i)	Frequency (f_i)	f_ix_i
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	$20f$
21-23	22	5	110
23-25	24	4	96
Total		$\Sigma f_i = 40+f$	$\Sigma f_ix_i = 704+20f$

We have,

$$\Rightarrow \Sigma f_i = 40 + f$$

$$\Rightarrow \Sigma f_ix_i = 704 + 20f$$

$$\therefore \text{Mean, } \bar{x} = \frac{\Sigma f_ix_i}{f_i}$$

$$\Rightarrow 18 = \frac{704+20f}{40+f}$$

$$\Rightarrow 720 + 18f = 704 + 20f$$

$$\therefore f = 8$$

2. We prepare following table to find value of k.

Class	x	f	$u_i = \frac{(x-50)}{20}$	f_iu_i
0-20	10	12	–2	–24
20-40	30	15	–1	–15
40-60	50	32	0	0
60-80	70	k	1	k
80-100	90	13	2	26
		$\Sigma f_i = 72+k$		$\Sigma f_iu_i = -13+k$

Assumed mean (A) = 50

width(h) = 20; $\bar{x} = 53$

$$\text{Now, } \bar{x}\ (\text{Mean}) = A + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

$$\Rightarrow 53 = \frac{50(72+k)+(-13+k)20}{72+k}$$

$$\Rightarrow 53(72+k) = 3600 + 50k - 260 + 20k$$

$\Rightarrow 3816 + 53k = 3340 + 70k$

$\Rightarrow 53k - 70k = 3340 - 3816$

$\Rightarrow -17k = -476$

$\therefore k = 28$

3. We prepare following table to find mean.

CI	x_i	f_i	$u_i = \frac{(x - f_i)}{h}$	f_iu_i
0-10	5	5	-3	-15
10-20	15	x	-2	$-2x$
20-30	25	10	-1	-10
30-40	35	12	0	0
40-50	45	7	1	7
50-60	55	8	2	16
		$\Sigma f_i = 42 + x$		$\Sigma f_iu_i = -2x - 2$

Let mid point of class 30-40 be assumed mean, $a = 35$.

We know the formula:

$$\text{Mean}(\bar{x}) = a + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

$$\Rightarrow 31.4 = 35 + \frac{(-2x - 2)}{(42 + x)} \times 10$$

$\Rightarrow (2x + 2)10 = (42 + x)3.6$

$\Rightarrow 20x + 20 = 151.2 + 3.6x$

$\Rightarrow 16.4x = 131.2$

$\therefore x = 8$

4. Maximum Frequency = 9

Therefore, Modal Class = 60 - 90

so, $f_0 = 6$; $f_1 = 9$; $f_2 = 6$

$l = 60$; $h = 30$;

We know that,

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$$

On putting the values:

$$\Rightarrow \text{Mode} = 60 + \frac{(9 - 6)}{(2 \times 9) - 6 - 6} \times 30$$

$$= 60 + 30\left(\frac{3}{6}\right)$$

$$= 60 + \frac{90}{6} = 75$$

$\therefore$ Mode = 75

5. We prepare following table to find value of f_1 and f_2.

Class	Frequency	Cumulative Frequency
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$
30-40	12	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
	$\Sigma f = 40$	

Median = 32.5

$\Rightarrow$ Median class = 30 - 40

$$\text{Now, } 32.5 = 30 + \frac{20 - 14 - f_1}{12} \times 10$$

$$32.5 - 30 = \frac{60 - 10f_1}{12} = 2.5 \times 12 = 60 - f_i$$

$\therefore f_1 = 3$ ...(i)

and $31 + f_1 + f_2 = 40$

$\Rightarrow 31 + 3 + f_2 = 40$ ...[From (i)

$\therefore f_2 = 6$

6. From the given frequency distribution, we have

$\Rightarrow$ Modal class = 60 - 80

$l = 60$; $f_1 = 12$;

$f_0 = 10$; $f_2 = 6$; $h = 20$

so,

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$$

$$\Rightarrow \text{Mode} = 60 + \frac{(12 - 10)}{24 - 10 - 6} \times 20$$

$$= 60 + \frac{2}{8} \times 20 = 60 + 5 = 65$$

7. Number of days remaining as $30 - 10 - 10 = 10$.

Sum of the temperature of all 30 days $= 34 \times 30 = 1020°$ C.

Sum of the temperature of first 10 days $= 30 \times 10 = 300°$ C.

Sum of the temperatures of next 10 days $= 35 \times 10 = 350°$ C.

$\therefore$ Sum of the temperatures of last 10 days $= 1020° - 300° - 350° = 370°$ C.

8. To calculate median, we form *c.f.* table.

Speed (in km/h)	*Number of players* (f_i)	*c.f.*
85-100	11	11
100-115	9	20
115-130	8	28
130-145	5	33

N = Number of observations = 33 (Odd)
Median of 33 observations
= 17^{th} observation, which lies in class 100 – 115.

$\therefore$ $l = 100, f = 9, c.f. = 11, h = 115 - 100 = 15$

We use the given formula:

$$Median = l + \frac{\left(\frac{n}{2} - c.f.\right)}{f} \times h$$

$$= 100 + \frac{\left(\frac{33}{2} - 11\right)}{9} \times 15$$

$$= 100 + \frac{(16.5 - 11)}{9} \times 15$$

$$= 100 + \frac{(5.5 \times 15)}{9} = 100 + \frac{82.5}{9}$$

$$= 100 + 9.16 = \mathbf{109.17\ km/h}$$

9. Complete the frequency distribution table as:

Rainfall (in mm)	*Frequency* (f_i)	*Class mark* (x_i)	f_ix_i
0 – 10	2	5	10
10 – 20	6	15	90
20 – 30	x	25	$25x$
30 – 40	7	35	245
40 – 50	4	45	180
Total	$\Sigma f_i = 19 + x$		$\Sigma f_ix_i = 525 + 25x$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow 27 = \frac{525 + 25x}{19 + x}$$

$$\Rightarrow 27(19 + x) = 525 + 25x$$

$$\Rightarrow 513 + 27x = 525 + 25x$$

$$\Rightarrow 27x - 25x = 525 - 513$$

$$\Rightarrow 2x = 12$$

$$\therefore x = \frac{12}{2} = 6$$

Long Answers

1.

Class interval	*Freq.* (f_i)	*Class mark* (x_i)	$d_i = x_i - a$	$u_i = d_i/h$	f_iu_i
120 – 130	38	125	–20	–2	–76
130 – 140	40	135	–10	–1	–40
140 – 150	46	145	0	0	0
150 – 160	23	155	10	1	23
160 – 170	25	165	20	2	50
170 – 180	8	175	30	3	24
Total	$\Sigma f_i = 180$				$\Sigma f_iu_i = -19$

By Using Step Deviation Method:

$$\bar{x} = A + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \times h\right)$$

$$\bar{x} = 145 + \left(\frac{-19}{180} \times 10\right)$$

$$\bar{x} = 145 - 1.06 = \mathbf{143.94\ cm}$$

2. We prepare following table to find missing frequencies x and y.

Marks	*No. of student* (f_i)	x_i	$d_i' = \frac{(x_i - 55)}{10}$	f_id_i'
0-10	1	5	–5	–5
10-20	3	15	–4	–12
20-30	7	25	–3	–21
30-40	10	35	–2	–20
40-50	15	45	–1	–15
50-60	x	a=55	0	0
60-70	9	65	1	9
70-80	27	75	2	54
80-90	18	85	3	54
90-100	y	95	4	$4y$
	$\Sigma f_i = (90+x+y)$			$\Sigma f_id_i = 44 + 4y$

Here, $\Sigma f_i = 120$

$\therefore$ $90 + x + y = 120$

$x = 120 - 90 - y = 30 - y$ ...(i)

$$\text{Mean }(\bar{x}) = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h$$

$$\Rightarrow 59 = 55 + \frac{44+4y}{120} \times 10 \quad \ldots Here \left[\begin{array}{ll} a & = 55; \\ \Sigma f_i & = 120; \\ h & = 10; \\ Mean & = 59; \end{array}\right.$$

$$so,\ 59 - 55 = \frac{4(11+y)}{12}$$

$\Rightarrow 4 \times 3 = 11 + y \Rightarrow y = 12 - 11 = 1$

From (*i*) we have

$x = 30 - 1 = 29$

Therefore, $x = 29$ and $y = 1$

3. We prepare following table to find the model weight

Weight (in kg)	*No. of students*	*Class Interval (C.I)*	*Frequency (f)*
Below 40	3	Below 40	3
Below 42	5	40 - 42	2
Below 44	9	42 - 44	4
Below 46	14	44 - 46	5
Below 48	28	46 - 48	14
Below 50	31	48 - 50	3
Below 52	35	50 - 52	4

Mode: The class which have highest frequency. In this case, class interval 46–48 is the modal class.

Now, Lower limit of modal class,

$l = 46;$ $h = 2;$

$f_1 = 14;$ $f_0 = 5;$ $f_2 = 4$

We know that,

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$$

$$= 46 + \frac{(14-5)}{(28-5-3)} \times 2$$

$$= 46 + \frac{(9 \times 2)}{20} = 46 + 0.9 = \mathbf{46.9}$$

Therefore, Modal weight = **46.9 kg**

4. When assumed mean, $a = 5$ plants

No. of Plants (Class interval)	***No. of Houses (Freq. (f_i))***	(X_i)	$u_i = X_i - a$	$f_i u_i$
0 - 2	4	1	-4	-16
2 - 4	3	3	-2	-6
4 - 6	2	5	0	0
6 - 8	5	7	2	10
8 - 10	6	9	4	24
Total	**20**			$\boldsymbol{\Sigma f_i u_i = 12}$

$$\therefore \text{Mean, } \overline{X} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} = 5 + \frac{12}{20}$$

$= 5 + 0.6 =$ **5.6 potted plants**

$\Rightarrow$ When assumed mean $(a) = 6$ plants

No. of Plants (Class interval)	***No. of Houses (Freq. (f_i))***	(X_i)	$u_i = X_i - a$	$f_i u_i$
0 - 2	4	1	-5	-20
2 - 4	3	3	-3	-9
4 - 6	2	5	-1	-2
6 - 8	5	7	1	5
8 - 10	6	9	3	18
Total	$\Sigma f_i = 20$			$\boldsymbol{\Sigma f_i u_i = -8}$

Calculating mean using assumed mean formula,

$$\overline{X} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} = 6 + \left(\frac{-8}{20}\right)$$

$= 6 - 0.4 =$ **5.6 potted plants**

Case Based Answers

1. (*a*) Class 60-80 has the maximum frequency 29, therefore this is modal class and lower limit of this class is 60.

(*b*) (*i*) Since numbers of students are 100, $15 + 18 + 21 + 29 + p = 100$

$\Rightarrow 83 + p = 100$

$\therefore$ **$p = 100 - 83 = 17$**

Or, (*ii*) Here,

$l = 60;$ $h = 20;$ $f_1 = 29;$

$f_0 = 21;$ $f_2 = p = 17$

$$\therefore \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 60 + \frac{8}{58-38} \times 20$$

$$= 60 + \frac{8}{20} \times 20 = 68$$

(c) **Given.** Mean = 53

So, 3 median = 2 mean + mode

$$\text{Median} = \frac{\textit{mode} + 2\ \textit{mean}}{3}$$

$$= \frac{68 + 2 \times 53}{3}$$

$$= \frac{68+106}{3} = \frac{174}{3} = 58$$

2. (a) Since, the highest frequency is 70, therefore the maximum number of children are of the age-group 10-12.

(b) (i) Since, the modal class is 10-12

$\therefore$ Lower limit of modal class = 10

Or, (ii) Here, $f_0 = 58$, $f_1 = 70$ and $f_2 = 42$.

Thus, the frequency of the class succeeding the modal class is 42.

(c) We know that,

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$$

$$= 10 + \frac{(70-58)}{(140-58-42)} \times 2$$

$$= 10 + \frac{12}{40} \times 2 = 10 + \frac{24}{40}$$

$$= 10.6 \textit{ years}$$

3. (a) The upper limit of a class and the lower class of its succeeding class differ by 1.

(b) (i)

Daily distance travel (in km)	199.5 - 209.5	209.5 - 219.5	219.5 - 229.5	229.5 - 239.5	239.5 - 249.5
No. of Buses	4	14	26	10	6
c.f.	4	18	44	54	60

As, $\Sigma f = N = 60$

$$\frac{N}{2} = 30$$

Class interval whose cumulative frequency is greater than 30 is 219.5 - 229.5.

$\therefore$ Median Class = 219.5 – 229.5

Now, the cumulative frequency of the class preceding the median class is **18**.

Or

(ii) We know that,

$$\textbf{Median} = l + \frac{\frac{n}{2} - c.f.}{f} \times h$$

$$= 219.5 + \frac{30-18}{26} \times 10$$

$$= 219.5 + 4.62 = \mathbf{224.12}$$

(c) We know,

Mode = 3 Median – 2 Mean

$$\text{Mean} = \frac{1}{2}(3\ \text{Median} - \text{Mode})$$

$$= \frac{1}{2}(672.36 - 223 - 78)$$

$$= \mathbf{224.29\ km}$$

(DAY 5 SWAHA)

* * * * *

3 Probability

What did CBSE ask last year?

MCQs & A/R	2 Questions (2 × 1 = 2 Marks)
Subjective	2 Very Short Questions (2 × 2 = 4 Marks)
	1 Short Question (1 × 3 = 3 Marks)
	No Long Answer Question
Case Based	No Case Based Questions Asked

Note: All the above typology of questions include 'Competency based Questions' labelled as COMPETENCY.

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Probability - A Theoretical Approach

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes of the experiment}}$$

Cards (52)

Black (26): ♠ Spades, ♣ Clubs

Red (26): ♦ Diamond, ♥ Heart

♠ Spades (13)	♣ Clubs (13)	♦ Diamond (13)	♥ Heart (13)
A	A	A	Ace
K	K	K	King
Q	Q	Q	Queen
J	J	J	Jack
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10

A, K, Q, J (Spades): Face Cards

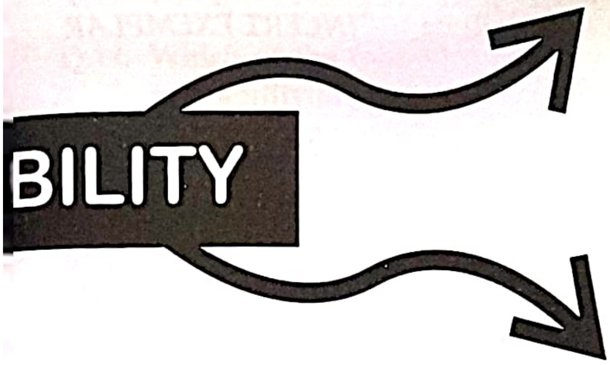
A, K, Q, J (Clubs): Honour cards

2 – 10 (Diamond): Pip cards

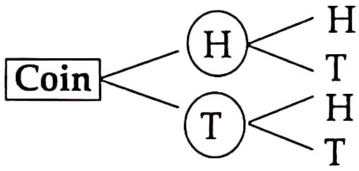
- Probability of a leap year having 53 Sundays $= \frac{2}{7}$
- 1 is neither prime nor composite.
- 2 dice throw : 6 elementary events

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Two dice results chart



Two Coins Result



Outcomes	No. of heads
HH	2
HT	1
TH	1
TT	0

Points to be noted

1. The probability of a sure event (or certain event) is 1.
2. The probability of an impossible event is 0.
3. The probability of an event E is a number P(E) such that

$$0 \le P(E) \le 1$$

4. An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
5. For any event E, $P(E) + P(\bar{E}) = 1$, where $\bar{E}$ stands for 'not E'. E and $\bar{E}$ are called complementary events.

OBJECTIVE QUESTIONS

(DAY 6)

Multiple Choice Questions

Q.1. A bag contains 5 pink, 8 blue and 7 yellow balls. One ball is drawn at random from the bag. What is the probability of getting neither a blue nor a pink ball? [CBSE 2023

(a) $\frac{1}{4}$ (b) $\frac{2}{5}$

(c) $\frac{7}{20}$ (d) $\frac{13}{20}$

Q.2. Box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a prime number less than 23 is COMPETENCY

(a) $\frac{7}{90}$ (b) $\frac{1}{9}$

(c) $\frac{4}{45}$ (d) $\frac{9}{89}$

Q.3. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is COMPETENCY

(a) $\frac{1}{13}$ (b) $\frac{9}{13}$

(c) $\frac{4}{13}$ (d) $\frac{12}{13}$

Q.4. Pratik has blue and green coins of the same size in a bag. He has 50 coins each of blue and green.

He is randomly picking up one coin at a time without replacement. He does not see which coin he has picked.

What is the MINIMUM number of coins he would have to pick to definitely get a pair of blue or green coins? COMPETENCY

(a) 2 (b) 3 (c) 4 (d) 5

Q.5. A library receives a shipment for a series of encyclopedias. The shipment includes volumes 31-40. These encyclopedias arrived in a box and are not ordered. One encyclopedia is picked at random from the box without looking into it.

What is the probability that the volume of the encyclopedia picked is a multiple of 2 or 5? COMPETENCY

(a) $\frac{1}{10}$ (b) $\frac{5}{10}$ (c) $\frac{6}{10}$ (d) $\frac{7}{10}$

Q.6. If the probability of an event is p, then the probability of its complementary event will be [NCERT EXEMPLAR

(a) $p - 1$ (b) $1 - p$

(c) p (d) $1 - \frac{1}{p}$

Q.7. Two fair coins are tossed. What is the probability of getting at most one head? COMPETENCY

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{8}$

FREE ADVICE: You can consider the possible outcomes where you get zero heads (two tails) or one head.

Q.8. For an event E, $P(E) + P(E') = x$, then the value of $x^3 - 3$ is [CBSE 2022

(a) −2 (b) 2 (c) 1 (d) −1

Q.9. The probability that the drawn card from a pack of 52 cards is neither an ace nor a spade is COMPETENCY

(a) $\frac{9}{13}$ (b) $\frac{35}{52}$ (c) $\frac{10}{13}$ (d) $\frac{19}{26}$

Q.10. A dice is rolled twice. The probability that 5 will not come up either time is COMPETENCY

(a) $\frac{11}{36}$ (b) $\frac{1}{3}$

(c) $\frac{13}{36}$ (d) $\frac{25}{36}$

Q.11. The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow? [CBSE 2020

(a) 0.14 (b) 0.15 (c) $\frac{1}{2}$ (d) 0.01

FREE ADVICE: The probability that it will not rain tomorrow is the complement of the probability that it will rain. In other words, it's 1 minus the probability that it will rain.

Q.12. A letter of English alphabet is chosen at random. What is the probability that the chosen letter is a consonant? COMPETENCY

(a) $\frac{21}{26}$ (b) $\frac{22}{26}$ (c) $\frac{23}{26}$ (d) $\frac{25}{26}$

Q.13. Which of the following cannot be the probability of an event? COMPETENCY

(a) $\frac{2}{3}$ (b) –1.5 (c) 15% (d) 0.7

FREE ADVICE: Probability kabhi bhi negative me nahi hoti hai or sabhi events ka sum karne pe value 1 aati hai

Q.14. Jyoti and Dara are playing a game of tic-tac-toe. The probability of Jyoti winning the game is 0.7.
What is the probability that Jyoti NOT winning the game? COMPETENCY

(a) 0.7
(b) 0.5
(c) 0.3
(d) Cannot be determined

Q.15. A card is drawn at random from a well shuffled standard deck of 52 cards.
What is the probability that the card drawn is neither a black card nor a three?
(Note: A deck of cards is divided into four suits - 2 black and 2 red. Each suit contains 13 ranks including numbered cards 2 through 10, and the face cards (jack, queen, king), along with the ace.) [CBSE 2024

(a) $\frac{22}{52}$ (b) $\frac{24}{52}$ (c) $\frac{26}{52}$ (d) $\frac{28}{52}$

Q.16. A box contains some new and weathered cricket balls of two colours. This data is shown in the table below.

Colour \ Condition	New	Weathered
White	5	5
Red	7	3

Pratik picks a new, white ball and puts it back in the box. If a ball is then picked randomly from the box, what is the probability that it is not the same variety as Pratik picked? [CBSE 2024

(a) $\frac{1}{15}$ (b) $\frac{5}{20}$ (c) $\frac{5}{15}$ (d) $\frac{15}{20}$

Q.17. 20 tickets on which number 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them, Find the probability that the number on the drawn ticket is a multiple of 3 or 7. COMPETENCY

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$

Q.18. An event is very unlikely to happen. Its probability is closest to [NCERT EXEMPLAR

(a) 0.0001 (b) 001 (c) 0.01 (d) 0.1

Q.19. When a die is thrown, the probability of getting an odd number less than 3 is COMPETENCY

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 0

Q.20. The P(A) denotes the probability of an event A, then COMPETENCY

(a) $P(A) < 0$ (b) $P(A) > 1$
(c) $0 \le P(A) \le 1$ (d) $-1 \le P(A) \le 1$

Q.21. The probability of guessing the correct answer to a certain test questions is $\frac{x}{12}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$ then x = ?

(a) 2 (b) 3 (c) 4 (d) 6

FREE ADVICE: Probability of not guessing the correct answer is $1-\left(\frac{x}{12}\right)$

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*
(*b*) *Both (A) and (R) are true, but (R) is not the correct explanation of (A).*
(*c*) *(A) is true but (R) is false.*
(*d*) *(A) is false, but (R) is true.*

Q.1. **Assertion (A):** The probability that a leap year has 53 Sundays is $\frac{2}{7}$. COMPETENCY
Reason (R): The probability that a non-leap year has 53 Sundays is $\frac{1}{7}$.

Q.2. **Assertion (A):** The probability of an event that cannot happen or which is impossible, is equal to zero. COMPETENCY
Reason (R): The probability lies between 0 and 1. Hence, it cannot be negative.

Q.3. **Assertion (A):** If P(E) = 0.07, then its probability of 'not E' is 0.93.
Reason (R): P(E) + P(not E) = 1

Q.4. **Assertion (A):** The sum of the probabilities of all the elementary events of an experiment is 1. COMPETENCY
Reason (R): If an event cannot occur, then its probability is 0.

Q.5. **Assertion (A):** Three coins are tossed simultaneously. The probability of getting all heads is $\frac{1}{8}$.
Reason (R): The chance of throwing 5 with an ordinary die is $\frac{1}{6}$.

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability of getting a red king. [CBSE 2020

Q.2. A die is thrown once. What is the probability of getting a prime number? [CBSE 2020

Q.3. Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5. COMPETENCY

Q.4. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game. [CBSE 2019

Q.5. If a number x is chosen at random from the number –3, –2, –1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

Q.6. A pair of fair 6-sided dice with numbers 1 - 6 written on them are thrown. What is the probability that the sum of the numbers shown on the pair of dice is greater than 1? Justify your answer. [CBSE 2024

Q.7. Out of 200 bulbs in a box, 12 bulbs are defective. One bulb is taken out at random from the box. What is the probability that the drawn bulb is not defective? COMPETENCY

Q.8. **Joel has exactly six 2-rupee coins, five 10-rupee coins and three 20-rupee coins in his pocket. He goes to the stationery store and buys a pen for ₹19. He takes out a coin from his pocket at random.**
(*i*) Find the probability that the coin will be sufficient to pay for the pen.
(*ii*) Find the probability that he will be able to give exactly ₹19 to the shopkeeper.
Show your work. [CBSE 2024

Q.9. If I toss a coin 3 times and get head each time, should I expect a tail to have a higher chance in the 4th toss? Give reason in support of your answer. COMPETENCY

Q.10. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a black jack nor a black ace. COMPETENCY

(DAY 7)

Short Answer Questions

Q.1. Two dice are tossed simultaneously. Find the probability of getting [CBSE 2019
(*a*) an even number on both dice.
(*b*) the sum of 2 numbers more than 9.

Q.2. **Three different coins are tossed together. Find the probability of getting:**
(*a*) exactly two heads;
(*b*) atleast two heads
(*c*) atleast two tails. COMPETENCY

Q.3. **Drish lives in India and Hugh lives in the USA. The date formats of both the countries is given below.** [CBSE 2024
India: day/month
USA: month/day
They wrote dates everyday in 2022. If a day in 2022 is randomly selected, what is the probability that:
(*i*) both their dates in the two formats are the same on that day?
(*ii*) the date written by Hugh is a valid date for Drish in India?
Show your work and give valid reasons.

Q.4. **A fruit basket contains 3 oranges, 1 apple, 5 pomegranate and 6 bananas.**
(*i*) Anirudh picks a fruit from the basket at random to eat. What is the probability that he picks an apple?
(*ii*) After Anirudh eats an apple, Aryan picks a fruit at random to eat. What is the probability that Aryan picks a banana?
(*iii*) After Anirudh and Aryan eat an apple and a banana respectively, Siddharth picks a fruit at random to eat. What is the probability that Siddharth picks an apple? COMPETENCY

Q.5. Two different dice are thrown together. Find the probability that the numbers obtained **COMPETENCY**

(*a*) have a sum less than 7.

(*b*) have a product less than 16.

(*c*) is a doublet of odd number.

Q.6. Savita and Hamida are friends. What is the probability that both will have **COMPETENCY**

(*a*) different birthdays?

(*b*) the same birthday?

(ignoring a leap year)

Q.7. In a family of three children, find the probability of having at least two boys. **COMPETENCY**

Q.8. All red face cards are removed from a pack of playing cards. The remaining cards were well shuffled and then a card is drawn at random from them. Find the probability that the drawn card is **COMPETENCY**

(*a*) a red card

(*b*) a face card

(*c*) a card of clubs

Q.9. An integer is chosen at random between 1 and 100. Find the probability that it is: *[CBSE 2018*

(*a*) divisible by 8.

(*b*) not divisible by 8.

FREE ADVICE: If question says, an integer is chosen between 1 and 100 then you should exclude 1 and 100 then number of outcomes = 98.

Long Answer Questions

Q.1. Peter thrown two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the number 25? *[CBSE 2017*

Q.2. Anjali has a jar where she saves coins. She has collected twelve 2-rupee coins, and eighteen 10-rupee coins.

One day, she added four 20-rupee coins

to it. If a coin is picked at random from this jar now, without looking, find the probability that it is:

(*i*) a 1-rupee coin
(*ii*) a 10-rupee coin
(*iii*) an even valued coin
(*iv*) anything except a 5-rupee or a 20-rupee coin.

Show your work. **COMPETENCY**

Q.3. **In a class, there are 18 girls and 16 boys. The class teacher wants to choose one pupil for the class monitor. What she does is she writes the name of each pupil on a card and puts them into a basket and mixes them thoroughly. A child is asked to pick one card from the basket. What is the probability that the name written on the card is** **COMPETENCY**

(*a*) The name of a girl?
(*b*) The name of a boy?

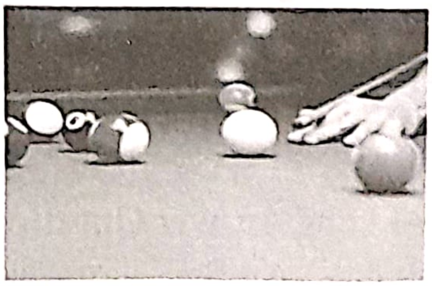
Q.4. **Aisha and Ahmad are about to play a board game. To decide who starts first they decide to throw two dice. The following was agreed upon:**

- Aisha goes first if the numbers on the dice are both prime numbers.
- Ahmad goes first if the sum of the numbers on the dice equals 6.
- They throw the dice repeatedly till one of these conditions is met.

If the first throw has decided who goes first, who is more likely to start first? List all the outcomes and show your steps. **COMPETENCY**

CASE BASED QUESTIONS

Q.1. There is a game played on a pool table with 15 balls numbered 1 to 15 and a cue ball that is solid white. Of the 15 numbered balls, 8 are a solid (non-white) colour and numbered 1 through 8 and seven are striped balls numbered 9 through 15. The fifteen numbered pool balls (no cue ball) are placed in a large bowl and mixed, then one is drawn out. *[CBSE 2023*



(*a*) What is the probability of drawing the eight ball?

(*b*) (*i*) What is the probability of drawing a number greater than fifteen? **COMPETENCY**

Or, (*ii*) What is the probability of drawing an even number?

(*c*) What is the probability of drawing a multiple of three?

Q.2. A survey was conducted in a college to find out how many students were interested in sports and music. The following table gives the result of the survey.

	Boys	***Girls***
Sports only	40	30
Music only	60	50
Both sports and music	20	15
Neither sports nor music	30	35

(*a*) **What is the probability that a student selected at random from the college is interested in sports only?**

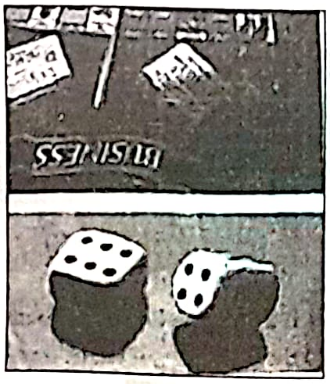
(*b*) (*i*) **What is the probability that a student selected at random from the college is interested in neither sports nor music?** **COMPETENCY**

Or

(*ii*) What is the probability that a student selected at random from the college is a girl who is interested in music only?

(*c*) What is the probability that a student selected at random from the college is a boy who is interested in either sports or music?

Q.3. Rahul and Ravi planned to play Business (board game) in which they were supposed to use two dice.



(*a*) Ravi got first chance to roll the dice. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is 8?

(*b*) (*i*) Rahul got next chance. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is 13? **COMPETENCY**

Or

(*ii*) Now it was Ravi's turn. He rolled the dice. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is less than or equal to 12?

(*c*) Now it was Ravi's turn. He rolled the dice. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is greater than 8?

*****

ANSWERS

Multiple Choice Answers

1. (*c*) We know that, Probability P(E)

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{7}{20}$

2. (*c*) Favourable condition possibility of a prime number less than 23 = {2,3,5,7,11,13,17,19} = 8.

Therefore probability of a prime number less than 23 = $\frac{8}{90} = \frac{4}{45}$.

3. (*d*) There will be 48 non-ace cards.

Number of favourable outcomes = 48

$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total Number of possible outcomes}}$

$P(E) = \frac{48}{52} = \frac{24}{26} = \frac{12}{13}$

4. (*b*) 3

5. (*c*) Total no. of multiples of 2 = 5,

i.e., [32, 34, 36, 38, 40]

Total no. of multiples of 5 = 2,

i.e., [35, 40]

But 40 is common in both.

Total favourable outcomes

= 5 + 2 − 1 = 6

Total no. of outcomes = 10

∴ **Required Probability** = $\frac{6}{10}$

6. (*b*) We know that,

Probability of an event + Probability of its complementary = 1

It can be written as Probability of its complementary event

= 1 − Probability of an event

= **1 − *p***

7. (*a*) When a coin is tossed two times.

The possible outcomes are [TT, HH, TH, HT].

Number of possible outcomes = 4

Favourable outcomes = [TT, HT, TH]

Number of favourable outcomes = 3

$\text{Probability} = \frac{\text{Number of favourable cases}}{\text{Total number of possible cases}}$

Probability of getting at most one head

$= \frac{3}{4}$

8. (*a*) We know, $p(E) + p(E') = 1$

$P(E) + P(E)' = x$

$\therefore x = 1; x^2 - 3 = 1^2 - 3 = 1 - 3 = \mathbf{-2}$

9. (*a*) Total number of cards will be one less than the sum of spades and aces.

= 4 + 13 − 1 = 16

The total number of cards in the pack that will neither be an ace or a spade

= 52 − 16 = 36

The probability of a card in the pack neither being an ace or a spade.

$= \frac{\text{Cards in the pack that are neither ace nor spade}}{\text{total number of cards}}$

$= \frac{36}{52} = \frac{9}{13}$

10. (*d*) Total number of outcomes when die is thrown twice = 6 × 6 = 36.

Number of possible outcomes when 5 will come up either time = (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5) = 11

Probability that 5 will come up either time

$= \frac{\text{Number of possible outcomes}}{\text{Total number of favourable outcomes}} = \frac{11}{36}$

∴ **P(5 will not come up either time)**

$= 1 - \frac{11}{36} = \frac{25}{36}$

11. (*b*) Sum of probabilities is one. Probability that it won't rain tomorrow

= 1 − 0.85 = **0.15**

12. (*a*) The probability of selecting a letter that is a consonant is given by

Favourable outcomes = *b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z*

Number of favourable outcomes = 21

Number of possible outcomes = 26

$\text{Prob.} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

∴ **Probability** = $\frac{21}{26}$

13. (*b*) The probability of an event cannot be negative in any case, thus −1.5 cannot be the probability of an event. The probability of happening of an event always lies between 0 to 1, *i.e.*, $0 \le P(E) \le 1$.

14. (*c*) As we know, $P(E) + P(\bar{E}) = 1$

⇒ 0.7 + P(Not winning) = 1

∴ **P(Not winning)** = 1 − 0.7 = **0.3**

15. (*b*) Remove 26 black cards and 2 cards of 3 from red cards suit

= 52 − 26 − 2 = 24

∴ **P(neither black card nor a three)** = $\frac{24}{52}$

16. (*d*) P(Variety that pratik picked) = $\frac{5}{20}$

∴ **P(Not same variety as Pratik Picked)**

$= 1 - \frac{5}{20} = \frac{15}{20}$

17. (*b*) *n*(S) = 20 and multiple of 3 or 7

= {3, 6, 9, 12, 15, 18, 7, 14}

n(A) = 8

∴ **P(ticket multiple 3 or 7)**

$= \frac{n(A)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

18. (*a*) The probability of an event which is very unlikely to happen is closest to zero and from the given options, it is 0.0001.

19. (*a*) Odd number is the number which is not divisible by 2.

Odd numbers = 3 = {1, 3, 5}

Odd numbers less than 3 = 1 = {1}

Total outcomes = 6 = {1, 2, 3, 4, 5, 6}

So the probability of getting an odd number which is less than 3

$= \frac{\text{Number of favourable outcomes}}{\text{Total outcomes}} = \frac{1}{6}$

20. (*c*) Higher the probability of an event, more likely it is for the event to occur.

So the probability of an event lies between 0 and 1.

Therefore, if P(A) denotes the probability of an event A, then $0 \leq P(A) \leq 1$.

21. (c) Probability of guessing the correct answer to certain test questions = $\frac{x}{12}$

Prob. of not giving correct answers

$= 1 - \frac{x}{12}$

$\Rightarrow 1 - \frac{x}{12} = \frac{2}{3}$ $\quad\quad \therefore x = 4$

Assertion Reason Answers

1. (b) *Both (A) and (R) are true, but (R) is not the correct explanation of (A).*

   Explanation: *Leap year = 366 days = 52 weeks and 2 days Thus, a leap year always has 52 Tuesdays.*

   The remaining 2 days can be:

   (i) Sunday and Monday

   (ii) Monday and Tuesday

   (iii) Tuesday and Wednesday

   (iv) Wednesday and Thursday

   (v) Thursday and Friday

   (vi) Friday and Saturday

   (vii) Saturday and Sunday

   Out of 7 cases, 2 have Tuesdays.

   P(53 Tuesdays) $= \frac{2}{7}$

   52 weeks can be either a Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, or a Sunday.

   $\therefore$ ***P(53 Sunday in non-leap year)*** $= \frac{1}{7}$.

2. (b) *Both A and R are true, but R is not the correct explanation of A.*

   Explanation: *We know that, all probabilities are between 0 and 1 inclusive.*

   A probability of 0 means an event is impossible, it cannot happen.

   A probability of 1 means an event is certain to happen, it must happen.

   Thus the probability of an event that cannot happen is 0.

3. (a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation:
Assertion: *If P(E) = 0.07, then its probability of 'not E' is 0.93.*
Reason: *P(E) + P(not E) = 1*
Since, P(E) = 0.07 and P(not E) = 1 − P(E)
P(not E) = 1 − 0.07 = 0.93

4. (b) *Both A and R are true, but R is not the correct explanation of A.*
Explanation: *The probability p of any event must be greater than or equal to 0. In other words, 0 ≤ p ≤ 1. Since 1 is the maximum limit, all probabilities must add to 1. Hence, the sum of the probabilities of all the elementary events of an experiment is 1.*

5. (b) *Both A and R are true, but R is not the correct explanation of A.*
Explanation: *The possible outcomes when three coins are tossed together are {HHH, TTT, THT, TTH, HHT, HTH, THH, HTT}. Therefore, the number of possible outcomes when two coins are tossed is 8. Now, the possible outcome of getting all heads is {HHH}, which means the number of favourable outcome is 1. Therefore, probability P of getting all heads is:*

$$P = \left(\frac{1}{8}\right)$$

Very Short Answers

1. Total no. of outcomes = 52
A red king = 2
∴ **Probability of getting a red king**
$$= \frac{2}{52} = \frac{1}{26}$$

2. The possible outcomes when a die is rolled are 1, 2, 3, 4, 5, 6. Among the outcomes, the prime numbers are 2, 3, 5. So, the no. of favourable outcomes = 3 and the no. of possible outcomes = 6.
∴ Probability of getting a prime number
$$= \frac{3}{6} = \frac{1}{2}$$

3. For two dies thrown together.
No. of times sum of them will be less than 5 be: (1,1),(1,2),(1,3),(2,1),(2,2),(3,1)
= Total 6 times

Total events for sum of less than 5

$= \frac{\text{Total events for sum of less than 5}}{\text{Total events}} = \frac{6}{36} = \frac{1}{6}$

4. Number of outcomes (S) = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}
Same result in all the tosses (success) can be obtained as (HHH), (TTT).
The no. of favourable outcomes is 2
So, $P(S) = \frac{2}{8} = \frac{1}{4}$
$\therefore$ P(F) loosing $= 1 - P(S) = 1 - \frac{1}{4} = \frac{3}{4}$

5. We have 7 possible outcomes. Thus $n(S) = 7$
So, our favourable outcomes are −2, −1, 0, 1, 2. $\therefore n(E) = 5$
$\therefore p(E) = \frac{n(E)}{N(S)} = \frac{5}{7}$

6. Total no. of outcomes = 36
Total no. of favourable outcome = 36
$\therefore$ **P(Sum of no. is greater than 1)**
$= \frac{36}{36} = 1$

7. Total No. of cases = 200
Favourable cases: 200 − 12 = 188
$\therefore$ **Required probability** $= \frac{188}{200} = \frac{47}{50}$

8. (*i*) Total number of outcomes
= 6 + 5 + 3 = 14
Number of favourable = coin will be sufficient to pay = 3.
(₹20 coins will be sufficient)
$\therefore$ **P(coin will be sufficient to pay for the Pen)** $= \frac{3}{14}$
(*ii*) **P(Joel will be able to give exactly ₹19 to the shopkeeper)** $= \frac{0}{36} = 0$

9. As we know that a coin has two equal chances always either head or tail. So next time on tossing he can get either tail or head. So, the given statement is false.

10. Total number of cards = 52
Numbers of black jacks = 2
Numbers of black aces = 2
Card is neither a jack nor an ace
= 52 − 2 − 2 = 48
$\therefore$ Required probability $= \frac{48}{52} = \frac{12}{13}$

Short Answers

1. There are 36 possible outcomes of rolling two dices. $n(S) = 36$
(*a*) An even number on both dice.
Favourable outcomes are (2, 2), (2, 4), (2, 6), (4, 2) (4, 4), (4, 6), (6, 2), (6, 4) & (6, 6).
No. of favourable outcomes $n(E) = 9$
$\therefore$ P(an even number on both dice)
$= \frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$
(*b*) When sum of two numbers more than 9, Favourable outcome are (4, 6), (5, 5), (5, 6), (6, 4), (6, 5) and (6, 6).
No. of favourable outcomes $n(E) = 6$
P(sum of two numbers more than 9)
$= \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

2. When we toss three coins simultaneously then the possible outcomes are:
HHH, HHT, HTH, THH, HTT, THT, TTH, TTT respectively.
$\therefore$ **Total number of outcomes are $2^3 = 8$**
(*a*) Favourable cases when exactly two heads = {HHT,HTH,THH}
Probability of getting exactly two heads
$= \frac{3}{8}$
(*b*) Favourable cases when at least two heads = {HHH, HHT, HTH, THH}
Probability of getting at least two heads
$= \frac{4}{8} = \frac{1}{2}$
(*c*) Favourable cases when at least two tails = {HTT,THT,TTH,TTT}
$\therefore$ **Probability of getting at least two tails**
$= \frac{4}{8} = \frac{1}{2}$

3. (*i*) There are 12 dates in 2022 where day and the month are interchangeable 01/01; 02/02; 03/03; 04/04; 05/05; 06/06; 07/07; 08/08; 09/09; 10/10, 11/11, 12/12.
$\therefore$ P(Both their dates in the two formats are same on that day)
$= \frac{12}{365}$
(*ii*) Since there are 12 months in a year, Hugh's dates, day can have value from 1-12 of every month such that it is valid for Drish.
Total no. favourable outcomes
= 12 × 12 = 144
$\therefore$ **P(date written by Hugh is valid date for Drish in India)** $= \frac{144}{365}$

4. Total no. of oranges = 3
Total no. of apples = 1
Total no. of pomegranate = 5
Total no. of bananas = 6
(*i*) Total no. of fruits = 15
$\therefore$ **P(Picking an apple)** $= \frac{1}{15}$.
(*ii*) After picking up banana by Anirudh total no. of fruits remaining = 15 − 1 = 14
$\therefore$ **P(Picking up a banana after apple)** $= \frac{6}{14}$ or $\frac{3}{7}$.
(*iii*) **P(Picking up an apple after an apple has been eaten) = 0**

5. **The outcomes when two dice are thrown together are:**
(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)
Total number of outcomes = 36
(*a*) Let A be the event of getting the numbers whose sum is less than 7.
The outcomes in favour of event A are (1, 1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2) and (5,1).
Number of favourable outcomes = 15
$\therefore P(A) = \frac{15}{36} = \frac{5}{12}$

whose product is less than 16.

The outcomes in favour of event B are (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (6,1) and (6,2).

Number of favourable outcomes = 25

$$\therefore \text{P(B)} = \frac{\textit{Number of favourable outcomes}}{\textit{Total number of outcomes}} = \frac{25}{36}$$

(c) Let C be the event of getting the numbers which are doublets of odd numbers.

The outcomes in favour of event C are (1,1), (3,3) and (5,5).

Number of favourable outcomes = 3

$$\therefore \text{P(C)} = \frac{\textit{Number of favourable outcomes}}{\textit{Total number of outcomes}} = \frac{3}{36} = \frac{1}{12}$$

6. (a) If Hamida's birthday is different from Savita's, the number of favourable outcomes for her birthday is 365 – 1 = 364

So, **P(Hamida's birthday is different from Savita's birthday)** $= \frac{364}{365}$

(b) **P(Savita and Hamida have the same birthday) = 1 – P (both have different birthdays)** $= \frac{364}{365} = \frac{1}{365}$

7. If there are three children in family and all possible outcomes are {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}.

So, the total number of outcomes,

$$n(S) = 2^3 = 8$$

At-least two of them are boys means all those cases in which we have either 2 or 3 boys. Thus favourable outcome are: {BBB, BBG, BGB, GBB}.

Number of favourable outcome, $n(E) = 4$

$\therefore$ Probability of having at least two boys, $P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

8. (a) There are 12 face cards in all in pack of 52 cards. Out of that there are 6 red face cards. (52 – 6 = 46). Out of 12 face cards, 6 red face cards are removed. Remaining red cards = 26 – 6

$\therefore$ **Probability of getting the red from remaining 46 cards** $= \frac{20}{46} = \frac{10}{23}$

(b) There are 2 black jacks, 2 black queens and 2 black kings, *i.e.*, total 6 face cards in 46 cards.

Probability of getting a face card, $\frac{6}{46} = \frac{3}{23}$.

(c) There are 13 cards of clubs.

$\therefore$ **Probability** $= \frac{13}{46}$

9. (a) We have $n(S) = 100$

Now, numbers , which are divisible by 8 are: 8 , 16 , 24 , 32, 40 , 48 , 56, 64, 72, 80, 88 , 96

So, total number which are divisible by 8 = 12.

So, number of possible event , $n(E) = 12$

Now, probability that it is divisible by 8.

P(divisible by 8) $= \frac{n(E)}{n(S)} = \frac{12}{100} = \frac{3}{25}$

(b) Probability that it is not divisible by 8

P(E') = 1 – Probability that it is divisible by 8.

$\therefore$ **P(not divisible by 8)** $= 1 - \frac{3}{25} = \frac{22}{25}$

Long Answers

1. Let us first write the all possible outcomes when Peter throws two different dice together. We get,

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Total number of outcomes = 36

The favorable outcome for getting the product of numbers on the dice equal to 25 is (5, 5).

$\therefore$ Favourable number of outcomes = 1

Probability that Peter gets the product of numbers as 25

$$= \frac{\textit{Favourable number of outcomes}}{\textit{Total number of outcomes}} = \frac{1}{36}$$

∴ The outcomes when Rina throws a die are 1, 2, 3, 4, 5, 6.

∴ Total number of outcomes = 6

Rina throws a die and squares the number, so to get the number 25, the favourable outcome is 5.

∴ Favourable number of outcomes = 1

Probability that Peter gets the product of numbers as 25

$$= \frac{\textit{Favourable number of outcomes}}{\textit{Total number of outcomes}} = \frac{1}{6}$$

Thus, $\frac{1}{6} > \frac{1}{36}$

So Rina has better chance to get the number 25.

2. Total coins in the Jar = 12 + 18 + 4 = 34

 (*i*) ∴ P(Picking a ₹1 coin) = 0

 (*ii*) ∴ P(Picking a ₹10 coin)

$$= \frac{18}{34} = \frac{9}{17}$$

 (*iii*) ∴ P(Picking an even valued coin)

$$= \frac{34}{34} = 1$$

 (*iv*) ∴ P(Picking anything except a ₹5 or a ₹20 coin)

$$= 1 - \frac{4}{34} = \frac{1}{1} - \frac{2}{17} = \frac{17-2}{17} = \frac{15}{17}$$

3. The total number of students in the class = 18 + 16 = 34

(*a*) The names of a girl are 18, so the number of favourable cases is 18.

We know that,

$$\text{Probability} = \frac{\textit{Number of favourable outcomes}}{\textit{Total number of outcomes}}$$

∴ **Probability (getting the name of a girl on the card)** $= \frac{18}{34} = \frac{9}{17}$

(*b*) The names of a boy are 16, so the number of favourable cases is 16. We know that,

$$\text{Probability} = \frac{\textit{Number of favourable outcomes}}{\textit{Total number of outcomes}}$$

∴ **Probability (getting the name of a boy on the card)** $= \frac{16}{34} = \frac{8}{17}$

4. Total outcomes = 36

Ayesha's condition = both prime numbers

Favourable outcomes for Ayesha

{(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (3, 5), (5, 2), (5, 3), (5, 5)}

Total favourable outcomes for Ayesha = 9

Ahmed's condition = sum of numbers on the dice equals 6.

Favourable outcomes for Ahmed {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}

Total favourable outcomes for Ahmed = 5

∴ P(Ayesha Starts first) $= \frac{9}{36}$

P(Ahmed Starts first) $= \frac{5}{36}$

As, $\frac{9}{36} > \frac{5}{36}$, **therefore, Ayesha is more likely to start first.**

Case Based Answers

1. (*a*) **P(drawing the eight ball)**

$$= \frac{\textit{favourable no. of balls}}{\textit{total no. of balls}} = \frac{1}{15}$$

(*b*) (*i*) Since no ball has numbered more than 15.

∴ **P(drawing a number greater than fifteen)**

$$= \frac{\textit{Number of favourable balls to the event)}}{\textit{total number of balls}}$$

$$= \frac{0}{5} = 0$$

Or

(*ii*) Here are 7 even number balls (2, 4, 6, 8, 10, 12, 14).

∴ **P(drawing an even number)**

$$= \frac{\textit{Number of favourable balls to the event}}{\textit{total number of balls}}$$

$$= \frac{7}{15}$$

(c) There are 5 balls having number multiple of 3.

P(drawing a multiple of three)

$$= \frac{\textit{Number of favourable balls to the event}}{\textit{total number of balls}}$$

$$= \frac{5}{15} = \frac{1}{3}$$

2. Total number of boys = 40 + 60 + 20 + 30
= 150

Total number of girls = 30 + 50 + 15 + 35
= 130

Total number students = 280

(a) Let E_1 be event of getting a student interested in sports only. Then, the number of favourable outcome = 40 + 30 = 70

$$\therefore\ P(E_1) = \frac{70}{280} = \frac{1}{4}$$

(b) (i) Let E_2 be event of getting a student who is interested neither in sports nor in music. Then, the number of favourable outcome = 30 + 35 = 65

$$\therefore\ P(E_2) = \frac{65}{280} = \frac{13}{56}$$

Or

(ii) Let E_3 be event of selecting a girl who is interested in music only. Then, the number of favourable outcome = 50

$$\therefore\ P(E_3) = \frac{50}{280} = \frac{5}{28}$$

(c) Let E_4 be event of selecting a boy who is interested in either sports or music. Then, the number of favourable outcome = 40 + 60 = 100

$$\therefore\ P(E_4) = \frac{100}{280} = \frac{5}{14}$$

3. (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Total number of outcomes = 36

(a) No. of outcomes where sum is 8 = 5

∴ Probability that sum of 2 numbers is 8

$$= \frac{\textit{Number of outcomes where sum is 8}}{\textit{Total number of outcomes}} = \frac{5}{36}$$

(b) (i) No. of outcomes where sum is 13, 0.

∴ Probability that sum of 2 numbers is 13

$$= \frac{\textit{Number of outcomes where sum is 13}}{\textit{Total number of outcomes}}$$

$$= \frac{0}{36} = \mathbf{0}$$

Or

(ii) Number of outcomes where sum is less than or equal to 12 = 36.

∴ Probability that sum of two numbers is less than or equal to 12

$$= \frac{\textit{Number of outcomes where sum is less than or equal to 12}}{\textit{Total number of outcomes}}$$

$$= \frac{36}{36} = \mathbf{1}$$

(c) Number of outcomes where sum is greater than 8 = 4 + 3 + 2 + 1 = 10

∴ Probability that sum of two numbers is greater than 8

$$= \frac{\textit{Number of outcomes where sum is greater than 8}}{\textit{Total number of outcomes}}$$

$$= \frac{10}{36} = \frac{5}{18}$$

(DAY 7 SWAHA)

*****

4 Circles

What did CBSE ask last year?

MCQs & A/R	2 Questions (2 × 1 = 2 Marks)
Subjective	No Very Short Questions
	2 Short Questions (2 × 3 = 6 Marks)
	No Long Questions
Case Based	No Case Based Questions

Note: All the above typology of questions include 'Competency based Questions' labelled as **COMPETENCY**.

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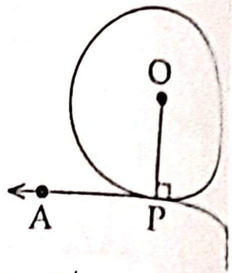
Introduction to the Chapter

- Tangent

  When a line meets the circle at one point or two coinciding points. The line is known as a tangent.

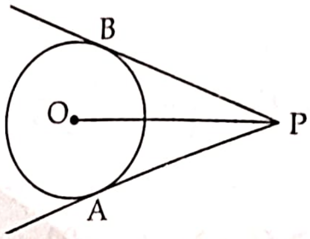
  The tangent to a circle is perpendicular to the radius through the point of contact.



  $\Rightarrow \quad OP \perp AB$

  The lengths of the two tangents from an external point to a circle are equal.

  $\Rightarrow \quad AP = PB$

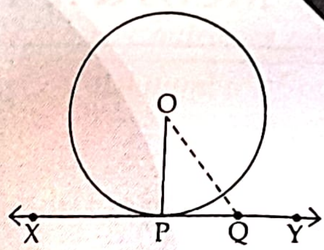


- **Length of Tangent Segment**

  PB and PA are normally called the lengths of tangents from outside point P.

- **Properties of Tangent to Circle**

  Theorem 1: Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.



  Given: XY is a tangent at point P to the circle with centre O.

  To prove: $OP \perp XY$

  Construction: Take a point Q on XY other than P and join OQ.

  Proof: If point Q lies inside the circle, then XY will become a secant and not a tangen the circle.

  $\therefore \quad OQ > OP$

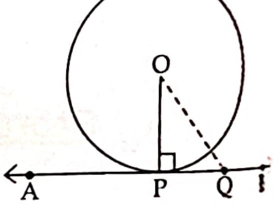
  This happens with every point on the line XY except the point P.

  OP is the shortest of all the distances of the point O to the points of XY.

  $\therefore \quad OP \perp XY$ ...[$\because$ Shortest side is the perpendicu

  Theorem 2: A line drawn through the end point of a radius and perpendicular to it the tangent to the circle.

  Given: A circle C(O, r) and a line APB is perpendicular to OP, where OP is radius.



  To prove: AB is tangent at P.

  Construction: Take a point Q on the line AB, different from P and join OQ

  Proof: Since $OP \perp AB$

  $\therefore \quad OP < OQ \quad \Rightarrow \quad OQ > OP$ ...[Giv

  $\therefore$ The point Q lies outside the circle.

  Therefore, every point on AB, other than P, lies outside the circle. This shows that A meets the circle at the point P.

  Hence, AP is a tangent to the circle at P.

Tangents

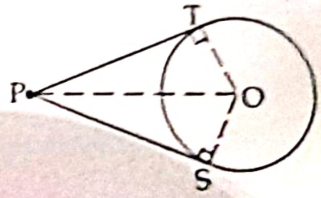
❑ **Theorem 3:** Prove that the lengths of tangents drawn from an external point to a circle are equal.

Given: PT and PS are tangents from an external point P to the circle with centre O.

To prove: PT = PS

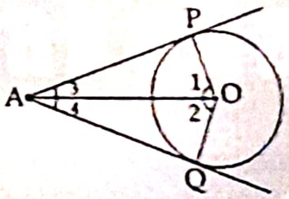
Construction: Join O to P, T and S.

Proof: In $\angle$OTP and $\angle$OSP



	OT = OS	...[Radii of same circle
	OP = OP	...[Common
	$\angle$OTP = $\angle$OSP	...[Each 90°
$\therefore$	ΔOTP $\cong$ ΔOSP	...[R.H.S.
$\therefore$	PT = PS	...[c.p.c.t.

***Note:** If two tangents are drawn to a circle from an external point, then:*



(i) They subtend equal angles at the centre,
i.e., $\angle 1 = \angle 2$.

(ii) They are equally inclined to the segment joining the centre to that point,
i.e., $\angle 3 = \angle 4$
i.e., $\angle OAP = \angle OAQ$

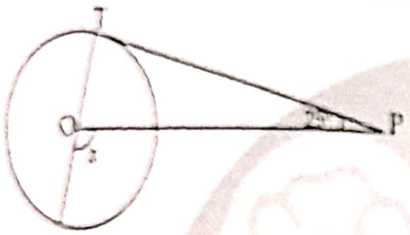
***Note:** CBSE loves questions that are based on 'figures circumscribing another figure'.*

OBJECTIVE QUESTIONS

(DAY 8)

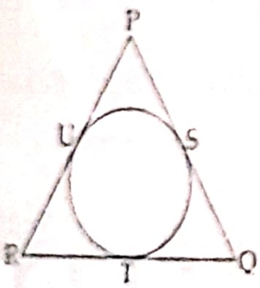
Multiple Choice Questions

Q.1. In the given figure, PT is a tangent at T to the circle with centre O.
If $\angle TPO = 25^\circ$, then x is equal to: **COMPETENCY**



(a) 25° (b) 65°
(c) 90° (d) 115°

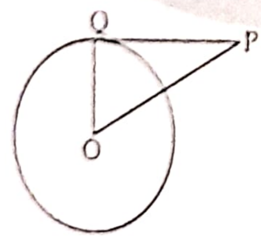
Q.2 In the given figure, ΔPQR is an isosceles triangle with PQ = PR, and the lengths of PU and UR are 5 units and 3 units respectively. **COMPETENCY**



(Note: The figure is not to scale.)
Which of the following is TRUE?

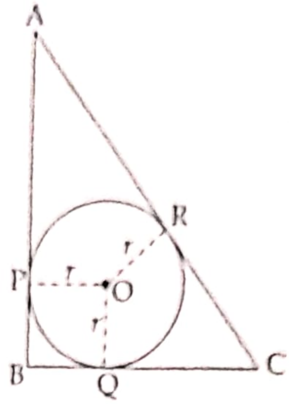
(a) PS = 3 units (b) SQ = 5 units
(c) QT = 3 units (d) QR = 8 units

Q.3. In the given figure, PQ is a tangent to the circle with centre O. If $\angle OPQ = x$, $\angle POQ = y$, then $x + y$ is: **COMPETENCY**



(a) 45° (b) 90°
(c) 60° (d) 180°

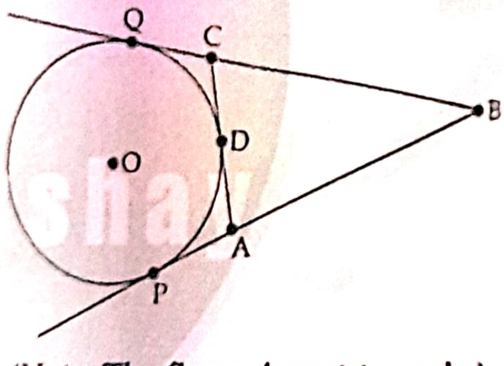
Q.4. ABC is right triangle, right angled at B, with BC = 6 cm and AB = 8 cm. A circle with centre O and radius r has been inscribed in triangle ABC as shown in the figure. Find the value of r.
[CBSE 2021



(a) 4 cm (b) 2 m
(c) 5 cm (d) 2 cm

FREE ADVICE: Answer crosscheck ke liye aap log pythagoras theorem ka use kar sakte hain.

Q.5. In the given, figure ΔABC is formed using three tangents to a circle centred at O.



(Note: The figure is not to scale.)
Based on the construction, which of the following statements is true?
[CBSE 2024

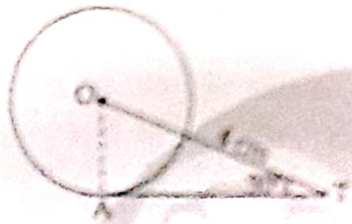
(a) The sum of the length of BP and BQ is less than the perimeter of ΔABC.
(b) The sum of the length of BP and BQ is same as the perimeter of ΔABC.
(c) The sum of the length of BP and BQ is greater than the perimeter of ΔABC.
(d) Length of the tangents should be known to compare it to the perimeter of ΔABC.

Q.6. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is: COMPETENCY

(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

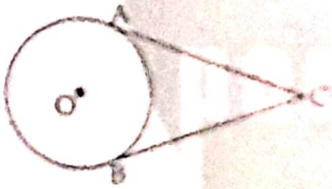
FREE ADVICE: Since AXY is a right angled triangle, we can use the Pythagorean theorem to find the length of XY (the base of the right triangle).

Q.7. In the given figure, TA is a tangent to the circle with centre O such that OT = 4 cm, ∠OTA = 30°, then length of TA is: COMPETENCY



(a) 4 cm (b) 2 cm
(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

Q.8. A circle with center O is shown here, where CA and CB are tangents to the circle.



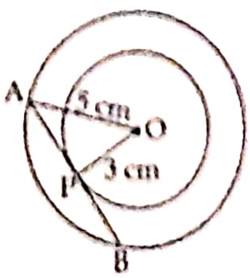
(Note: Figure is not to scale)
If measure of ∠ACB = 50°, find the measure of ∠AOB. [CBSE 2024]

(a) 40° (b) 50° (c) 130° (d) 140°

Q.9. Two concentric circles have radii 10 cm and 6 cm. Find the length of the chord of the larger circle which touches the smaller circle. [CBSE 2021]

(a) 12 cm (b) 16 m
(c) 18 cm (d) 16 cm

Q.10. In given figure, the length PB = ____ cm. COMPETENCY



(a) 2 cm (b) 4 cm
(c) 6 cm (d) 8 cm

Q.11. PQ is a tangent to a circle with centre at the point P on the circle. If ΔOPQ is an isosceles triangle, then find ∠OQP. [CBSE 2021]

(a) 40° (b) 45°
(c) 50° (d) 75°

Q.12. In the given figure, O is the center of the circle. PQ, QR and RP are tangents of the circle. TS is parallel to QR.

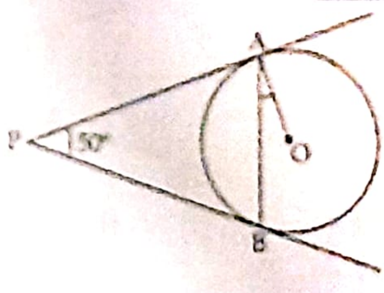
(Note: The figure is not to scale.) Which of these is the measure of ∠RPQ? COMPETENCY

(a) 30° (b) 55°
(c) 62.5° (d) 70°

Q.13. In the given figure, PQ is a chord of a circle and PT is tangent at P such that angle QPT = 60° then the measure of ∠PRQ is: COMPETENCY

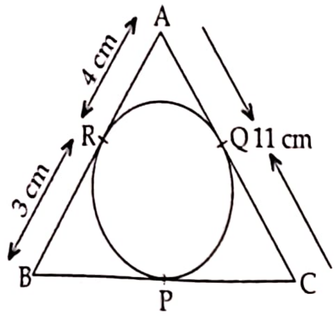
(a) 150° (b) 160° (c) 120° (d) 240°

Q.14. In the given figure, PA and PB are tangents to the circle with centre O such that ∠APB = 50°, then the measure of ∠OAB is: COMPETENCY



(a) 20° (b) 25° (c) 30° (d) 45°

Q.15. In the figure, ΔABC is circumscribing a circle, the length of BC is _____ cm. [CBSE 2020



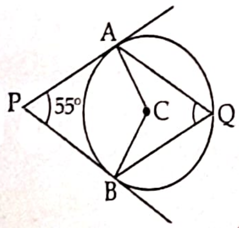
(a) 10 cm (b) 20 cm
(c) 15 cm (d) 25 cm

Q.16. From an external point P, tangent PA and PB are drawn to a circle with centre O. If angle PAB = 50°, find $\angle AOB$. [CBSE 2016

(a) 100° (b) 180° (c) 160° (d) 120°

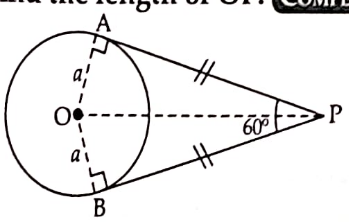
FREE ADVICE: Since PA and PB are tangents to the circle, they are both perpendicular to the radius drawn from the center of the circle to the point of tangency, which is point A and B in this case.

Q.17. In the given figure, PA and PB are tangents from external point P to a circle with centre C and Q is any point on the circle. Then the measure of $\angle AQB$ is: **COMPETENCY**



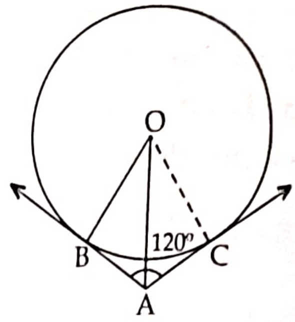
(a) $62\frac{1}{2}°$ (b) 125° (c) 55° (d) 90°

Q.18. If the angle between two tangents drawn from an external point P to a circle of radius r and centre O, is 60°, then find the length of OP. **COMPETENCY**



(a) a (b) a^2
(c) $2a$ (d) $4a$

Q.19. In the given figure, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC$ = 12... then OA is equal to: **COMPETENCY**



(a) 2AB (b) AB (c) AB^2 (d) 3AB

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*
(b) *Both (A) and (R) are true, but (R) is not the correct explanation of (A).*
(c) *(A) is true and (R) is false.*
(d) *(A) is false, but (R) is true.*

Q.1. **Assertion (A):** All angles formed by a chord on the same side of the circumference of a circle are equal to each other.
Reason (R): The sum of any two angles formed by a chord on the opposite sides of the circumference of a circle is 180°. **COMPETENCY**

Q.2. **Assertion (A):** Perpendicular bisectors of two chords of a circle intersect at its centre.
Reason (R): A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

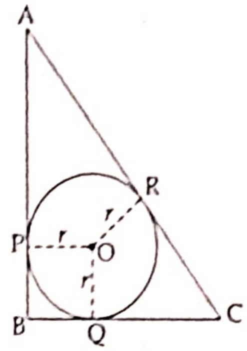
Q.3. **Assertion (A):** Angles in the same segment of a circle are equal.
Reason (R): In cyclic quadrilateral, opposite angles are supplementary. **COMPETENCY**

Q.4. **Assertion (A):** The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre. **COMPETENCY**
Reason (R): A parallelogram circumscribing a circle is a rhombus.

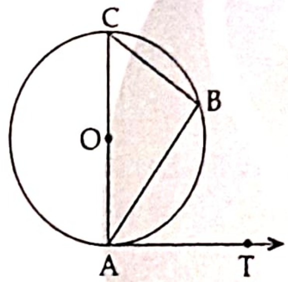
SUBJECTIVE QUESTIONS

— Very Short Answer Questions —

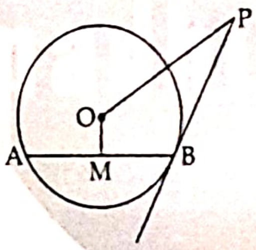
Q.1. ABC is right triangle, right angled at B, with BC = 6 cm and AB = 8 cm. A circle with centre O and radius r has been inscribed in triangle ABC as shown in the figure. Find the value of r. *[CBSE 2021*



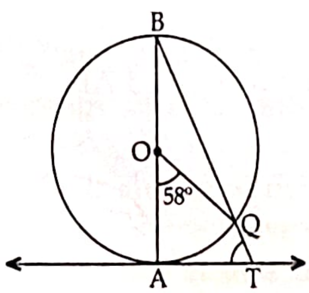
Q.2. In given figure AB is a chord of circle with centre O, AOC is diameter and AT is tangent at A. Prove that
$\angle BAT = \angle ACB$ *[CBSE 2020*



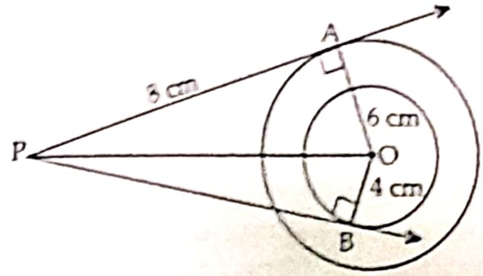
Q.3. PB is a tangent to the circle with center O. AB is a chord of length 24 cm at a distance of 5 cm from the center. If the tangent is of length 20 cm, find the length PO. **COMPETENCY**



Q.4. In figure, AB is the diameter of a circle with center O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$. **COMPETENCY**

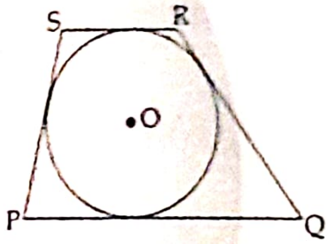


Q.5. In the figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to larger circle, and BP to the smaller circle and length of AP is 8 cm, find length of BP. *[CBSE 2019*



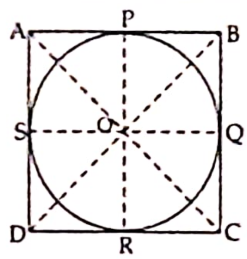
FREE ADVICE: *Answer confirm karne ke liye aap log trigonometric ratio ka bhi use kar sakte hain.*

Q.6. Shown below is a circle O inscribed in a trapezium such that PQ $\parallel$ RS. The radius of the circle is 6 cm.

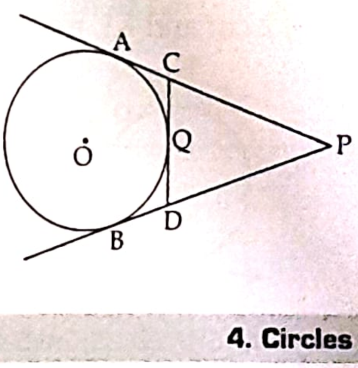


(Note: The figure is not to scale.)
What is the height of the trapezium? Show your work and give valid reasons. **COMPETENCY**

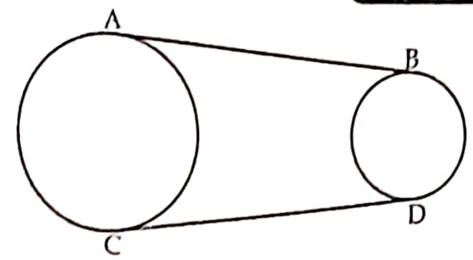
Q.7. A circle touches all the four sides of a quadrilateral ABCD, prove that
AB + CD = AD + BC. *[CBSE 2017*



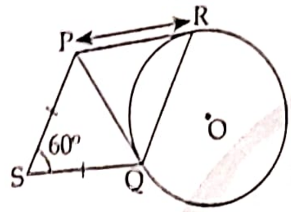
Q.8. In the given figure, PA and PB are tangents to the circle from an external point P. CD is tangent touching circle at Q. If PA = 12 cm, QC = QD = 3 cm then find PC + CD. **COMPETENCY**



Q.9. In the figure, AB and CD are common tangents to two circles of unequal radii. Prove AB = CD. **COMPETENCY**



Q.10. In the figure below, PQ and PR are tangents to the circle.

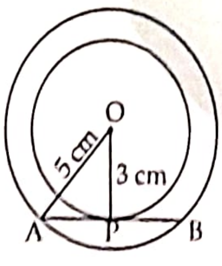


(Note: The figure is not to scale.)
Find the perimeter of ΔSQP. Show your work. **COMPETENCY**

(DAY 9)

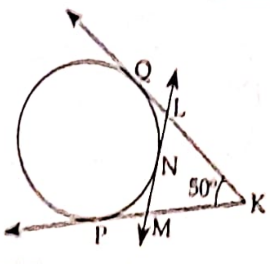
Short Answer Questions

Q.1. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. *[CBSE 2023]*



Q.2. Prove that, a tangent to a circle is perpendicular to the radius through the point of contact. *[CBSE 2020]*

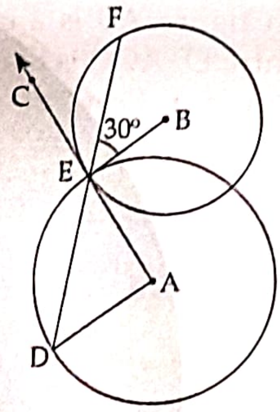
Q.3. Shown below is a circle with 3 tangents KQ, KP and LM. **COMPETENCY**
QL = 2 cm and KL = 6 cm



(Note: The figure is not to scale.)
If PM : KL = 1 : 2, find the measure of ∠LMK. Show your steps. **COMPETENCY**

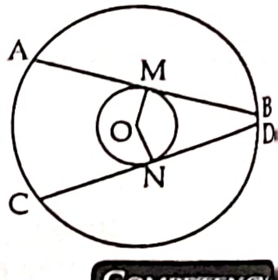
Q.4. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre. *[CBSE 2023]*

Q.5. Shown below are two circles with centres A and B. The circle with centre A has a radius of 5 cm. AC is a tangent to circle with centre B. FED is a straight line.



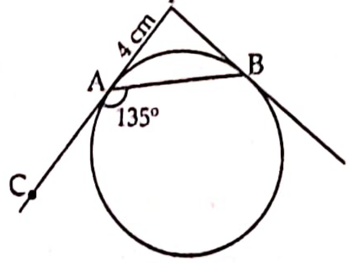
(Note: The figure is not to scale.)
What is the length of chord DE? Show your work and give valid reasons. *[CBSE 2024]*

Q.6. In two concentric circles, prove that all chords of the outer circle which touch the inner circle, are of equal length.



COMPETENCY

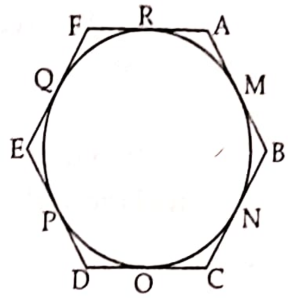
Q.7. In the figure, PA and PB are tangents to a circle from an external point P such that PA = 4 cm, angle BAC = 135°. Find length of chord AB.



[CBSE 2024]

Q.8. If a hexagon ABCDEF circumscribes a circle, prove that AB + CD + EF = BC + DE + FA.

COMPETENCY

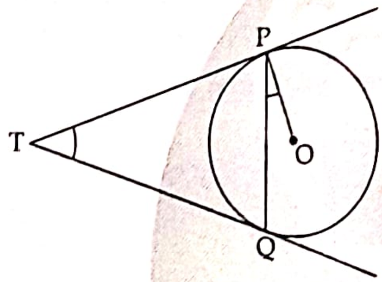


FREE ADVICE: *Isme Tangent property use kar ke aasani se solve kar sakte hai.*

Long Answer Questions

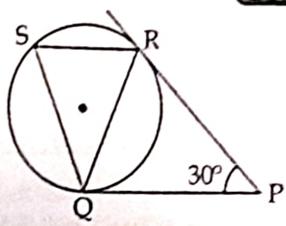
Q.1. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

[CBSE 2023



Q.2. In the given figure, tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find the measure of $\angle RQS$.

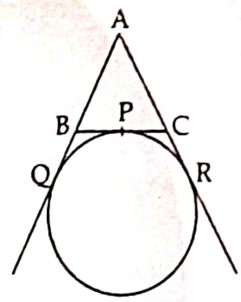
COMPETENCY



Q.3. A circle touches the side BC of a ΔABC at a point P and touches AB and AC when produced at Q and R respectively. Show that

$$AQ = \frac{1}{2} \text{ (Perimeter of } \Delta ABC)$$

COMPETENCY



Q.4. Prove that the parallelogram circumscribing a circle is rhombus.

[NCERT Exemplar

CASE BASED QUESTIONS

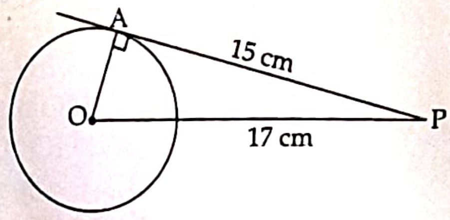
Q.1. In an online test, Akshay comes across the statement. "If a tangent is drawn to a circle from an external point, then the square of length of tangent drawn is equal to difference of squares of distance of the tangent from the centre of circle and radius of the circle."

Help Akshay, in answering the following questions based on the above statement.

(*a*) If AB is a tangent to a circle with centre O at B such that AB = 10 cm and OB = 5 cm, then find OA.

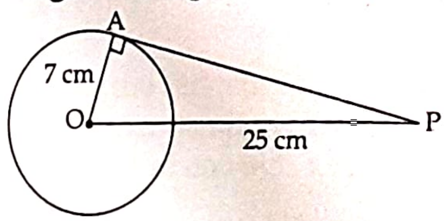
COMPETENCY

(*b*) (*i*) In the adjoining figure, find the radius of the circle.

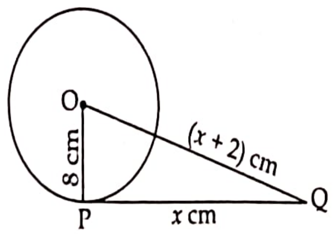


Or

(*ii*) In the adjoining figure, find the length of tangent AP.



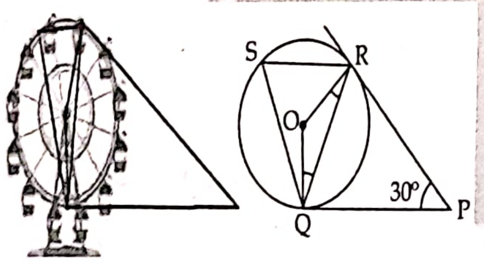
(c) In the adjoining figure, what is the length of the tangent? **COMPETENCY**



Q.2. A Ferris wheel (or a Giant wheel) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Priya came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as shown in the question.

Based on the above information, answer the following questions:



(a) In the given figure, find ∠ROQ.

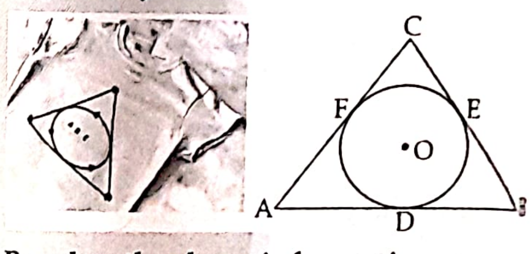
(b) (i) Find ∠RQP.

Or

(ii) Find ∠RSQ

(c) Find reflex ∠ROQ. **COMPETENCY**

Q.3. Ayan has been selected by his School to design logo for Sports Day T-shirts for students and staff . The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.



Based on the above information, answer the following questions.

(a) Find the length of AD.

(b) (i) Find the Length of BE.

Or

(ii) Find the length of CF.

(c) If radius of the circle is 4cm, Find the area of ΔOAB. **COMPETENCY**

*****

ANSWERS

Multiple Choice Answers

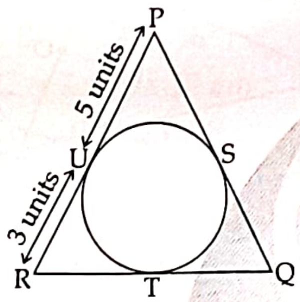
1. (*d*) Since tangent is perpendicular to radius at the point of contact.

$\therefore$ $\angle PTO = 90°$

Hence, by the exterior angle formula, in ΔOTP, we get

$x = 90° + 25° = \mathbf{115°}$

2. (*c*)



$PU = PS = 5$ units

...[$\because$ tangents drawn from an external point are equals

$PR = PQ$

$PR = PS + SQ$

$\Rightarrow 8 = 5 + SQ$

$\Rightarrow SQ = 3$ units

$\therefore$ **SQ = QT = 3 units**

...[$\because$ tangents drawn from an external point are equal

3. (*b*) Since the sum of angles in a triangle is 180°, we have

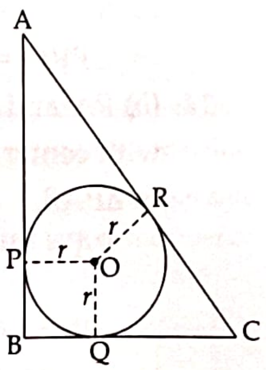
$x + \angle OQP + y = 180°$

$\Rightarrow x + y + 90° = 180°$

...[Here $\angle OQP = 90°$ (Tangent is $\perp$ to the radius)

$\therefore$ $\mathbf{x + y} = 180° - 90° = \mathbf{90°}$

4. (*d*) **Given:** A circle with centre O has been inscribed inside the triangle with BC = 6 cm and AB = 8 cm.



$AR = AP$

...[Tangents of circle

and $AP = AB - BP$

$= (8 - r)$ cm ...(*i*)

Again $CR = CQ$ ...[Tangents of circle

and $CQ = CB - BQ$

$= (6 - r)$ cm

$\therefore$ $AC = AR + RC$

$= 8 - r + 6 - r = 14 - 2r$ cm

By pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$(14 - 2r)^2 = 8^2 + (6)^2$

$= 64 + 36 = 100$

Taking squaring root both sides, we get

$14 - 2r = 10$

$14 - 10 = 2r$

$\therefore$ $r = 2$ cm

Hence, radius of circle (*r*) = 2 cm.

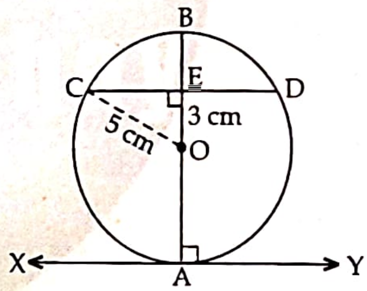
5. (*b*) The sum of the length of BP and BQ is same as the perimeter of ΔABC.

6. (*d*) We know that, $AE = AO + OE$

$8 = 5 + OE$

$OE = 8 - 5$

$\therefore$ $OE = 3$ cm



OE is perpendicular to the chord CD.

So, $\angle OEC = 90°$

$CE = DE$

In triangle OED,

$OC^2 = OE^2 + CE^2$

$(5)^2 = (3)^2 + CE^2$

$CE^2 = 25 - 9 = 16$

$CE = 4$ cm

$\therefore$ **Length of the chord, CE** = 2(CE)

= 2(4) = **8 cm**

7. (*c*) Join OA.

We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

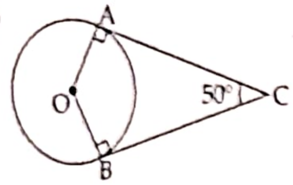
$\therefore$ $\angle OAT = 90°$

In ΔOAT, $\cos 30^\circ = \frac{AT}{OT}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4}$

$\therefore$ AT = $2\sqrt{3}$ cm

8. (c)



As, OA $\perp$ AC and OB $\perp$ BC

...[$\because$ Radius is perpendicular to tangent through point of contact.

In Quadrilateral BOAC

$\angle OAC + \angle ACB + \angle CBO + \angle AOB = 360^\circ$

...[$\angle$s sum property of quadrilateral

$\Rightarrow 90^\circ + 50^\circ + 90^\circ + \angle AOB = 360^\circ$

$\Rightarrow 230^\circ + \angle AOB = 360^\circ$

$\therefore \angle AOB = 360^\circ - 230^\circ = 130^\circ$

9. (d) Let the two concentric have the center O and Let AB be the chord of an outer circle whose length is 10 cm and which will also be tangent to the inner circle at point D because it is given that the chord touches the inner circle.

Using Pythagoras theorem,

$OD^2 + BD^2 = OB^2$

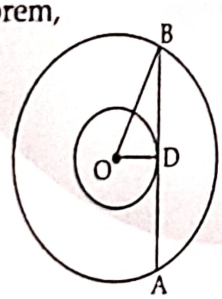
$\Rightarrow (6)^2 + BD^2 = (10)^2$

$\Rightarrow 36 + BD^2 = 100$

$\Rightarrow BD^2 = 100 - 36$

$\Rightarrow BD^2 = 64$

$\Rightarrow BD = \pm 8$



As length cannot be negative.

$\Rightarrow$ BD = 8 cm

$\Rightarrow AB = 2 \times 8$ ...[Since AB = 2BD

$\therefore$ AB = 16 cm

10. (b) In ΔAOP,

Using Pythagoras theorem,

$(AO)^2 = (AP)^2 + (OP)^2$

$\Rightarrow 25 = (AP)^2 + 9$

$\therefore$ AP = 4 cm

...[AP = PB $\perp$ from centre, bisects the chord.

$\therefore$ PB = 4 cm

11. (b) ΔOPQ is isosceles whose vertex is P

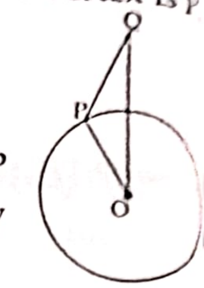
$\therefore$ OP = PQ

$\Rightarrow \angle OQP = \angle QOP$

$\Rightarrow \angle OQP + \angle QOP$

$= 2\angle OQP$

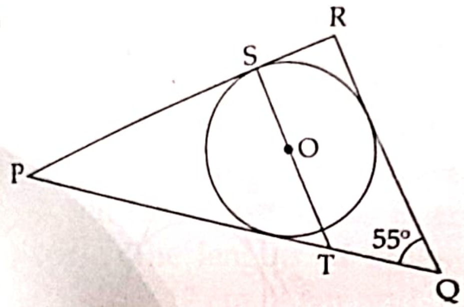
$\therefore$ By angle sum property of triangle,



$\therefore \angle OPQ + 2\angle OQP = 180^\circ$

$\Rightarrow 90^\circ + 2\angle OQP = 180^\circ \Rightarrow$ **OQP = 45°**

12. (a)



As, ST $\parallel$ RQ

$\Rightarrow \angle RQT = \angle STP = 55^\circ$

...[Corresponding angles are equal

$\Rightarrow$ OS $\perp$ PR

$\Rightarrow \angle OSP = 90^\circ$

...[Radius is perpendicular to tangent through point of contact

Now, In ΔSPT by using angle sum property of triangle,

$\angle SPT + \angle STP + \angle PST = 180^\circ$

...[Angle-sum property of triangles

$\Rightarrow \angle SPT + 55^\circ + 90^\circ = 180^\circ$

$\Rightarrow \angle SPT + 145^\circ = 180^\circ$

$\Rightarrow \angle SPT = 180^\circ - 145^\circ$

$\therefore \angle RPQ = 35^\circ$

13. (c) $\angle PSQ + \angle PRQ = 180^\circ$

$\Rightarrow 60^\circ + \angle PRQ = 180^\circ$

$\Rightarrow \angle PRQ = 120^\circ$

14. (b) PA and PB are tangent to the circle with center O.

In ΔPAB,

PA = PB ...[Length of tangent from external point to circle are equal

$\angle PBA = \angle PAB$ ...[Isosceles triangle

Now $\angle PAB + \angle PBA + \angle APB = 180^\circ$

$2\angle PAB = 180^\circ - 50^\circ = 130^\circ$

$\angle PBA = \angle PAB = 65^\circ$ ...(i)

Now PA is tangent and OA is radius at point A.

$\angle OAP = 90^\circ$ ...[Tangent at any point is $\perp$ to radius

$\angle OAB = \angle OAP - \angle PAB$

$= 90^\circ - 65^\circ = 25^\circ$

Hence $\angle OAB$ is 25°.

15. **(*a*)** $CQ = AC - AQ = 11 - 4 = 7$ cm

$CQ = CP = 7$ cm ...[Tangents from the same point are equal

$BR = BP = 3$ cm ...[Tangents from the same point are equal

Now, $BC = CP + BP$

$BC = 7 + 3 = 10$ cm

16. **(*a*)** Given that $\angle PAB$ is 50°,

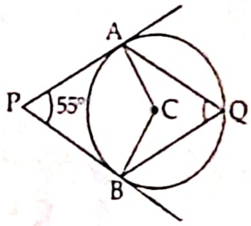
$\angle AOB$ (the angle at the center of the circle) is twice this value.

$\Rightarrow AOB = 2\angle PAB$

$\Rightarrow AOB = 2 \times 50^\circ$

$\therefore$ **$\angle AOB = 100^\circ$**

17. **(*a*)** In quadrilateral CAPB, we have



$\angle APB = 55^\circ$

$\angle CAP = \angle CBP = 90^\circ$

$\Rightarrow \angle ACB = 125^\circ$

Now, we know that angle subtended at the center is twice the angle subtended at any point on the arc, so we have

$\angle AQB = \frac{1}{2}\angle ACB$

$= \frac{1}{2} \times 125^\circ = 62\frac{1}{2}^\circ$

18. **(*c*)** $AP = BP$ ...[Length of tangents from external point to circle are equal

$\angle A = \angle B = 90^\circ$ ...[Tangent is $\perp$ to radius

$OP = OP$ ...[Common side

$\therefore \Delta AOP \cong \Delta BOP$ ...[RHS test of congruence

$\angle APO = \angle BPO = 30^\circ$

$\angle AOP = \angle BOP = 60^\circ$

In ΔAOP,

$\cos 60^\circ = \frac{OA}{OP}$

$\therefore$ **$OP = 2a$**

19. **(*a*)** $\angle BAC = 120^\circ$ ...[Given

$AB = AC$ ...[Tangent Property

$OB = OC$ ...[Radius

$\Rightarrow \angle BAO = \angle CAO = 60^\circ$ ...[$\angle$Bisector

In ΔABO,

$\cos A = \frac{AB}{OA}$

$\Rightarrow \cos 60^\circ = \frac{AB}{OA}$

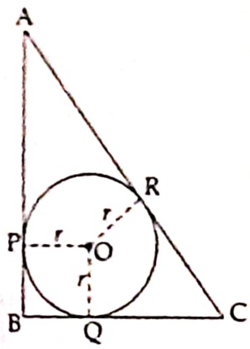
$\therefore OA = 2AB$

Assertion Reason Answers

1. (*a*) Both (A) and (R) are true and (R) is the correct explanation for (A).
2. (*b*) Both (A) and (R) are true and (R) is not the correct explanation of (A).
   Explanation: *The perpendicular bisectors of chords indeed intersect at the centre of the circle, and a line drawn from the centre to bisect a chord is always perpendicular to that chord.*
3. (*b*) Both (A) and (R) are true and (R) is not the correct explanation of (A).
4. (*b*) Both (A) and (R) are true and (R) is not the correct explanation of (A).

Very Short Answers

1. In right ΔABC,



$\angle B = 90^\circ$, $BC = 6$ cm and $AB = 8$ cm

Let r be the radius circle whose centre is O and touches the sides AB, BC and CA at P, Q and R respectively.

Since AP and AR are the tangents to the circle, $AR = AP$.

Similarly, $CQ = CR$ and $BP = BQ$

Hence, BPOQ is a square.

Thus, $BP = BQ = r$ (sides of a square)

So, $AR = AP = AB - BP = 8 - r$ and

$CR = CQ = BC - BQ = 6 - r$

But $AC^2 = AB^2 + BC^2$

...[By Pythagoras Theorem

$= (8)^2 + (6)^2 = 64 + 36 = 100$

$= (10)^2$

So, AC = 10 cm

$\Rightarrow AR + CR = 10$

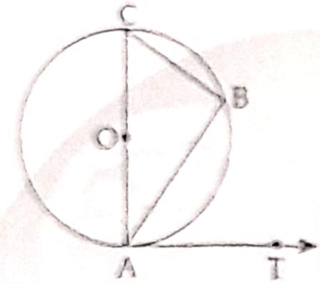
$\Rightarrow 8 - r + 6 - r = 10$

$\Rightarrow 14 - 2r = 10$

$\Rightarrow 2r = 14 - 10 = 4 \quad \Rightarrow r = 2$

$\therefore$ **The radius of the circle = 2 cm**

2. We know that,



$\angle BAC + \angle ACB = 90^\circ$

$\Rightarrow \angle ACB = 90^\circ - \angle BAC$ ...(i)

According to the property of triangle, the radius of the circle is perpendicular to the tangent at the point of contact.

$OA \perp AT$

$\angle OAT = \angle CAT = 90^\circ$

From the figure,

$\angle CAT = \angle BAT + \angle BAC$

$\Rightarrow 90^\circ = \angle BAT + \angle BAC$

$\Rightarrow \angle BAT = 90^\circ - \angle CAB$ ...(ii)

Comparing (i) and (ii),

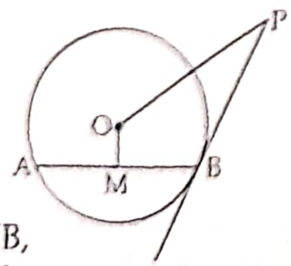
Since RHS are same

$\angle ACB = \angle BAT$

$\therefore$ It is proved that $\angle ACB = \angle BAT$.

3. **Given:** AB = 24 cm,

OM = 5 cm



So, $MB = \frac{1}{2} AB$

$= 12$ cm

In right-angled ΔOMB,

$OB^2 = OM^2 + MB^2$

$\Rightarrow OB^2 = (5)^2 + (12)^2$

$\Rightarrow OB^2 = 25 + 144$

$\Rightarrow OB^2 = 169$ cm

$\therefore$ OB = 13 cm

As BP is tangent to circle at B, $OB \perp BP$.

$OP^2 = OB^2 + BP^2$

$\Rightarrow OP^2 = (13)^2 + (20)^2$

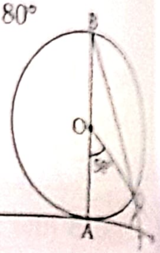
$\Rightarrow OP^2 = 169 + 400$

$\Rightarrow OP^2 = 569$

$\Rightarrow OP = \sqrt{569}$ cm

Hence, the length of OP = $\sqrt{569}$ cm

4. AB is the straight line.



$\angle AOQ + \angle BOQ = 180°$

$\angle BOQ = 180° - 58°$

$= 122°$

In triangle BOQ,

OB = OQ

...[OB = OQ given

So $\angle OBQ = \angle OQB$

...[Since angle opposite to equal sides are equal

$\Rightarrow \angle OBQ + \angle OQB + \angle BOQ = 180°$

$\Rightarrow 122° + 2(\angle OBQ) = 180°$

$\therefore \angle OBW = 29°$

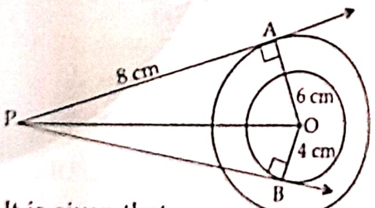
In ΔABT,

$\angle ABT + \angle BAT + \angle BTA = 180°$

$\Rightarrow 29° + 90° + \angle BAT = 180°$

$\therefore$ **$\angle ATQ = 61°$**

5.



It is given that

OA = 6 cm, OB = 4 cm, AP = 8 cm

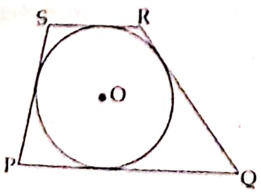
$OP^2 = OA^2 + AP^2 = 36 + 64 = 100$

OP = 10 cm

$BP^2 = OP^2 - OB^2 = 100 - 16 = 84$

$\therefore$ **BP = $2\sqrt{21}$ cm**

6.



Given.

(i) PQRS is a trapezium

(ii) PQ || SR

(iii) Radius of circle = 6 cm

To Find. Height of the trapezium.

Proof. As, Diameter is the shortest distance between PQ and RS

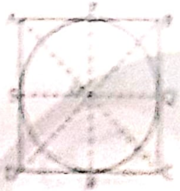
Height of trapezium = Diameter of circle

$= 2 \times$ Radius

$= 2 \times 6 = 12$ cm

Hence, Height of trapezium = 12 cm

7. **Given:** Circle touching sides of ABCD at P, Q, R and S.



To prove: AB + CD = AD + BC

Proof: AP = AS ...(i)

...[Tangents from an external point to a circle are equal in length

PB = BQ ...(ii)

DR = DS ...(iii)

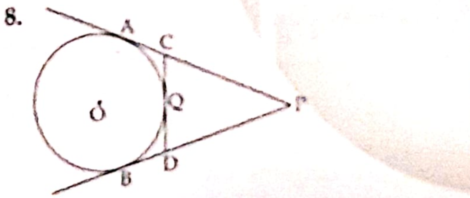
CR = CQ ...(iv)

Adding equation (i), (ii), (iii) & (iv)

AP + BP + DR + CR = AS + DS + BQ + CQ

AB + DC = AD + BC (Hence proved)

8.



Given that PA = PB = 12 cm (length of tangent)

Similarly,

QC = AC = 3 cm

QD = BD = 3 cm

PC = PA − AC = 12 − 3 = 9 cm

Similarly,

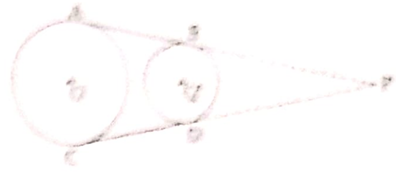
PD = PB − BD = 12 − 3 = 9 cm

Hence, PC + PD = 9 + 9 = 18 cm

9. **Given:** AB and CD are common tangents.

To Prove: AB = CD

Construction: Extend the tangents AB and CD to point P.



Proof: The tangents are PB and PD

So, PB = PD ...(i)

Considering the larger circle, the tangents are PA and PC

So, PA = PC ...(ii)

Subtracting (i) and (ii),

PA − PB = PC − PD

From the figure,

PA − PB = AB

PC − PD = CD

$\therefore$ **AB = CD (Hence Proved)**

10.

As, PQ = PR = 5 cm

and $\angle PSQ = 60°$ and $\angle SPQ = \angle PQS = x$

...[Angle opposite to equal sides are equal

So, in ΔPSQ by angle sum property of Δ,

$\angle PSQ + \angle PQS + \angle SPQ = 180°$

$\Rightarrow 60° + x + x = 180°$

$\Rightarrow 2x = 180° - 60°$

$\Rightarrow 2x = 120°$

$x = 120°/2 = 60°$

As each angle measure 60° hence ΔPSQ is an equilateral triangle.

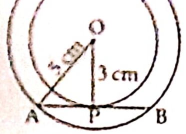
PS = SQ = PQ = 5 cm

$\therefore$ **Perimeter of $\Delta SQP = 3 \times 5 = 15$ cm**

Short Answers

1. In ΔOPA (Right-angled triangle)

By the Pythagoras Theorem,



$OA^2 = OP^2 + PA^2$

$(5)^2 = (3)^2 + PA^2$

$PA^2 = 25 - 9$

$PA^2 = 16$

$PA = \pm 4$

PA is the length of the tangent and cannot be negative.

Hence, PA = 4 cm.

PB = PA

...[Perpendicular from center bisects the chord considering QP to be the larger circle's chord

Therefore, PA = PB = 4 cm

Length of the chord, AB = PA + PB

= 4 + 4 = 8

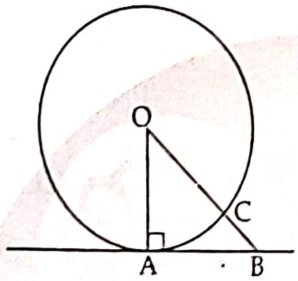
AB = 8 cm

Therefore, the length of the chord of the larger circle is 8 cm.

2. In the figure,

OA = OC

...[Radii of circle



Now,

OB = OC + BC

$\therefore$ OB > OC

...[OC is radii and CB is extra length in OC

$\Rightarrow$ OA < OB

B is an arbitrary point on the tangent.

Thus, OA is shorter than any other line segment joining O to any point on tangent.

Shortest distance of a point from a given line is the perpendicular distance from that line.

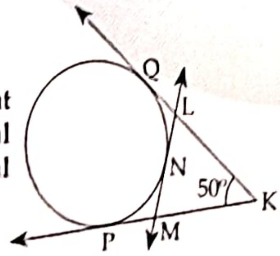
Hence, the tangent at any point of circle is perpendicular to the radius.

3. **Given.** QL = 2 cm, KL = 6 cm

KQ = 6 + 2 = 8 cm

KP = KQ

...[The lengths of tangent drawn from an external point to a circle are equal



$\Rightarrow$ KP = 8 cm

$PM = \frac{1}{2} \times KL$

$\Rightarrow PM = \frac{1}{2}(6) = 3$ cm

MK = PK - 3 = 8 - 3 = 5 cm

Now, PM = MN = 3 cm

QL = LN = 2 cm

LM = 3 + 2 = 5 cm

In ΔKLM

MK = 5 cm, LM = 5 cm and KL = 6 cm

$\therefore$ ΔKLM is an isosceles triangle.

So, ∠MKL = ∠KLM

∠MKL + ∠KLM + ∠LMK = 180°

...[Angle sum property of triangle

$\Rightarrow$ 50° + 50° + ∠LMK = 180°

$\Rightarrow$ 100° + ∠LMK = 180°

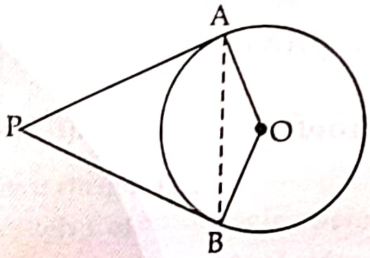
$\therefore$ ∠LMK = 180° – 100° = **80°**

4. Let us consider O as the centre point of the circle.

Construction: Draw a line segment between points A and B such that it subtends ∠AOB at centre O of the circle.

Here ∠OAP = ∠OBP = 90° ...[From (i)

In a quadrilateral, the sum of interior angles is 360°.



In OAPB,

∠OAP + ∠APB + ∠PBO + ∠BOA = 360°

Using Equation (*i*), we can write the above equation as

90° + ∠APB + 90° + ∠BOA = 360°

∠APB + ∠BOA = 360° – 180°

$\therefore$ ∠APB + ∠BOA = 180°

Where,

[∠APB = Angle between the two tangents PA and PB from external point P.

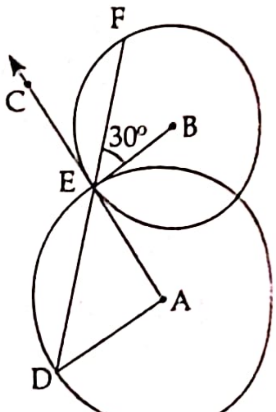
∠BOA = Angle subtended by the line segment AB joining the point of contacts at the centre.

Hence, proved the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

5. As, BE ⊥ AC

$\Rightarrow$ ∠BEC = 90°

...[Radius is perpendicular to tangent through point of contact.



$\Rightarrow$ ∠FEC = 90° – 30°

$\Rightarrow$ ∠FEC = 60°

$\Rightarrow$ ∠AED = ∠FEC

= 60°

...[Vertically opposite angles are equal

As, AE = AD ...[Radii of the same circle

$\Rightarrow \angle AED = \angle ADE = 60^\circ$

...[Angles opposite to equal sides are equal

So, In ΔAED,

$\angle AED + \angle EAD + \angle ADE = 180^\circ$

$\Rightarrow 60^\circ + \angle EAD + 60^\circ = 180^\circ$

$\Rightarrow 120^\circ + \angle EAD = 180^\circ$

$\Rightarrow \angle EAD = 180^\circ - 120^\circ$

$\therefore$ **$\angle EAD = 60^\circ$**

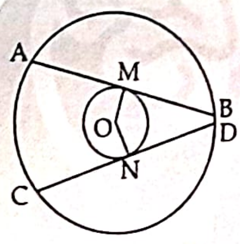
As, In ΔEAD, each angle measure = 60°

$\Rightarrow \Delta EAD$ is an equilateral triangle.

$\Rightarrow$ EA = AD = ED = 5 cm

$\therefore$ **Length of chord DE = 5 cm**

6. **Given:** Two concentric circles with centre O. Two chords AB and CD of the outer circle which touch the inner circle at M and N, respectively.



To prove: AB = CD

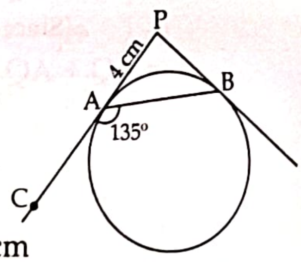
Proof: Clearly, AB and CD are tangents to the inner circle.

Thus, OM = ON = Radius of the inner circle

Thus, AB and CD are two chords of the outer circle which are equidistant from the centre.

$\therefore$ **AB = CD (Proved)**

7. It is given that PA and PB are tangents drawn from an external point P to the circle.



$\therefore$ PA = PB = 4 cm

...[Lengths of tangents drawn from an external point to a circle are equal

Also, $\angle BAC = 135^\circ$

Now, $\angle BAC + \angle PAB = 180^\circ$

$\therefore 135^\circ + \angle PAB = 180^\circ$

$\Rightarrow \angle PAB = 180^\circ - 135^\circ = 45^\circ$

In ΔPAB, PA = PB

$\therefore \angle PBA = \angle PAB = 45^\circ$

...[Equal sides have equal angles opposite to them

Also, $\angle PBA + \angle PAB + \angle APB = 180^\circ$

$\Rightarrow 45^\circ + 45^\circ + \angle APB = 180^\circ$

$\Rightarrow \angle APB = 180^\circ - 90^\circ = 90^\circ$

So, ΔPAB is a right angled triangle at P.

Using Pythagoras theorem, we have

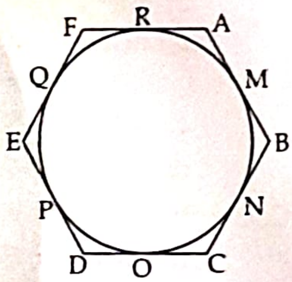
$AB^2 = PA^2 + PB^2$

$\Rightarrow AB = \sqrt{[(4)^2 + (4)^2]} = \sqrt{32} = 4\sqrt{2}$

Thus, the length of the chord AB is $4\sqrt{2}$ cm.

8. **Given:** A Hexagon ABCDEF circumscribes a circle.

To prove: AB + CD + EF = BC + DE + FA



Proof: Tangents drawn from an external point to a circle are equal.

Hence, we have

AM = RA ...(*i*) [Tangents from point A

BM = BN ...(*ii*) [Tangents from point B

CO = NC ...(*iii*) [Tangents from point C

OD = DP ...(*iv*) [Tangents from point D

EQ = PE ...(*v*) [Tangents from point E

QF = FR ...(*vi*) [Tangents fro`m point F

[eq. (*i*)] + [eq. (*ii*)] + [eq. (*iii*)] + [eq. (*iv*)] + [eq. (*v*)] + [eq. (*vi*)]

AM + BM + CO + OD + EQ + QF

= RA + BN + NC + DP + PE + FR

On rearranging, we get,

(AM + BM) + (CO + OD) + (EQ + QF)

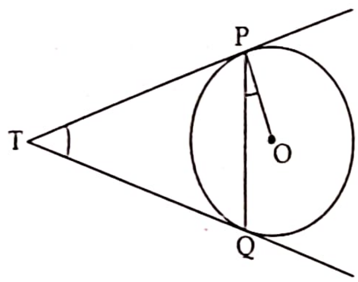
= (BN + NC) + (DP + PE) + (FR + RA)

AB + CD + EF = BC + DE + FA

Hence Proved.

Long Answers

1.



We know that length of tangents drawn from an external point to a circle are equal.

$\therefore$ TP = TQ ...(*i*)

$\therefore$ $\angle TQP = \angle TPQ$ ...(*ii*)

[Angles of equal sides are equal

Now, PT is tangent and OP is radius.

$\therefore$ OP $\perp$ TP

...[Tangent at any point of circle is perpendicular to the radius through point of contact

$\therefore$ $\angle OPT = 90°$

or, $\angle OPQ + \angle TPQ = 90°$

or, $\angle TPQ = 90° - \angle OPQ$ ...(*iii*)

In ΔPTQ, $\angle TPQ + \angle PQT + \angle QTP = 180°$

...[$\therefore$ Sum of angles of a triangle is 180°

or, $90° - \angle OPQ + \angle TPQ + \angle QTP = 180°$

...[From (*ii*) and (*iii*)

or, $2(90° - \angle OPQ) + \angle QTP = 180°$

or, $180° - 2\angle OPQ + \angle OQT = 180°$

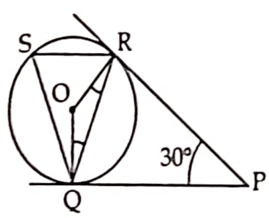
$\therefore$ $\angle PTQ = 2\angle OPQ$

Hence Proved.

2. PR = PQ ...[$\because$ Tangents drawn from an external point are equal

$\Rightarrow \angle PRQ = \angle PQR$

...[$\because$ Angles opposite to equal sides are equal



In ΔPQR,

$\Rightarrow \angle PRQ + \angle RPQ + \angle PQR = 180°$

...[Δ Rule

$\Rightarrow 30° + 2\angle PQR = 180°$

$\Rightarrow \angle PQR = \dfrac{(180 - 30)°}{2} = 75°$

$\Rightarrow$ SR || QP and QR is a transversal

$\because$ $\angle SRQ = \angle PQR$ ...[Alternate interior angle

$\therefore$ $\angle SRQ = 75°$ $\Rightarrow \angle ORP = 90°$

$\therefore$ $\angle ORP = \angle ORQ + \angle QRP$

$\Rightarrow 90° = \angle ORQ + 75°$

$\Rightarrow \angle ORQ = 90° - 75° = 15°$

Similarly, $\angle RQO = 15°$

In ΔQOR,

$\angle QOR + \angle QRO + \angle OQR = 180°$

...[Δ Rule

$\therefore$ $\angle QOR + 15° + 15° = 180°$

$\angle QOR = 180° - 30° = 150°$

$\Rightarrow QSR = \dfrac{1}{2}\angle QOR$

$\Rightarrow \angle QSR = \dfrac{150°}{2} = 75°$

...[Used $\angle SRQ = 75°$ as solved above

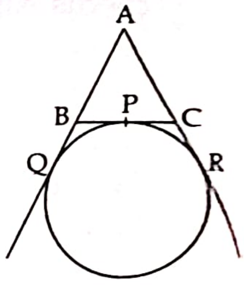
In ΔRSQ, $\angle RSQ + \angle QRS + \angle RQS = 180°$

...[Δ Rule

$\therefore$ $75° + 75° + \angle RQS = 180°$

$\therefore$ **$\angle RQS = 180° - 150° = 30°$**

3. **Given:** A circle touching the side BC of ΔABC at P and AB, AC produced at Q and R respectively.



Proof: Lengths of tangents drawn from an external point to a circle are equal.

$\Rightarrow$ AQ = AR, BQ = BP, CP = CR

Perimeter of ΔABC

= AB + BC + CA

= AB + (BP + PC) + (AR – CR)

= (AB + BQ) + (PC) + (AQ – PC)

...[Since AQ = AR, BQ = BP, CP = CR

= AQ + AQ = 2AQ

$\Rightarrow AQ = \dfrac{1}{2}$(Perimeter of ΔABC)

$\therefore$ AQ is the half of the perimeter of ΔABC.

4. Let parellelogram ABCD circumscribe a circle.

So,

BP = BQ ...(*i*) [Tangents from point B

CR = CQ ...(*ii*) [Tangents from point C

DR = DS ...(*iii*) [Tangents from point D

AP = AS ...(*iv*) [Tangents from point A

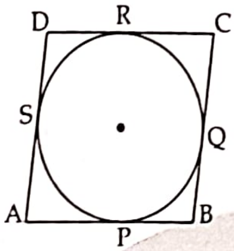
On Adding,

(*i*) + (*ii*) + (*iii*) + (*iv*) we get,

BP + CR + DR + AP

= BQ + CQ + DS + AS

On rearranging,



BP + AP + CR + DR

= BQ + CQ + DS + AS

AB + CD = BC + AD

Substitute, CD = AB and AD = BC

Since ABCD is a parallelogram, then

AB + AB = BC + BC

2AB = 2BC

AB = BC

$\therefore$ AB = BC = CD = DA

This implies that all the four sides are equal.

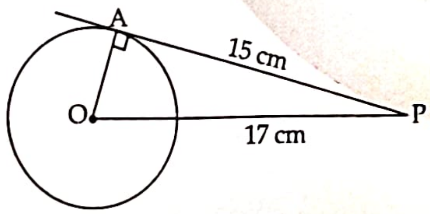
Therefore, the parallelogram circumscribing a circle is a rhombus.

Case Based Answers

1. (*a*) Here, $OA^2 = AB^2 + OB^2$

$\therefore$ **OA** = $\sqrt{10^2 + 5^2}$ = $\mathbf{5\sqrt{5}}$

(*b*) (*i*)



Here $OA^2 = OP^2 - AP^2$

OA = $\sqrt{OP^2 - AP^2}$

= $\sqrt{17^2 - 15^2}$

= $\sqrt{64}$ = **8 cm**

Or

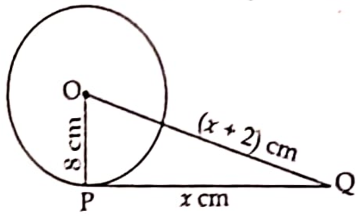
(*ii*) Length of tangent,

AP = $\sqrt{OP^2 - OA^2}$

= $\sqrt{25^2 - 7^2} = \sqrt{576}$

= **24 cm**

(*c*) We have, OP + PQ = OQ



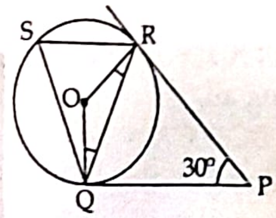
$\Rightarrow (8)^2 + x^2 = (x + 2)^2$

$\Rightarrow 64 = 4x + 4$

$\Rightarrow x = 15$ cm

So, length of tangent, **PQ = 15 cm**

2. (*a*)



In Quadrilateral ORPQ,

Sum of all angles is 360°

So, $\angle ROQ + \angle ORP + \angle QPR + \angle OQP = 360°$

$\angle ROQ + 90° + 90° + 30° = 360°$

$\therefore \angle ROQ = 360° - 210° = \mathbf{150°}$

(*b*) (*i*) In ΔPQR,

$\angle PRQ = \angle RQP = x$

Two tangents drawn from an external point to a circle are equal.

$\therefore \; x + x + \angle QPR = 180°$

$\Rightarrow 2x + 30° = 180°$

$\mathbf{x = 75°}$

Or

(*ii*) Here we have

$\angle RSQ = \frac{1}{2}$ of $\angle ROQ$

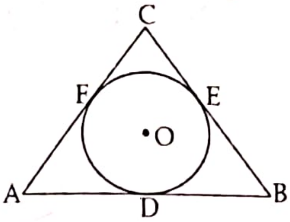
$= \frac{1}{2}(150°) = \mathbf{75°}$

(*c*) The sum of all angles of a quadrilateral = 360°

Reflex $\angle ROQ = 360 - \angle ROQ$

$= 360° - 150° = \mathbf{210°}$

3. (*a*)



Let,

AD = AF = x cm

BD = BE = y cm

CF = CE = z cm

So,

AB = $x + y = 12$ cm ...(*i*)

BC = $y + z = 8$ cm ...(*ii*)

CA = $z + x = 10$ cm ...(*iii*)

Adding all,

AB + BC + CA = 12 + 8 + 10

$(x + y) + (y + z) + (z + x) = 30$

$\Rightarrow 2(x + y + z) = 30$

$\Rightarrow x + y + z = 15$ cm

Then, $(x + y + z) - (y + z) = x$

$\Rightarrow 15 - 8 = 7$ cm = AD

(*b*) (*i*) $(x + y + z) - (x + z) = y$

$\Rightarrow 15 - 10 = 5$ cm = **BE**

Or

(*ii*) $(x + y + z) - (x + y) = z$

$\Rightarrow 15 - 12 = 3$ cm = **CF**

(*c*) Now, given that,

Radius of circle = OD = **4 cm**

∴ **Area ΔOAB**

$= \left(\frac{1}{2}\right) \times$ Perpendicular height × Base

$= \left(\frac{1}{2}\right) \times OD \times AB$

$= \left(\frac{1}{2}\right) \times 4 \times 12$

$= 24$ **cm²**

(DAY 9 SWAHA)

*****

5 Areas Related to Circles

What did CBSE ask last year?

MCQs & A/R	2 Questions ($1 \times 2 = 2$ Marks)
Subjective	No Very Short Questions
	1 Short Question ($1 \times 3 = 3$ Marks)
	1 Long Question ($1 \times 5 = 5$ Marks)
Case Based	1 Question ($1 + 1 + 2 = 4$ Marks)

Note: *All the above typology of questions include 'Competency based Questions' labelled as* **COMPETENCY**.

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Introduction to the Chapter

- Tangent

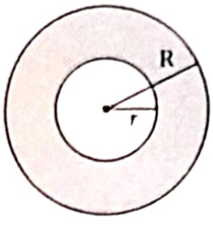
Circumference of a circle $= 2\pi r$

Area of a circle $= \pi r^2$ ...[where 'r' is the radius of circle

Area of a semi-circle $= \dfrac{\pi r^2}{2}$

Area of a circular path or ring:

Let 'R' and 'r' be radii of two circles



Then area of shaded part $= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

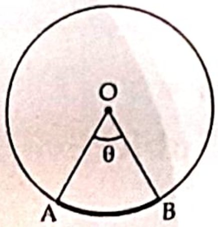
$= \pi(R + r)\ (R - r)$

Minor arc and major arc: An arc length is called a major arc, if the arc length enclosed by the two radii is greater than a semi-circle.

If the arc subtends angle 'θ' at the centre, then the

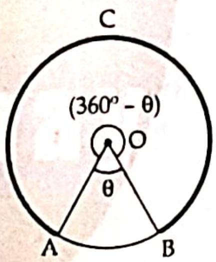
$$\text{Length of minor arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{\theta}{180^\circ} \times \pi r$$



Minor Arc

$$\text{Length of major arc} = \left(\frac{360^\circ - \theta}{360^\circ}\right) \times 2\pi r$$



Major Arc

- Time of revolution $= t = \dfrac{2\pi R v}{\text{Speed}}$
- Number of revolutions $= v = \dfrac{\text{Distance covered}}{2\pi R}$

CLOCK

- Area swept by minute hand
  - ➡ in 60 minutes $= 360^\circ$
  - ➡ in 1 minute $= 6^\circ$
- Area swept by hour hand
  - ➡ in 12 hours $= 360^\circ$
  - ➡ in 1 hour $= 30^\circ$
  - ➡ in 1 minute $= 0.5^\circ$

Sectors and Segments

- Sector of a Circle and its Area

A region of a circle is enclosed by any two radii and the arc intercepted between two radii is called the sector of a circle.

(i) A sector is called a minor sector if the minor arc of the circle is part of its boundary.

$\therefore$ $\overparen{OAB}$ is minor sector.

Area of minor sector $= \frac{\theta}{360°} \cdot (\pi r^2)$;

Perimeter of minor sector $= 2r + \frac{\theta}{360°}(2\pi r)$

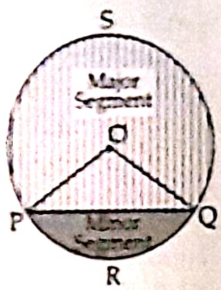
(ii) A sector is called a major sector if the major arc of the circle is part of its boundary.

$\therefore$ $\overparen{OACB}$ is the major sector.

Area of major sector $= \left(\frac{360° - \theta}{360°}\right)(\pi r^2)$

Perimeter of major sector $= 2r + \left(\frac{360° - \theta}{360°}\right)(2\pi r)$

Minor Segment: The region enclosed by an arc and a chord is called a segment of the circle. The region enclosed by the chord PQ and minor arc PRQ is called the minor segment (shown as shaded portion).



$\therefore$ Area of Minor segment
= Area of corresponding sector
− Area of corresponding triangle

$$= \left[\frac{\theta}{360°}\pi r^2 - \frac{1}{2}r^2 \sin\theta\right] = \frac{1}{2}r^2\left[\frac{\theta}{180°}\pi - \sin\theta\right]$$

Major Segment: The region enclosed by the chord PQ and major arc $\overparen{PSQ}$ is called the major segment.

$\therefore$ Area of Major segment = Area of circle − Area of minor segment
Or Area of major sector + Area of triangle

$$= \pi r^2 - \frac{\theta}{360°}\pi r^2 + \frac{1}{2}r^2 \sin\theta$$

$$= r^2\left[\pi - \frac{\theta}{360°}\pi + \frac{\sin\theta}{2}\right]$$

OBJECTIVE QUESTIONS

(DAY 10)

Multiple Choice Questions

Q.1. The area of a quadrant of a circle where the circumference of circle is 176 m is [CBSE 2022]

(a) 2464 m^2 (b) 1232 m^2
(c) 616 m^2 (d) 308 m^2

Q.2. The diameter of a car wheel is 42 cm. The number of complete revolutions it will make in moving 132 km is: COMPETENCY

(a) 10^4 (b) 10^6
(c) 10^5 (d) 10^3

Q.3. The minute hand of a clock is 84 cm long. The distance covered by the tip of minute hand from 10:10 am to 10:25 am is: COMPETENCY

(a) 44 cm (b) 88 cm
(c) 132 cm (d) 176 cm

FREE ADVICE: The circumference of the circle traced by the minute hand can be calculated using the length of the minute hand (radius). The formula for the circumference of a circle is $2\pi r$, where r is the length of the minute hand.

Q.4. If the perimeter of a circle is half of that of a square, then the ratio of the area of the circle to the area of the square is:

(a) 22 : 7 (b) 11 : 7
(c) 7 : 11 (d) 7 : 22

FREE ADVICE: If the perimeter of the circle is half that of a square, then we can write the equation as:

$$2\pi r = \left(\frac{1}{2}\right) \times 4s$$

Q.5. If the radius of the circle is 6 cm and the length of an arc 12 cm. Find the area of the sector. [CBSE 2014]

(a) 36 cm^2 (b) 42 cm^2
(c) 53 cm^2 (d) 24 cm^2

Q.6. The area of the circle that can be inscribed in a square of side 6 cm is COMPETENCY

(a) 36π cm^2 (b) 18π cm^2
(c) 12π cm^2 (d) 9π cm^2

FREE ADVICE:
Diameter of the circle = Side of the square

Q.7. The area of a circular path of uniform width d surrounding a circular region of radius r is COMPETENCY

(a) $\pi d(2r + d)$ (b) $\pi(2r + dr)$
(c) $\pi(d+r)r$ (d) $\pi(d+r)d$

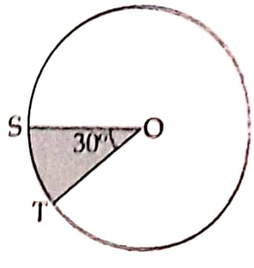
Q.8. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. The length of the arc is:

(a) 20 cm (b) 21 cm
(c) 22 cm (d) 25 cm

Q.9. Area of the largest triangle that can be inscribed in a semi-circle of radius r units is COMPETENCY

(a) r^2 sq. units (b) $\frac{1}{2}r^2$ sq. units
(c) $2r^2$ sq. units (d) $\sqrt{2r^2}$ sq. units

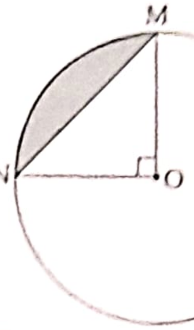
Q.10. Shown here is a circle with centre O. The area of the minor sector SOT is 7 cm^2.
(Note: The figure is not to scale.)



What is the area of the circle? COMPETENCY

(a) $84\ \pi$ cm^2 (b) $\frac{84}{11}\ \pi$ cm^2
(c) 84 cm^2 (d) $\frac{\sqrt{84}}{\sqrt{\pi}}$ cm^2

Q.11. In the circle shown here, O is the centre. MN is a chord which subtends an angle of 90° at the centre. The area of the shaded region is 72 cm^2.
What is the radius of the circle?
(Note: Take π as $\frac{22}{7}$.) COMPETENCY



(*a*) $6\sqrt{7}$ cm (*b*) $6\sqrt{28}$ cm
(*c*) 84 cm (*d*) 252 cm

Q.12. A circular pond needs to be fenced along its circumference. One-fourth of the fencing is already done, which cost ₹750 at the rate of ₹50 per metre.
How many metres of the pond still need to be fenced? **COMPETENCY**

(*a*) 15 (*b*) 20 (*c*) 45 (*d*) 60

Q.13. If the circumference of a circle and the perimeter of a square are equal, then **COMPETENCY**

(*a*) Area of the circle = Area of the square
(*b*) Area of the circle > Area of the square
(*c*) Area of the circle < Area of the square
(*d*) Nothing definite can be said about the relation between the areas of the circle and square.

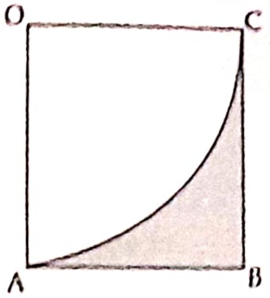
Q14. If the radius of a circle is halved, how does its area change? **COMPETENCY**

(*a*) It becomes half.
(*b*) It becomes one-fourth.
(*c*) It doubles.
(*d*) It remains the same.

Q.15. Area of a sector of angle p (in degrees) of a circle with radius R is

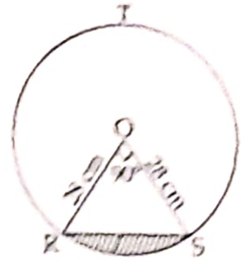
(*a*) $\frac{p}{180°} \times 2\pi R$ (*b*) $\frac{p}{180°} \times \pi R^2$
(*c*) $\frac{p}{360°} \times 2\pi R$ (*d*) $\frac{p}{360°} \times \pi R^2$

Q.16. In the given figure, OABC is a square of side 7 cm. OABC is a quadrant of a circle with O as centre. The area of the shaded region is **COMPETENCY**



(*a*) 10.5 cm^2 (*b*) 38.5 cm^2
(*c*) 49 cm^2 (*d*) 11.5 cm^2

Q.17. Shown here is a circle with centre O and radius 28 cm. Chord RS subtends an angle of 90° at O. (Note: The figure is not to scale.)



What is the area of the segment RTS?
(Note: Take π as $\frac{22}{7}$) **COMPETENCY**

(*a*) 224 cm^2 (*b*) 616 cm^2
(*c*) 1848 cm^2 (*d*) 2240 cm^2

Q.18. A circle is inscribed in an equilateral triangle. If the radius of the circle is 6 cm, what is the side length of the triangle?

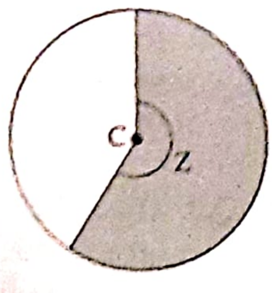
(*a*) $6\sqrt{3}$ cm (*b*) 12 cm
(*c*) 18 cm (*d*) $36\sqrt{3}$ cm

Q.19. If the radius of a circle is increased by 50%, how does its area change? **COMPETENCY**

(*a*) It increases by 25%
(*b*) It increases by 50%
(*c*) It increases by 75%
(*d*) It increases by 100%

FREE ADVICE: If the radius of a circle is increased by 50%, the new radius becomes 1.5 times the original radius.

Q.20. In the figure shown here, C is the centre of the circle. The area of the shaded sector is $\frac{5}{8}$ of the area of the circle.



(2024)

(Note: The figure is not to scale.)
What is the measure of angle z?

(*a*) 135°
(*b*) 200°
(*c*) 225°
(*d*) Cannot be determined as the radius of circle is not given [CBSE 2024

Q.21. A circle with radius 'r' has an area equal to the area of a square with side length 'a'. What is the relationship between 'r' and 'a'? COMPETENCY

(a) $\frac{r}{a} = \frac{1}{\sqrt{\pi}}$ (b) $r = 2a$

(c) $r = \frac{a}{2}$ (d) $r = \sqrt{a}$

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

(b) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*

(c) *(A) is true and (R) is false.*

(d) *(A) is false, but (R) is true.*

Q.1. Assertion: If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason: Circumference = $2\pi r$ COMPETENCY

Q.2. Assertion: In a circle of radius 6 cm, the angle of a sector is 60°. Then the area of the sector is $\frac{132}{7}$ cm^2.

Reason: Area of the circle with radius r is πr^2. [CBSE 2024]

Q.3. Assertion: The length of the minute hand of a clock is 7 cm. Then the area swept by the minute hand in 5 minutes is $\frac{77}{6}$ cm^2.

Reason: The length of an arc of a sector of angle θ and radius r is given by

$l = \frac{\theta}{360°} \times 2\pi r.$

Q.4. Assertion: If a wire of length 22 cm is bent in the shape of a circle, then area of the circle so formed is 40 cm^2.

Reason: Circumference of the circle = length of the wire. COMPETENCY

SUBJECTIVE QUESTIONS

Very Short Answer Questions

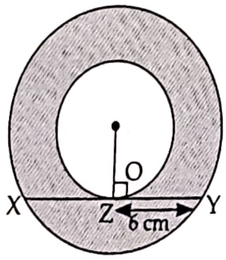
Q.1. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes. COMPETENCY

Q.2. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of a circle of a circle which has circumference equal to sum of their circumferences. [CBSE 2020]

Q.3. The radius of a circle is 17.5 cm. Find the area of the sector of the circle enclosed by two radii and an arc 44 cm in length.

Q.4. Shown here are two concentric circles with centre O. XY is tangent to the inner circle at Z.

(Note: The figure is not to scale.)



What is the area of the shaded region in terms of π? Show your work. COMPETENCY

Q.5. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Q.6. Find the area of a quadrant of a circle, where the circumference of circle is 44 cm. COMPETENCY

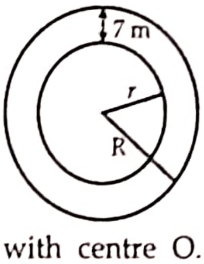
Q.7. Find the area of the sector of a circle with radius 4 cm and of angle 60°. Also, find the area of the corresponding major sector. (Use π = 3.14) [NCERT

Q.8. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at the speed of 66 km per hour? COMPETENCY

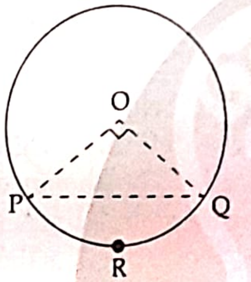
(DAY 11)

Short Answer Questions

Q.1. A road which is 7 m wide surrounds a circular park whose circumference is 88 m. Find the area of the road. *[CBSE 2020*



Q.2. Given here is a circle with centre O. The length of arc PRQ is 22 cm and it subtends an angle of 90° at the centre. A triangle POQ is cut along the dotted lines as shown here.

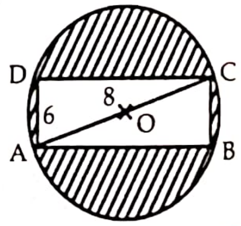


(Note: The figure is not to scale.)

Find the area of the remaining circle after the triangle is cut. Show your work. (**Note:** Take π as $\frac{22}{7}$) *[CBSE 2024*

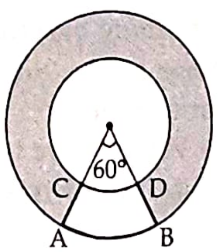
Q.3. Find the area of the shaded region in figure if ABCD is a rectangle with sides 8 cm and 6 cm. O is the centre of circle.

(Take $\pi = 3.14$) **COMPETENCY**



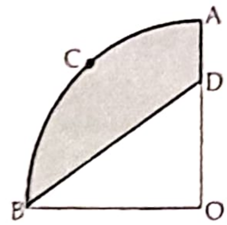
> ***FREE ADVICE:*** *Circle ka area nikal ke isme se rectangle ka area minus kar do, easy!*

Q.4. In figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. **COMPETENCY**

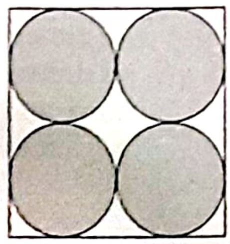


> ***FREE ADVICE:*** *Pay attention! 60° wale sector ka area nikalte time yahi angle dono hi circle ke liye comman hoga.*

Q.5. In the given figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the shaded region. **COMPETENCY**



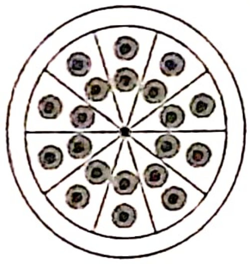
Q.6. On a white sheet of square paper, 4 identical shaded circles are drawn such that the circles inside the square touch the boundaries of two other circles and the two sides of the square as shown here.



If the area of the square sheet is 576 cm^2, what is the area that is NOT covered by the circles? Show your work.

(Note: Take π as 3.14.) **COMPETENCY**

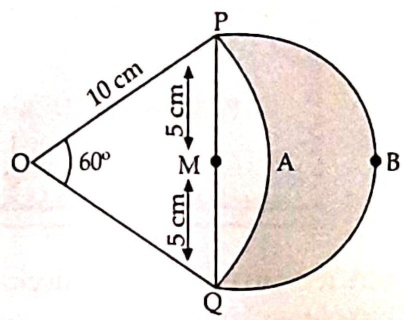
Q.7. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:



(*a*) the total length of the silver wire required.

(*b*) the area of each sector of the brooch. *[NCERT*

Q.8. In the given figure, point A is showing two arcs PAQ and PBQ.



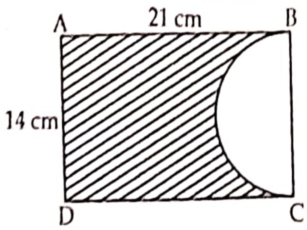
Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M.

If OP = PQ = 10 cm, show that area of shaded region is $25\left(\sqrt{3}-\frac{\pi}{6}\right)$ cm^2.

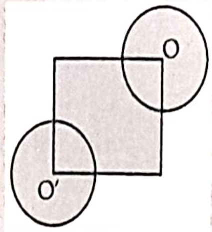
COMPETENCY

Long Answer Questions

Q.1. In the given figure, ABCD is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure. *[CBSE 2017*



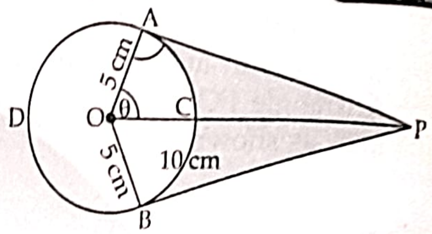
Q.2. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where O and O′ are



centres of the circles. Find the area of the shaded region. *[CBSE 20*

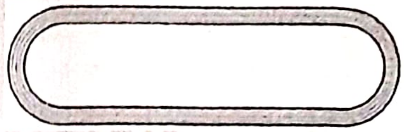
Q.3. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, elastic belt is directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. (Use π = 3.14 and $\sqrt{3}$ = 1.73)

COMPETENCY



Q.4. Figure below depicts a racing track whose left and right ends are semi-circular. The distance between the two inner parallel lines segments is 60 m and they are each 106 m long.

If the track is 10 m wide, find:



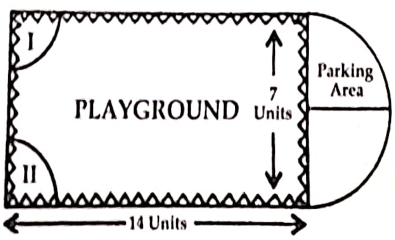
(*a*) the distance around the track along its inner edge.

(*b*) the area of the track.

COMPETENCY

CASE BASED QUESTIONS

Q.1. Governing council of a Local Public Development Authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.



After survey, it was decided to build rectangular playground, with a semi-circular area allotted for parking at one end of the playground.

The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.

Based on the above information, answer the following questions:

(*a*) What is the total perimeter of the parking area?

(*b*) (*i*) What is the total area of parking and the two quadrants?

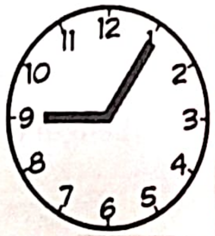
[CBSE 2024

Or

(*ii*) **What is the ratio of area of playground to the area of parking area?**

(*c*) **Find the cost of fencing the playground and parking area at the rate of 2 per unit.**

Q.2. A clock is a device used to measure, keep and indicate time. The clock is one of the oldest human inventions, meeting the need to measure intervals of time shorter than the natural units—the day, the lunar month and the year.



Based on the above information, answer the following questions:

(*a*) **What is the angle through which the minute hand turns in one minute?**

(*b*) (*i*) **By how much does the tip of the minute hand move in one hour? if the length of the minute hand is 3.5 cm.** COMPETENCY

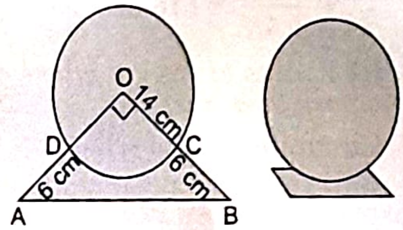
Or

(*ii*) **By how much does the tip of the minute hand move in 1 hour?**

(*c*) **Find the area swept by the hour hand in three hours if the length of the hour hand is 2.1 cm.**

COMPETENCY

Q.3. Prize Distribution Director of a company select a round glass trophy for awarding their employees on annual function. Design of each trophy is made as shown in the figure, where its base ABCD is golden plated from the front side at the rate of ₹6 per cm².



Based on the above information, answer the following questions:

(*a*) **Find the area of sector ODCO.**

(*b*) (*i*) **Find the area of AOB.**

Or

(*ii*) **Find the total cost of golden plating.**

(*c*) **Find the area of major sector formed in the given figure.** COMPETENCY

*****

ANSWERS

Multiple Choice Answers

1. (*c*) Circumference of circle = $2\pi r = 176$

$\Rightarrow 2 \times \frac{22}{7} \times r = 176$

$\Rightarrow r = 7 \times 4 = 28$ m

Area of circle $= \pi r^2 = \frac{22}{7} \times 28 \times 28$

$= 88 \times 28$

$= \mathbf{2464\ m^2}$

Now, **Area of quadrant of circle**

$= \frac{2464}{4} = \mathbf{616\ m^2}$

2. (*c*) Radius of the car wheel = r

= 42 cm = 21 cm ...[Given

Circumference of wheel

= Circumference of circle

$= 2\pi r = 2 \times \frac{22}{7} \times 21 = 132$

Circumference of wheel is 132 cm.

No. of revolutions in 132 km

$= \frac{Distance}{Circumference}$

Convert 132 km into cm.

132 km = 1,32,00,000 cm

As 1 km = 1,00,000 cm

So, No. of revolutions is $\frac{1,32,00,000}{132}$

i.e., 1,00,000, *i.e.*, 10^5.

3. (*c*) Minute hand = 84 cm

= Radius of the clock

Circumference of the circle (clock) = $2\pi r$

$= 2 \times \frac{22}{7} \times 84 = 528$ cm

The hand travelled from 10:10 to 10:25 which is 15 minutes, which is a quarter ($^1/_4$) of an hour.

$\left(\frac{1}{4}\right) \times 528 = \mathbf{132\ cm}$

4. (*d*) Let the side of the square is x cm.

So perimeter of the square is $4x$ cm.

Area of the square is x^2 sq cm.

Perimeter of the circle is $\frac{4x}{2}$ cm = $2x$ cm

If the radius of the circle is r cm

then, $2\pi r = 2x$

then, $r = \frac{x}{\pi}$

So area of the circle is πr^2 sq. cm.

$= \frac{\pi x^2}{\pi^2}$ sq. cm $= \frac{x^2}{\pi}$ sq. cm

∴ **Required Ratio**

= Area of the circle : Area of the squ

$= \frac{x^2}{\pi} : x^2 = \frac{1}{\pi} : 1$

$= \frac{7}{22} : 1 = \mathbf{7 : 22}$

5. (*a*) Area of the sector

$= \frac{1}{2} \times$ (Length of the correspondi area) $\times$ Radi

$= \frac{1}{2} \times l \times r$

$= \frac{1}{2} \times 12 \times 6 = \mathbf{36\ cm^2}$

6. (*d*) We have, $2r = 6$ cm

$\Rightarrow r = 3$ cm

Area of the inscribed circle = πr^2

$= \pi \times (3)^2 = \mathbf{9\pi\ cm^2}$

7. (*a*) Area of a circular path,

A = Area of outer circle

– Area of inner circl

$A = \pi R^2 - \pi r^2$

$A = \pi(R^2 - r^2)$

$A = \pi\{(r + d)^2 - r^2\}$

$A = \pi\{r^2 + d^2 + 2rd - r^2\}$

$A = \pi\{r^2 - r^2 + d^2 + 2rd\}$

...$[(a + b)^2 = a^2 + b^2 + 2a$

$A = \pi\{d^2 + 2rd\} = \pi d(2r + d)$

∴ **Area of a circular path = $\pi d(2r + d)$**

8. (*c*) Radius (r) of circle = 21 cm

Angle subtended by the given arc = 60°

Length of an arc of a sector of angle θ.

$= \frac{\theta}{360^\circ} \times 2\pi r$

Length of arc, ACB $= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$

$= \frac{1}{6} \times 2 \times 22 \times 3$

$= \mathbf{22\ cm}$

9. (*a*) In a semi circle, the diameter is the base of the semi-circle.
This is equal to $2 \times r$.

$$A = \frac{1}{2} \times b \times h$$

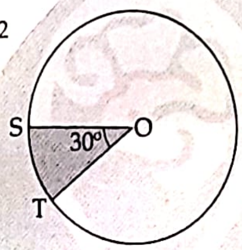
$$A = \frac{1}{2} \times 2 \times r \times r$$

Because the base of the triangle is equal to $2 \times r$.
The height of the triangle is equal to r.

$$A = \frac{1}{2} \times 2 \times r \times r \text{ becomes}$$

$\therefore$ **$A = r^2$ sq. units**

10. (*c*) **Given:** Angle formed by sector, $(\theta) = 30$
Area of sector = 7 cm²
As we know,
Area of sector



$$= \frac{\theta}{360^\circ} \times \pi r^2$$

$$\Rightarrow 7 = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times r^2$$

$$\Rightarrow r^2 = \frac{7 \times 360^\circ \times 7}{30^\circ \times 22} \qquad \therefore r^2 = \frac{294}{11}$$

Now, **Area of circle** $= \pi r^2$

$$= \frac{22}{7} \times \frac{294}{11} = \mathbf{84\ cm^2}$$

11. (*a*) Area of segment (Area of shaded region) = Area of sector – Area of triangle

$$72 \text{ cm}^2 = \left(\frac{\theta}{360^\circ} \times \pi r^2\right) - \left(\frac{1}{2} \times b \times h\right)$$

$$\Rightarrow 72 \text{ cm}^2 = \left(\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times r^2\right) - \left(\frac{1}{2} \times r^2\right)$$

$$\Rightarrow 72 = r^2\left(\frac{11}{14} - \frac{1}{2}\right)$$

$$\Rightarrow 72 = r^2\left(\frac{11-7}{14}\right) = r^2\left(\frac{4}{14}\right)$$

$$\Rightarrow r^2 = \frac{72 \times 14}{4} \qquad \Rightarrow r^2 = 18 \times 14$$

$$\Rightarrow r = \sqrt{2 \times 3 \times 3 \times 2 \times 7}$$

$\therefore$ **$r = 6\sqrt{7}$cm**

12. (*c*) Let the circumference of pond = x m

Fencing already done = $\frac{x}{4}$ m

Total cost = Cost per metre × Amount of fencing

$$\Rightarrow 50 \times \frac{x}{4} = 750$$

$$\Rightarrow x = \frac{750 \times 4}{50} = \mathbf{60\ m}$$

$\therefore$ **Amount of fencing still left**

$$= 60 - \left(\frac{60}{4}\right) = 60 - 15 = \mathbf{45\ m}$$

13. (*b*) Let the radius of the circle be r.
$\therefore$ Area of Circle $= \pi r^2$
and the circumference $= 2\pi r$
The perimeter of the square is equal to circumference of circle.

$$\therefore \text{ Its one side} = \frac{1}{4} \times 2\pi r = \frac{1}{2}\pi r$$

$$\text{So Area of Square} = \left(\frac{\pi r}{2}\right)^2$$

$$\therefore \frac{\text{Area of Circle}}{\text{Area of Square}} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2} = \frac{4}{\pi} = \frac{4}{22} \times 7 = \frac{14}{11} > 1$$

So, **Area of Circle > Area of Square**

14. (*b*) Initially, let the radius be 'r'.
Area $= \pi r^2$
The radius is then reduced to $\frac{r}{2}$.

$$\text{New area} = \pi \times \left(\frac{r}{2}\right)^2 = \frac{1}{4} \times \pi r^2$$

The area reduces to one-fourth of its initial area.

15. (*d*) ***Formula:*** Area of sector $= \frac{\pi r^2 \theta}{360^\circ}$
Area of the sector of angle p

$$= \frac{p}{360^\circ} \times \pi R^2$$

16. (*a*) Area of the shaded part = Area of the square OABC – Area of the sector OAC

$$= 7^2 - \frac{90}{360} \times \frac{22}{7} \times 7^2$$

$$= 49 - 38.5 = \mathbf{10.5\ cm^2}$$

17. (*d*) Radius (r) = 28 cm
Angle formed $(\theta) = 90^\circ$
Area of minor segment
= Area of sector – Area of triangle

$$= \left(\frac{\theta}{360^\circ} \times \pi r^2\right) - \left(\frac{1}{2} \times b \times h\right)$$

$$= \left(\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28\right) - \left(\frac{1}{2} \times 28 \times 28\right)$$

$= 616 - 392 = \mathbf{224\ cm^2}$

$\therefore$ **Area of major segment RTS**

= Area of circle

- Area of minor segment

$$= \left(\frac{22}{7} \times 28 \times 28\right) - 224$$

$= 2464 - 224 = \mathbf{2240\ cm^2}$

18. (*a*) D is mid-point of BC and OB and OC are bisectors of $\angle B$ and $\angle C$ respectively.

$\angle OBD = 30^\circ$ and OB = 6 cm.

$\Rightarrow BC = 2\,BD = 2(3\sqrt{3})$ cm $= \mathbf{6\sqrt{3}\ cm}$

19. (*a*) Let the radius = r units.

Then Area = πr^2

Increased radius = $\frac{150}{100} r = 1.5\, r$ units

$\therefore$ Increased area = $\pi \times (1.5r)^2 = 2.25\pi r^2$

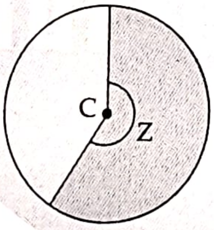
% increase $= (2.25\,\pi r^2 - \frac{\pi r^2}{\pi r^2} \times 100)\%$

$= (1.25 \times 100)\% = \mathbf{125\%}$

20. (*c*) A.T.Q.,

$\Rightarrow$ Area of sector

$= \frac{5}{8}$ (Area of circle)



$\Rightarrow \frac{\theta}{360^\circ} \times \pi r^2 = \frac{5}{8} \times \pi r^2$

$\therefore\ \theta = \frac{360^\circ \times 5}{8} = \mathbf{225^\circ}$

21. (*a*) Let the radius of circle = r

Let the side of square = a

Given, Area of circle = area of square

$\pi r^2 = a^2$

$\Rightarrow \frac{r^2}{a^2} = \frac{1}{\pi}$ $\quad\Rightarrow r : a = \mathbf{1 : \sqrt{\pi}}$

Assertion Reason Answers

1. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: *To check if the assertion is true, we can substitute the given circumference value (176 cm) into formula for the circumference of a circle.*

$C = 2\pi r \qquad \Rightarrow 176 = 2\pi r$

Now, we need to solve the equation to find the value of r (radius). Dividing both sides of the equation by

$$\frac{176}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\Rightarrow r = \frac{176 \times 7}{2 \times 22} = \frac{56}{2} = 28$$

Therefore, the radius of the circle is approximately 28 cm, which matches the assertion.

2. (*b*) Both (A) and (R) are true and (R) is not the correct explanation of (A).

Explanation: *Radius of the circle = 6 cm*

Also, angle of the sector = 60°

So, Area of the sector $= \left(\frac{\theta}{360^\circ}\right) \times \pi r^2$

$$= \left(\frac{1}{6}\right) \times (\pi \times 6 \times 6) = \left(\frac{132}{7}\right)$$

Therefore, the given assertion is true.

We know, area of circle is equal to πr^2, *but by using this formula, we can't find the given area of the sector.*

Therefore, it is not the correct explanation of the given assertion.

3. (*b*) Both (A) and (R) are true and (R) is not the correct explanation of (A).

Explanation: $l = \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{30}{360^\circ} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{6}$$

4. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

Very Short Answers

1. Given: Length of minute hand = 12 cm

So, minute hand covers a total of 60 minutes in one round of the clock.

So, 60 minutes = 360°

1 minute $= \frac{360^\circ}{60^\circ} = 6^\circ$

35 minutes = $(35 \times 6)^\circ = 210^\circ$

So in 35 minutes, minute hand subtends an angle of 210°.

Now Area of segment $= \frac{\theta}{360^\circ} \times \pi r^2$

Where θ is the angle subtended.

Therefore, the area covered by minute hand $= \frac{210^\circ}{360^\circ} = \frac{7}{12}\pi r^2$

The radius of the circle = Length of the minute hand = 12 cm

The area covered by minute hand

$$= \frac{7}{12} \times \frac{22}{7} \times 12^2$$

The area covered by minute hand

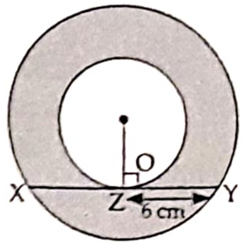
$= (22 \times 12)\ cm^2$ Area $= 264\ cm^2$

Hence, The area covered by minute hand in 35 minutes is $264\ cm^2$.

2. Using the formula of the circumference of circle $C = 2\pi r$.
   Radius (r_1) of the 1^{st} circle = 19 cm
   Radius (r_2) of the 2^{nd} circle = 9 cm
   Let the radius of the 3^{rd} circle be r.
   Circumference of the 1^{st} circle
   $= 2\pi r_1 = 2\pi(19) = 38\pi$
   Circumference of the 2^{nd} circle
   $= 2\pi r_2 = 2\pi(9) = 18\pi$
   Circumference of the 3^{rd} circle $= 2\pi r$
   Given that,
   Circumference of the 3^{rd} circle
   = Circumference of the 1^{st} circle
   + Circumference of the 2^{nd} circle
   $\Rightarrow 2\pi r = 38\pi + 18\pi = 56\pi$
   $\therefore r = \frac{56\pi}{2\pi} = 28$
   Therefore, the radius of the circle that has a circumference equal to the sum of the circumference of the two given circles is **28 cm**.

3. Radius of a circle = 17.5 cm
   Length of arc of circle = 44 cm
   Now, Area of Sector
   $= \frac{1}{2} \times$ Arc Length $\times$ Radius
   $= \frac{1}{2} \times 44 \times 17.5 = 385$
   $\therefore$ **Area of Sector is $385\ cm^2$.**

4. Let the radius of outer and inner circles be R and r respectively.



   Area of Shaded Region = Area of outer circle − Area of inner circle
   $= \pi R^2 - \pi r^2$
   $= \pi(R^2 - r^2)$ ...(*i*)
   Now, In ΔOZY by using pythagoras theorem,
   $(H)^2 = (B)^2 + (P)^2$
   $OY^2 = OZ^2 + ZX^2$
   $(R)^2 = (6)^2 + r^2$
   $\Rightarrow R^2 = 36 + r^2 \quad \Rightarrow R^2 - r^2 = 36$
   Now, substituting value of $R^2 - r^2 = 36$ in (*i*), we have
   $\therefore$ **Area of shaded region**
   $= \pi(R^2 - r^2) = \pi(36) = \mathbf{36\pi\ cm^2}$

5. Given: Length of minute hand $r = 14$ cm
   So, area swept by the minute hand in 5 minutes.
   Here, $\theta = \frac{5}{60^\circ} \times 360^\circ = 30^\circ$
   The area swept by the minute hand
   $= \frac{\theta}{360^\circ}\pi r^2 = \pi r^2 \times \frac{30^\circ}{360^\circ}$
   $= \frac{22}{7} \times 14^2 \times \frac{30}{360^\circ} = 51.33\ cm^2$
   Therefore, area swept by the minute hand is $51.33\ cm^2$.

6. We have, Circumference $= 2\pi r = 44$
   $\Rightarrow \frac{22}{7} \times r = 22 \quad \Rightarrow r = 7$ cm
   So, area of quadrant $= \frac{1}{4}\pi r^2$
   $= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} = \mathbf{38.5\ cm^2}$

7. **Given.** Radius of circle (r) = 4 cm
   Angle of sector $r(\theta) = 60°$
   Area of sector is defined as $A = \pi r^2 \frac{\theta}{360^\circ}$
   $= \frac{(3.14 \times 4 \times 4 \times 60^\circ)}{360^\circ} = 8.37\ cm^2$
   Angle of corresponding major sector
   $= 360° - 60° = 300°$

$\Rightarrow$ Area of corresponding major sector

$= \frac{\pi r^2 \theta}{360°} = \frac{(3.14 \times 4 \times 4 \times 300°)}{360°}$

$= 41.87 \text{ cm}^2$

Thus, **Area of sector = 8.37 cm^2 (approx)**
Area of major sector = 41.87 cm^2 (approx)

8. The circumference of the wheel
= Distance travelled by the wheel in one revolution

The diameter of the wheel of the car
= 80 cm

The radius of the wheel of the car = 40 cm

Distance travelled after 1 revolution
= Circumference of the wheel

$= 2\pi r$

$= 2\pi(40) = 80\pi$ cm ...(*i*)

Speed of car = 66 km/hour

Car travels 66 kms in 60 minutes,

Distance travelled in 10 minutes = 11 kms ...(*ii*)

Let the number of revolutions of the wheel of the car be equal to 'n'.

$n \times$ *Distance travelled in 1 revolution*
= *Distance travelled in 10 minutes*

$n \times 80\pi = 11{,}00{,}000$

...[From equation (*i*) and (*ii*)

$\therefore\ n = \frac{1100000 \times 7}{80 \times 22} = \mathbf{4375}$

Hence, each of the wheels of the car will make 4375 revolutions.

Short Answers

1. Width (w) = 7 m, $2\pi r = 88$ m

Circumference $= 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow r = 2 \times 7 = 14$

So, radius = 14

$R = r + w = 14 + 7 = 21$

$\therefore$ Area of road $= \pi R^2 - \pi r^2$

$= \pi(R + r)(R - r)$

$= \frac{22}{7} \times (21 + 14) \times (21 - 14)$

$= \frac{22}{7} \times (35) \times (7) = \mathbf{770\ m^2}$

2. **Given.** Sides of triangular field is 20 m, 34 m and 42 m.

Semi-perimeter $= \frac{20\text{m} + 34\text{m} + 42\text{m}}{2}$

$= \frac{96}{2} = 48$ m

Area of the field

$= \sqrt{[48(48-20)(48-34)(48-42)]}$

$= \sqrt{[48 \times 28 \times 14 \times 6]} = \sqrt{112896}$

$= 336 \text{ m}^2$

We know that sum of angles of triangles = 180°

Thus, Area grazed

= Area of semi-circle with radius 7 m

$= \frac{\pi}{2} \times (7 \text{ m})^2 = \frac{22}{7} \times \frac{1}{2} \times 49 = 77 \text{ m}^2$

$\therefore$ **Area of field-Area of grazed**

$= (336 - 77) \text{ m}^2 = \mathbf{259\ m^2}$

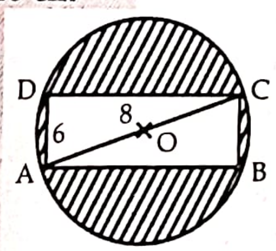
3. In right triangle ABC

$AC^2 = AB^2 + BC^2$

$= (8)^2 + (6)^2 = 64 + 36 = 100$

$\Rightarrow AC^2 = 100$

$\Rightarrow AC = 10$ cm



Now, Radius of circle is

$= \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm

Area of the shaded region = Area of circle – Area of rectangle OABC

Area of circle $= \pi r^2 = \pi(5)^2$

$\Rightarrow 25\pi = 78.5 \text{ cm}^2$

Area of rectangle $= l \times b = 6 \times 8 = 48 \text{ cm}^2$

Area of the shaded region = 78.5 – 48
= **30.5 cm^2**

Hence, the area of the shaded region is 30.5 cm^2.

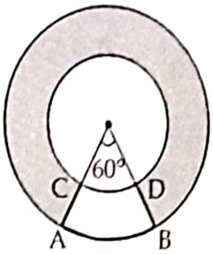
4. From the given figure we have,

R = 42 cm, $r = 21$ cm and $\theta = 60°$.

Area of the ring $= \pi(R^2 - r^2)$

$= \pi(42^2 - 21^2)$

$= 4158 \text{ cm}^2$



Now, Area of shaded part = Area of ring – sector formed by bigger circle – sector formed by smaller circle

$$= 4158 - \pi(42)^2 \times \frac{60°}{360} - \pi(21)^2 \times \frac{60°}{360}$$

$$= 4158\left[\left(\frac{\pi}{6}\right) \times (42^2 - 21^2)\right] = \mathbf{3465\ cm^2}$$

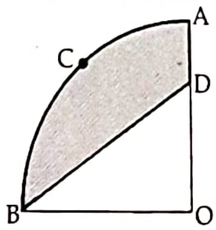
5. Area of the quadrant OACB = $\frac{1}{4}\pi r^2$

We can get the area of quadrant OACB with radius r = 3.5 cm

Area of shaded region = Area of quadrant OACB – Area of ΔBDO

Since ∠BOD = 90°,

ΔBDO is a right-angled triangle.



Area of quadrant, OACB = $\frac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (3.5\ \text{cm})^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\ \text{cm}^2 = \frac{77}{8}\ \text{cm}^2$$

In ΔBDO,

OB = r = 3.5 cm = $\frac{7}{2}$ cm and OD = 2 cm

Area of ΔBDO = $\frac{1}{2}$ × base × height

$$= \frac{1}{2} \times OB \times OD = \frac{1}{2} \times \frac{7}{2}\ \text{cm} \times 2\ \text{cm}$$

$$= \frac{7}{2}\ \text{cm}^2$$

From figure, it is observed that,

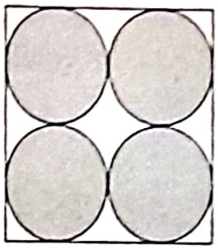
Area of shaded region = Area of Quadrant OACB – Area of ΔBDO

$$= \frac{77}{8}\ \text{cm}^2 - \frac{7}{2}\ \text{cm}^2$$

$$= \frac{77 - 28}{8}\ \text{cm}^2 = \frac{49}{8}\ \text{cm}^2$$

6. Area of square sheet = 576 cm²

Side² = 576



⇒ Side = $\sqrt{576}$ ∴ Side = 24 cm

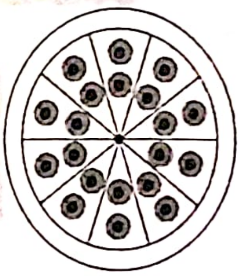
Radius of each circle = $\frac{24}{4}$ = 6 cm

Area of each circle = πr^2
= (3.14 × 6 × 6)
= 113.04 cm²

Area of 4 circles = 113.04 × 4
= 452.16 cm²

∴ **Area of square sheet not covered by the circle = 576 – 452.16 = 123.84 cm²**

7. (*a*) Given. d = 35 mm



Total length of silver wire required
= circumference of brooch + 5 × diameter

$$= \pi d + 5d = (\pi + 5) \times 35$$

$$= \left(\frac{22}{7} + 5\right) \times 35 = \frac{(22 + 35)}{7} \times 35$$

= 57 × 5 = **285 mm**

(*b*) Radius of the brooch (r) = $\frac{35}{2}$ mm

The wire divides the brooch into 10 equal sectors.

So, the angle of the sector (θ)

$$= \frac{360°}{10} = 36°$$

$\therefore$ Area of each sector of the brooch

$= \frac{36°}{360°} \times \pi r^2$

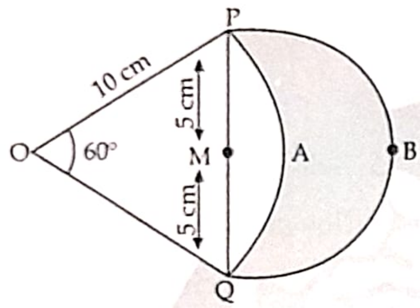
$= \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \text{ mm} \times \frac{35}{2} \text{ mm}$

$= \mathbf{96.25 \text{ mm}^2}$

8. Since OP = PQ = QO

$\Rightarrow$ POQ is an equilateral triangle.

$\therefore$ POQ = 60°



Area of segment PAQM

$= \pi r^2 \frac{\theta}{360°} - \frac{\sqrt{3}}{4} a^2$

$= \frac{60° \pi \times 10^2}{360°} - \frac{\sqrt{3}}{4} a^2$

$= \frac{100\pi}{6} - \frac{100\sqrt{3}}{2}$

$= \frac{50\pi}{3} - 25\sqrt{3} \text{ cm}^2$

Area of semicircle with M as centre is

$= \frac{\pi}{2} \times (5)^2 = \frac{25\pi}{2} \text{ cm}^2$

$\therefore$ **Area of shaded region**

$= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$

$= \frac{25\pi}{2} - \frac{50\pi}{3} + 25\sqrt{3} = \mathbf{25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ cm}^2}$

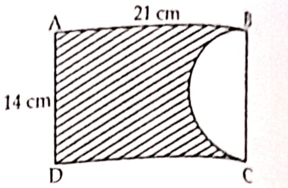
Long Answers

1. Area of shaded region

= *Area of rectangle – Area of semi-circle*

$= (21 \times 14) - \frac{1}{2} \times \pi \times 7 \times 7$

$= 294 - 77 = 217 \text{ cm}^2$



Perimeter of shaded region

$= 21 + 14 + 21 + \pi \times 7$

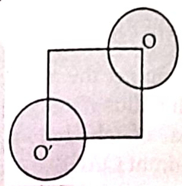
$= 56 + 22 = 78$ cm

Hence, area of shaded region = 217 c

and perimeter = 78 cm.

2. **Given.** Side = 28 cm

Radius = $\frac{28}{2}$ cm = **14 cm**



The area of the shaded region

= Area of square + 2(Area of major sector)

$= (\text{Side})^2 + 2\left(\frac{360° - \theta}{360°}\pi r^2\right)$

$= (28)^2 + 2\left(\frac{360° - 90°}{360°} \times \frac{22}{7} \times 14 \times 14\right)$

$= 784 + 2\left(\frac{270°}{360°} \times \frac{22}{7} \times 14 \times 14\right)$

$= 784 + 924 =$ **1708 cm²**

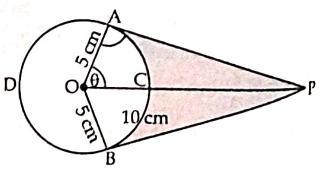
3. $\angle OAP = 90°$ ...[Tangent is ⊥ to the center through the point of contact

In rt. ΔOAP,

$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} = \cos 60°$

$\therefore \theta = 60°$

$\therefore AOB = 60° + 60° = 120°$ ...(i)



Reflex $\angle AOB = 360° - \angle AOB$

$= 360° - 120° = 240°$

$\alpha = 240°$

...[Let Reflex $\angle AOB = \alpha$

Them length of the belt that is still in contact with the pulley

= ADB = length of major arc

$= \left(\frac{\alpha}{360°}\right)2\pi r = \frac{240°}{360°} \times 2 \times 3.14 \times 5$

$= \frac{62.8}{3} =$ **20.93 cm**

In rt. ΔOAP, $\sin 60° = \frac{AP}{OP}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AP}{10}$ $\Rightarrow 2AP = 10\sqrt{3}$

$\Rightarrow AP = 5\sqrt{3}$ cm

Area of $\Delta OAP = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times AP \times OA$

$= \frac{1}{2} \times 5\sqrt{3} \times 5 = \frac{25}{2}\sqrt{3}$ cm²

Area(ΔOAP) = Area(ΔOBP) = $\frac{25}{2}\sqrt{3}$ cm²

Area of minor sector OACB = $\frac{\theta}{360°}\pi r^2$

$= \frac{120°}{360°} \times 3.14 \times (5)^2$ ...[From (i)

$= \frac{78.5}{3} =$ **26.16 or 26.17 cm² (approx.)**

$\therefore$ Shaded area = ar(ΔOAP) + ar(ΔOBP) – area of minor sector OACB

$= \frac{25\sqrt{3}}{2} + \frac{25\sqrt{3}}{2} - 26.17$

$= 25\sqrt{3} - 26.17$

$= 25(1.73) - 26.17$ ...[$\because \sqrt{3} = 1.73$

$= 43.25 - 26.17 =$ **17.08 cm²**

4. Radii of the inner semicircles HIJ and KLG (r_1) $= \frac{60}{2} = 30$ m

Radii of the outer semicircles BCD and EFA = 30 m + 10 m = 40 m

JK = GH = 106 m

DJ = HB = 10 m

(a) The distance around the track along its inner edge.

= GH + arc HIJ + JK + arc KLG

$= 106 + \frac{2\pi r_1}{2} + 106 + \frac{2\pi r_1}{2}$

$= 106 + \pi \times 30 + 106 + \pi \times 30$

$= 212 + \frac{1320}{7} = \frac{(1484 + 1320)}{7} = \frac{2804}{7}$ m

(b) Radius of semicircle BCD

= Radius of semicircle EFA

(r_2) = 30 m + 10 = 40 m

Area of the track = *Area of rectangle ABHG + Area of rectangle KJDE + (Area of semicircle BCD – Area of semicircle HIJ) + (Area of semicircle EFA – Area of semicircle KLG)*

$= (106 \times 10) + (106 \times 10) +$

$\left[\frac{1}{2}\pi(40)^2 - \frac{1}{2}\pi(30)^2\right] + \left[\frac{1}{2}\pi(40)^2 - \frac{1}{2}\pi(30)^2\right]$

$= 1{,}060 + 1{,}060 + \left[\frac{1}{2}\pi(1600 - 900)\right] +$

$\left[\frac{1}{2}\pi(1600 - 900)\right]$

$= 1{,}060 + 1{,}060 + \frac{\pi}{2} \times 700 + \frac{\pi}{2} \times 700$

$= 2{,}120 + 700\pi$

$= 2{,}120 + 700 \times \frac{22}{7}$

$= 2{,}120 + 2{,}200 =$ **4,320 m²**

Case Based Answers

1. **(a)** Total perimeter of parking area

$= 7 + (2\pi r)\frac{1}{2}$ units

$= 7 + \left[2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{1}{2}\right]$

$= 7 + 11 =$ **18 units**

(b) (i) The area of the parking and the two quadrants can be found by adding the area of the semi-circle and the area of the two quadrants.

Area of the semi-circle is $\frac{1}{2}\pi r^2$.

$$= \frac{1}{2}\pi(2)^2 = 2\pi \text{ square units}$$

Total area = *Area of semi-circle + Area of two quadrants*

$$= 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$$

$$= \mathbf{7.85 \text{ square units}}$$

Or

(ii) Area of playground : Area of parking area

$$= 98 : \left(\frac{5\pi}{2}\right) = 98 : 7.85 = \mathbf{12.46 : 1}$$

Therefore, the area of the playground is about 12.46 times greater than the area of the parking area.

(c) The perimeter of the playground is 2(*length* + *breadth*) = 2(14 + 7) = 42 units. The perimeter of the parking area is the same as we found in part (i), which is approximately 33.85 units.

Total perimeter = *Perimeter of playground + Perimeter of parking area*

Total perimeter = 42 + 33.85

= **75.85 units**

Multiplying this by ₹2 per unit, we get the cost of fencing the playground and parking area.

Cost of fencing = *Total perimeter* × *₹2 per unit*

Cost of fencing = 75.85 × 2 = **₹151.70**

Therefore, the cost of fencing the playground and parking area at the rate of ₹2 per unit is approximately ₹151.70.

2. (a) In one hour, the min hand turns 360° now, in one min the min hand turns

$$= \frac{360^\circ}{60^\circ} = 60^\circ$$

(b) (i) The length of the minute hand in r = 3.5 cm. The distance moved by the tip of the minute hand in one

hour is the circumference of circle.
So, the circumference of circle

$= 2\pi r = 2 \times \frac{22}{7} \times 3.5 = 22$ cm

Or

(*ii*) Distance moved by the tip = $2\pi r$

$= 2 \times \frac{22}{7} \times \frac{7}{2} = 22$ cm

(*c*) We have to find the area swept by the hour hand in three hours if the length of the hour hand is $r = 2.1$ cm.
In 12 hours, the hour hand will turn 360°
In three hours, the hour hand will turn $\frac{360°}{12} \times 3 = 90°$ angle subtended $\theta = 90°$.

Now, the area swept by the hour hand in three hours

$= \frac{\theta}{360°} \times \pi r^2 = \frac{90}{360°} \times \frac{22}{7} \times 2.1 \times 2.1$

$= \frac{1}{4} \times 22 \times 0.63 = 3.465 \text{ cm}^2$

3. (*a*) Area of the sector ODCO $= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$

(*b*) (*i*) Area of the $\Delta AOB = \frac{1}{2} \times OA \times OB$

$= \frac{1}{2}(20 \times 20) = 200 \text{ cm}^2$

Or

(*ii*) Area of region which is golden plated
= area of ∠OAB
− area of sector ∠DCO
$= 200 - 154 = 46 \text{ cm}^2$
Therefore, Total cost of golden plating = ₹(6 × 46) = ₹276

(*c*) Area of the major sector = Area of the circle − Area of the minor sector

$= \pi r^2 - \frac{1}{4}\pi r^2$

$= \frac{3\pi r^2}{4} = \frac{3}{4} \times \frac{22}{7} \times 14 \times 14 = 462 \text{ cm}^2$

(DAY 11 SWAHA)

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6 Polynomials

What did CBSE ask last year?

MCQs		1 Question (1 × 1 = 1 Mark)
Subjective		1 Very Short Question (1 × 2 = 2 Marks)
		No Short Questions
		1 Long Question (1 × 5 = 5 Marks)
Case Based		No Case Based Questions

Note: All the above typology of questions include 'Competency based Questions' labelled as COMPETENCY

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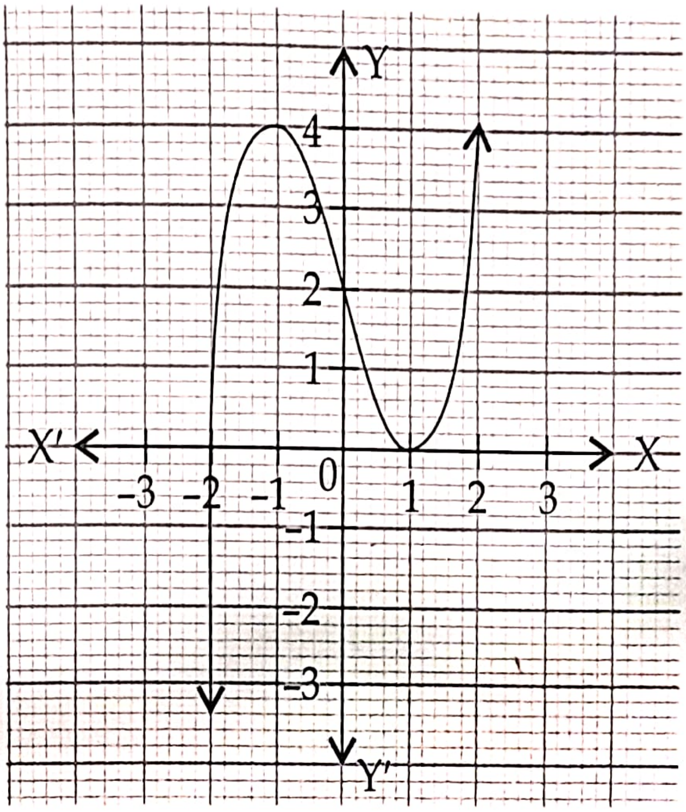
Introduction to the chapter

- 'Polynomial' comes from the word 'Poly' (Meaning Many) and 'nomial' (in this case meaning Term)—so it means many terms.
- A polynomial is made up of terms that are only added, subtracted or multiplied.
- A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
- Degree. The highest exponent of the variable in the polynomial is called the degree of polynomial.

  For Example: $3x^3 + 4$; here degree = 3.
- Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomial respectively. A polynomial can have terms which have Constant like 3, –20 etc., Variables like x and y and Exponents like 2 in y^2.
- These can be combined using addition, subtraction and multiplication but Not DIVISION.

Note: *MCQs are asked frequently from this topic.*

POLYN



MIALS

Relationship between the Zeroes of a polynomial

If a and b are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then

$$\alpha+\beta=-\frac{b}{a} \text{ and } \alpha\beta=\frac{c}{a}$$

Note: *CBSE's most loved topic for very short & short answer questions.*

OBJECTIVE QUESTIONS

(DAY 12)

Multiple Choice Questions

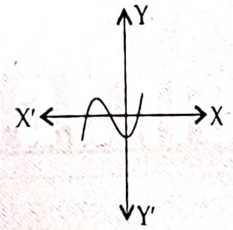
Q.1. The number of quadratic polynomials having zeroes -5 and -3 is [CBSE 2023]

(a) 1
(b) 2
(c) 3
(d) more than 3

Q.2. The graph of a polynomial $p(x)$ cuts the x-axis at 3 points and touches it at 2 other points. The number zeroes of $p(x)$ is: COMPETENCY

(a) 1 (b) 2
(c) 4 (d) 5

Q.3. In fig., the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is: COMPETENCY



(a) 1
(b) 2
(c) 3
(d) 4

Q.4. If $x - 1$ is a factor of the polynomial $p(x) = x^3 + ax^2 + 2b$ and $a + b = 4$, then [CBSE 2022]

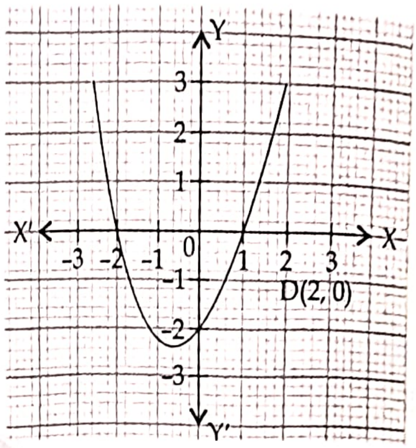
(a) $a = 5, b = -1$
(b) $a = 9, b = -5$
(c) $a = 7, b = -3$
(d) $a = 3, b = 1$

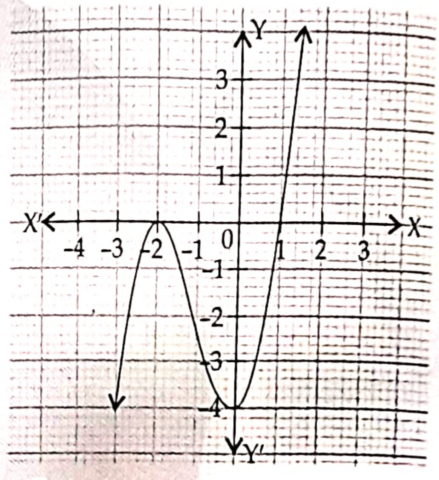
Q.5. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is: [CBSE 2020]

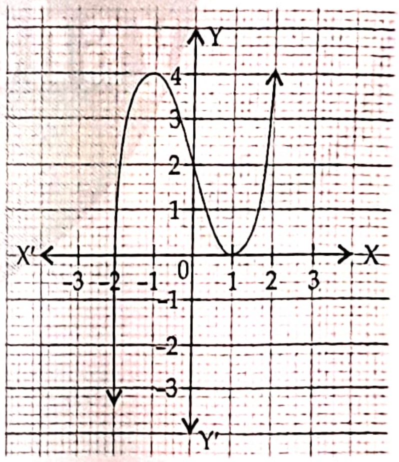
(a) 10 (b) -10
(c) -7 (d) -2

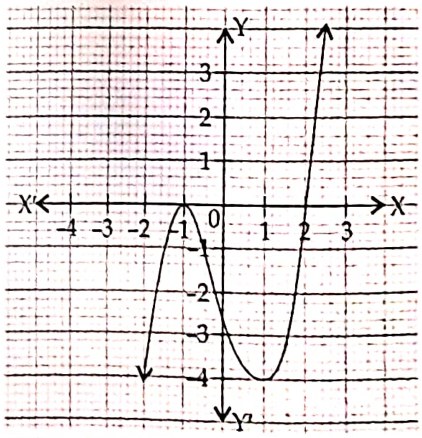
Q.6. Which of the following could be the graph of the polynomial? $(x - 1)(x + 2)$? COMPETENCY

FREE ADVICE: Zeroes of the polynomial will cut the x-axis of the graph.

(a)


(b)


(c)


(d)


Q.7. At which point will the graph of the polynomial $p(x) = -x + 6x^2 - 1$ intersects the negative x-axis?

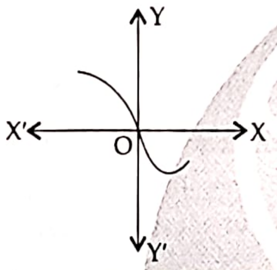
(a) only $\frac{-1}{3}$

(b) only $\frac{-1}{2}$

(c) both $\frac{-1}{3}$ and $\frac{-1}{2}$

(d) none, it never intersects negative x-axis

Q.8. Find the number of the zeros of the polynomial $y = f(x)$. COMPETENCY

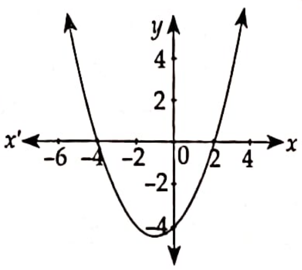


(a) 1 (b) 2
(c) 3 (d) 4

Q.9. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3, then COMPETENCY

(a) $a = -7; b = -1$ (b) $a = 5; b = -1$
(c) $a = 2; b = -6$ (d) $a = 0; b = -6$

Q.10. Shown below is the graph of a quadratic polynomial.



Which of these is the polynomial graphed above?

(a) $(x - 2)(x + 4)$

(b) $(x - 4)(x + 2)$

(c) $\frac{1}{2}(x - 2)(x + 4)$

(d) $\frac{1}{2}(x - 4)(x + 2)$

Q.11. Which of these are the zeros of the polynomial $x(x - 7)$?

(a) only 0 (b) only 7
(c) both 0 and 7
(d) the polynomial does not have any 0

Q.12. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are COMPETENCY

(a) both $+ve$ (b) both $-ve$
(c) one $+ve$ and one $-ve$
(d) both equal

Q.13. The zeroes of the quadratic polynomial $x^2 + kx + k, k \neq 0$ is COMPETENCY

(a) both cannot be $+ve$
(b) both cannot be $-ve$
(c) are always unequal
(d) are always equal

Q.14. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then COMPETENCY

(a) c and a have opposite signs
(b) c and b have opposite signs
(c) c and a have Same signs
(d) c and b have same signs

Q.15. The quadratic polynomial, sum of whose zeroes is -5 and their product is 6 is: [CBSE 2020

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$
(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Q.16. If α, β are the zeroes of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$ then the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$, is: COMPETENCY

(a) -7 (b) 7 (c) 3 (d) -3

Q.17. $p(x)$ is a polynomial given by:

$p(x) = -2x + 8x^2 - 1$

At which of the following points will the graph of $p(x)$ intersect the positive x-axis? [CBSE 2024

(i) $\frac{1}{2}$ (ii) $\frac{1}{4}$

(a) only (i) (b) only (ii)
(c) both (i) and (ii)
(d) none, it never intersects at positive x-axis

Q.18. Which of these are the zeroes of $x^2 + 7x + 12$?

(a) 3 and 4 (b) (-3) and (-4)

(c) (-3) and 4 (d) 3 and (-4)

Q.19. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

COMPETENCY

(a) $b - a + 1$ (b) $b - a - 1$

(c) $a - b + 1$ (d) $a - b - 1$

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

(b) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*

(c) *(A) is true and (R) is false.*

(d) *(A) is false, but (R) is true.*

Q.1. Assertion: $x^2 + 7x + 12$ has no zeroes.

Reason: A quadratic polynomial can have at the most two zeroes.

Q.2. Assertion: If the sum of the zeroes of the quadratic polynomial $x^2 - 2kx + 8$ is 2 then value of k is 1.

Reason: Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$.

Q.3. Assertion: If both zeroes of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is $\frac{1}{2}$.

COMPETENCY

Reason: Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$.

Q.4. Assertion: $x^3 + x$ has only one real zero.

Reason: A polynomial of n^{th} degree must have n real zeroes.

COMPETENCY

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

[CBSE 2013

Q.2. $p(x) = (x + 5)^2 - 7(x - k)$; where k is a constant.

If $p(x)$ is divisible by x, find the value k. Show your steps. [CBSE 2024

Q.3. If α, β are zeroes of the quadratic polynomial $ax^2 + bx + c$ then find the value of $\alpha^2 + \beta^2$. COMPETENCY

Q.4. Find the value of k, if the quadratic equation $kx(x - 2) + 6$ has two equal roots. [NCERT

Q.5. $f(x) = x^2 + 10x + 21$

Find the zeroes of the above polynomial. Show your work.

Q.6. Find the condition that zeroes of polynomial $g(x) = ax^2 + bx + c$ are reciprocal of each other. [CBSE 2017

Q.7. Find the quadratic polynomial, sum and product of whose zeroes are -1 and -20 respectively. Also, find the zeroes of the polynomial so obtained.

COMPETENCY

Q.8. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

COMPETENCY

(DAY 13)

Short Answer Questions

Q.1. Find the quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomials $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$. [CBSE 2020

Q.2. Find the zeroes of the polynomial $x^2 - 3x - m(m + 3)$. [CBSE 2020

Q.3. Find the zeroes of the polynomials $3x^2 + 4x - 4$ by factorisation method.

COMPETENCY

Q.4. If the sum of the squares of zeroes of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k. *[CBSE 2020*

Q.5. Aasira multiplied a variable with 4, subtracted 12 and added the square of the original variable. She expressed the final expression as a product of 2 factors.

Her friend, Rishi, said that the factors will always have a difference of 8.

Is Rishi right? Show your work.

Q.6. Find the quadratic polynomial, sum and product of whose zeroes are –1 and –20 respectively. Also find the zeroes of the polynomial so obtained. *[CBSE 2019*

FREE ADVICE: *Padhle Gang! Quadratic ka equation ka general form jaroor likhna*

Q.7. $p(x)$ is a polynomial given by $ax^2 - 4x + 3$, where a is a non-zero real number. One of the zeroes of $p(x)$ is 3 times the other zero.

(*i*) Find the value of a.
Show your work.

(*ii*) Based on the value of a, what would be the shape of the graph of $p(x)$? Give a reason for your answer.

Q.8. A polynomial is given by $p(x) = x^3 + 3x^2 - 4x + c$, where c is a constant.

The sum of two zeroes of $p(x)$ is zero. Using the relationship between the zeroes and coefficients of a polynomial, find the:

(*i*) zeroes of $p(x)$. (*ii*) value of c.

Show your steps.

Long Answer Questions

Q.1. If α and β are the zeroes of $x^2 + px + q$ then find the value of $\left(\frac{\alpha}{\beta} + 2\right)\left(\frac{\beta}{\alpha} + 2\right)$.

COMPETENCY

Q.2. If one zero of $f(x) = 4x^2 - 8kx + 8x - 9$ is negative of the other, find zeroes of $kx^2 + 3kx + 2$. *[CBSE 2015*

Q.3. If α and β are the zeroes of quadratic polynomial $f(x) = kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, find the value of k.

COMPETENCY

Q.4. $g(x) = px^2 + qx + 152$ is a polynomial where p and q are real numbers. The zeroes of $g(x)$ are distinct prime numbers. **Find the:** *[CBSE 2024*

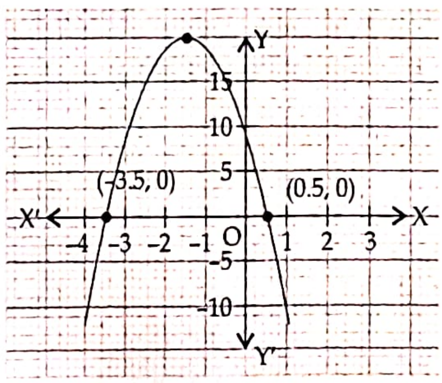
(*i*) zeroes of $g(x)$.

(*ii*) values of p and q.

Show your work and give valid reasons.

CASE BASED QUESTIONS

Q.1. Just before the morning assembly a teacher of New RSJ public school observes some clouds in the sky and so she cancels the assembly. She also observes that the clouds has a shape of the polynomial. The mathematical representation of a cloud is shown in the figure.



Based on the above information, answer the following questions.

(*a*) Find the zeroes of the polynomial represented by the graph.

(*b*) (*i*) What will be the expression for the polynomial represented by the graph? **COMPETENCY**

Or

(*ii*) What will be the value of polynomial represented by the graph when $x = 3$?

(*c*) If α and β are the zeroes of the polynomial $f(x) = x^2 + 2x - 8$ then what will be the value of $\alpha^4 + \beta^4$? **COMPETENCY**

Q.2. The figure given alongside shows the path of a diver, when she takes a jump from the diving board. Clearly it is a parabola.

Annie was standing on a diving board, 48 feet above the water level. She took a dive into the pool. Her height (in feet) above the water level at any time 't' in seconds is given by the polynomial $h(t)$ such that $h(t) = -16t^2 + 8t + k$.

Based on the above information, answer the following questions.

(*a*) What is the value of the k?

(*b*) (*i*) At what time will she touch the water in the pool?

Or

(*ii*) Rita's height (in feet) above the water level is given by another polynomial $p(t)$ with zeroes 1 and 2. Then write $p(t)$.

(*c*) A polynomial $q(t)$ with sum of zeroes as 1 and the product as is modeling Anu's height in feet above the water at any time t (in seconds). Then $q(t)$ is given by

Q.3. A local sports complex is planning to build a new stadium with a seating capacity for various events. The stadium's shape follows a parabolic design, and its seating capacity can be modeled by the quadratic polynomial similarly there are other examples as well related to polynomials.

Based on the above information, answer the following questions.

(*a*) Find the value of x for which the graph of $p(x) = x^2 - 2$ intersects the x-axis.

(*b*) (*i*) If α and β are the zeroes of the polynomial $p(x) = ax^2 + bx + c$ then find $\alpha^2\beta + \beta^2\alpha$. **COMPETENCY**

Or

(*ii*) If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $p(x) = 3x^2 - 2x + 5k$ then find the value of k.

(*c*) Find quadratic polynomial having sum and product of zeroes as α and $\frac{-1}{\alpha}$ respectively: **COMPETENCY**

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ANSWERS

Multiple Choice Answers

1. **(a)** We know that if a quadratic polynomial has zeroes at -5 and -3, $(x + 5)$ and $(x + 3)$ are the factors of the resultant polynomial, and the resultant polynomial can be written in the form $(x + 5)(x + 3) = x^2 + 8x + 15$. So, there is only **one** quadratic polynomial that has zeroes -5 and -3 and it is $x^2 + 8x + 15$.

2. **(d)** Number of Zeroes = Number of times graph touches x-axis
$= 3 + 2 = 5$

3. **(c)** 3

4. **(b)** Since $x - 1$ is a factor of polynomial $p(x) = x^3 + ax^2 + 2b$

$\therefore$ $x = 1$ is a factor of polynomial $p(x)$.

Since $x = 1$ is a zero,

$\therefore$ $p(1) = 0$

$(1)^3 + a(1)^2 + 2b = 0$

$1 + a + 2b = 0$

$a + 2b = -1$

Now, our equations are

$a + b = 4$ ...(i)

$a + 2b = -1$ ...(ii)

Doing (ii) – (i)

$(a + 2b) - (a + b) = -1 - 4$

$a + 2b - a - b = -5$

$\therefore$ $b = -5$

From (i), $a - 5 = 4$

$\Rightarrow a = 9$

5. **(b)** Given polynomial $x^2 + 3x + k = f(x)$

Also, given that 2 is one of the zeroes of the polynomial.

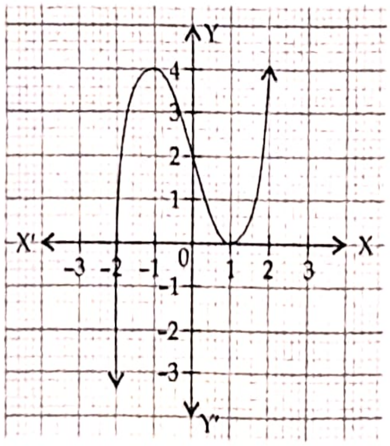
$\therefore$ $f(2) = 0$

$\Rightarrow 4 + 6 + k = 0$

$\Rightarrow k = -10$

Hence, the value of k is -10.

6. **(c)**



7. **(a)** We have, $-x + 6x^2 - 1$

$\Rightarrow 6x^2 - x - 1$

$\Rightarrow 6x^2 - 3x + 2x - 1$

$\Rightarrow (6x^2 - 3x) + (2x - 1)$

$\Rightarrow 3x(2x - 1) + 1(2x - 1)$

$\Rightarrow 3x + 1 = 0$ or $2x - 1 = 0$

$\Rightarrow 3x = -1$ or $2x = 1$

$\therefore$ $x = \frac{-1}{3}$ or $x = \frac{1}{2}$

As $x = \frac{-1}{3}$

it will intersect at negative x-axis.

8. **(a)** The number of zeroes of $p(x)$ is the number of times the curve intersects the x-axis, *i.e.*, attains the value 0.

Here, the polynomial $p(x)$ meets the x-axis at 1 point.

So, number of zeroes = 1.

9. **(d)** Given. $f(x) = x^2 + (a + 1)x + b$.

2 and -3 are zeroes of $f(x)$.

$\therefore$ $f(2) = 0$

$\Rightarrow (2)^2 + (a + 1) \times 2 + b = 0$

$\Rightarrow 4 + 2a + 2 + b = 0$

$\Rightarrow 6 + 2a + b = 0$ ...(i)

and $f(-3) = 0$

$\Rightarrow (-3)^2 + (a + 1) \times (-3) + b = 0$

$\Rightarrow 9 - 3a - 3 + b = 0$

$\Rightarrow 6 - 3a + b = 0$ ...(ii)

On subtracting Eq. (*ii*) from Eq. (*i*),
We get

$5a = 0 \quad \Rightarrow a = 0$

$\Rightarrow 6 - 3a + b = 0$ ...(*ii*)

$\Rightarrow 6 + b = 0$ [From Eq. (*i*)

$\Rightarrow b = -6$

Hence, $a = 0$ and $b = -6$

10. (*a*) Sum of roots $(\alpha + \beta) = -4 + 2 = -2$

Product of roots $(\alpha\beta) = -4 \times 2 = -8$

$x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow x^2 - (-2)x - 8$

$\Rightarrow x^2 + 2x - 8$

$\Rightarrow x^2 + 4x - 2x - 8$

$\Rightarrow x(x + 4) - 2(x + 4)$

$\therefore$ **$(x + 4)(x - 2)$**

11. (*c*) $x(x - 7) = 0$

$\therefore x = 0;$ and $x = 7$

12. (*b*) Given polynomial is $x^2 + 99x + 27$

Sum of zeroes = -99

Product of zeroes = 27

$\because$ Sum of zeroes is negative and product of zeroes is positive, this is only possible if both should be negative.

$\therefore$ **Both zeros will be negative.**

13. (*a*) Product of zeroes = k

The sign is positive it means both the zeroes have same sign.

Sum of zeroes = $-k$

The sign is negative and both have same sign hence the zeroes are both negative.

14. (*c*) Given that the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal.

$\Rightarrow$ Value of the discriminant (D) has to be zero.

$\Rightarrow b^2 - 4ac = 0 \quad \Rightarrow b^2 = 4ac$

Since L.H.S b^2 cannot be negative, thus, R.H.S. can also be never negative.

Therefore, a and c must be of the same sign.

15. (*a*) Sum of zeroes = -5

Product of zeroes = 6

Equation of quadratic polynomial =

x^2 -(sum of zeroes)x + product of zeroes = 0

$x^2 - (-5)x + 6 = 0$

$\therefore$ **$x^2 + 5x + 6 = 0$**

16. (*b*) α, β are the zeroes of $x^2 - (k + 6)x + 2(2k - 1)$ *i.e.*, $\alpha + \beta = k + 6$ (Sum of roots of quadratic)

and $\alpha\beta = 2(2k - 1)$ (Product of roots)

According to question, $\alpha + \beta = \frac{1}{2}\alpha\beta$

$\Rightarrow k + 6 = \frac{2(2k-1)}{2} = 2k - 1$

$\Rightarrow k + 6 = 2k - 1$

$\Rightarrow k = 7$

Hence, the answer is 7.

17. (*a*) $-2x + 8x^2 - 1$

$\Rightarrow 8x^2 - 2x - 1$

$\Rightarrow 8x^2 - 4x + 2x - 1$

$\Rightarrow (8x^2 - 4x) + (2x - 1)$

$\Rightarrow 4x(2x - 1) + 1(2x - 1)$

$\Rightarrow 4x + 1 = 0$ or $2x - 1 = 0$

$\Rightarrow 4x = -1$ or $2x = 1$

$\therefore x = \frac{-1}{4}$ or $x = \frac{1}{2}$

So, at point $x = \frac{1}{2}$ it will intersect at positive x-axis.

18. (*b*) We have, $x^2 + 7x + 12$

$\Rightarrow x^2 + 4x + 3x + 12$

$\Rightarrow (x^2 + 4x) + (3x + 12)$

$\Rightarrow x(x + 4) + 3(x + 4)$

$\Rightarrow (x + 4)(x + 3)$

$\Rightarrow x + 4 = 0$ or $x + 3 = 0$

$\therefore x = -4$ or $x = -3$

So, zeroes are -4 and -3.

19. (*a*) Given, the cubic polynomial is $x^3 + ax^2 + bx + c$.

One of the zeroes of the polynomial is -1.

We have to find the product of the other two zeroes.

We know that, if α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

Given, $\alpha = -1$

Here, $a = 1$, $b = a$, $c = b$ and $d = c$

...*Where* $\left[\begin{array}{l} a = \text{coefficient of } x^3 \text{ term} \\ b = \text{coefficient of } x^2 \text{ term} \\ c = \text{coefficient of } x \\ d = \text{coefficient of constant term} \end{array}\right.$

By the property of polynomials,

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$(-1) + \beta + \gamma = \frac{-a}{1}$$

$$\beta + \gamma = -a + 1 \qquad ...(i)$$

$$\text{Now, } \alpha\beta + \beta\gamma + \gamma\alpha = \frac{b}{1}$$

$$(-1)\beta + \beta\gamma + \gamma(-1) = b$$

$$\beta\gamma - \beta - \gamma = b$$

$$\beta\gamma - (\beta + \gamma) = b$$

$$\beta\gamma - (-a + 1) = b \qquad ...[\text{From } (i)$$

$$\beta\gamma + a - 1 = b$$

$$\therefore\ \boldsymbol{\beta\gamma = b - a + 1}$$

Assertion Reason Answers

1. **(*d*)** (A) is false, but (R) is true.
2. **(*a*)** Both (A) and (R) are true and (R) is the correct explanation of (A).
3. **(*d*)** (A) is false, but (R) is true.

   Explanation: *As the polynomial is $x^2 - 2kx + 2$ and its zeroes are equal but opposition sign.*

   Therefore, sum of the zeroes of the polynomial $= \frac{-b}{a} = \frac{-(-2k)}{1} = 0.$

   $\Rightarrow 2k = 0 \qquad \Rightarrow \boldsymbol{k = 0}$
4. **(*c*)** (A) is true and (R) is false.

   Explanation:

   It is given by the $D = b^2 - 4ac$

   If $D > 0$ *it has real roots.*

Very Short Answers

1. By splitting the middle term, we get

   $$f(x) = \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$$
   $$= \sqrt{3}\ x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$$
   $$= (\sqrt{3}x - 2)(x - 2\sqrt{3})$$

   On putting $f(x) = 0$, we get

   $$(\sqrt{3}x - 2)(x - 2\sqrt{3}) = 0$$

   $\Rightarrow \sqrt{3}x - 2 = 0$ or $x - 2\sqrt{3} = 0$

   $\therefore\ x = \boldsymbol{\frac{2}{\sqrt{3}}}$ or $\boldsymbol{x = 2\sqrt{3}}$
2. $p(x) = (x + 5)^2 - 7(x - k)$

   If $p(x)$ is divisible by x, then $x = 0$

   Now, $p(0) = (0 + 5)^2 - 7(0 - k)$

   $= 25 - 7(-k) = 25 + 7k$

   As, $p(x)$ is divisible by x,

   $25 + 7k = 0$

   $\Rightarrow 7k = -25 \qquad \therefore\ \boldsymbol{k = \frac{-25}{7}}$
3. As given α and β are the zeroes of the polynomial $ax^2 + bx + c$.

   Sum of the zeroes, $\alpha + \beta = \frac{-b}{a}$

   Product of the zeroes, $\alpha\beta = \frac{c}{a}$

   $\Rightarrow (\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

   $\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

   $\Rightarrow \alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}$

   $\Rightarrow \alpha^2 + \beta^2 = \frac{b^2}{a^2} - \frac{2c}{a}$

   $\therefore\ \alpha^2 + \beta^2 = \boldsymbol{\frac{(b^2 - 2ac)}{a^2}}$
4. **Given.** $kx(x - 2) + 6 = 0$

   $kx^2 - 2kx + 6 = 0$

   Since the roots are equal,

   $\therefore\ D = b^2 - 4ac = (-2k)^2 - 4(k)(6) = 0$

   $\Rightarrow 4k^2 = 4k(6) \qquad \therefore\ \boldsymbol{k = 6}$
5. $f(x) = x^2 + 10x + 21 = x^2 + 3x + 7x + 21$

   $\Rightarrow (x^2 + 3x) + (7x + 21)$

   $\Rightarrow x(x + 3) + 7(x + 3)$

   $\Rightarrow (x + 3)(x + 7)$

   $\therefore\ x = -3$ or $x = -7$

   So, **Zeroes are -3 and -7.**

6. The product of the roots of the quadratic equation is the ratio of the coefficient of x^0 and x^2.

Here, the coefficient of x^2 is a, while the coefficient of x^0 is c.

Hence, $m \times \frac{1}{m} = ca$

$1 = ca$

$\Rightarrow c = \frac{1}{a}$

Hence, $c = \frac{1}{a}$ is one of the conditions that the zeroes of the polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

7. As we know, Quadratic polynomial, $p(x)$
$= [x^2 - (\text{sum of zeores})\ x + (\text{product of zeroes})]$
$= [(x^2 - (-1)\ x + (-20)]$
$= (x^2 + 1\ x - 20)$
$= (x^2 + 5x - 4x - 20) = 0$ ...[On solving
$= [x(x + 5) - 4(x + 5)] = 0$
$(x - 4)\ (x + 5) = 0$
$x - 4 = 0$ or $x + 5 = 0$
$\therefore x = 4$ or $x = -5$

8. $2x^2 - 3x + 1 = 0$
$2x^2 - 2x - x + 1 = 0$
$\Rightarrow 2x(x - 1) - 1(x - 1) = 0$
$\Rightarrow (x - 1)(2x - 1) = 0$
$\therefore x = 1$ or $x = \frac{1}{2}$
$\therefore \alpha = 1;\quad \beta = \frac{1}{2}$
$3\alpha = 3 \times 1 = 3$
$3\beta = 3 \times \frac{1}{2} = \frac{3}{2}$

So, polynomial is
$x^2 - (3\alpha + 3\beta)x + (3\alpha)(3\beta) = 0.$
$x^2 - \left(3 + \frac{3}{2}\right)x + (3)\left(\frac{3}{2}\right) = 0$
$x^2 - \frac{9x}{2} + \frac{9}{2} = 0$
$2x^2 - 9x + 9 = 0$

So, K[$2x^2 - 9x + 9 = 0$] is the required polynomial.

Short Answers

1. Let p and q be zeroes of $ax^2 + bx + c$.

$\therefore p + q = \frac{-b}{a}$ & $pq = \frac{c}{a}$

Let p & q be zeroes of required polynomial.

It is given that $P = \frac{1}{p}$ and $Q = \frac{1}{q}$
Then,

$P + Q = \frac{1}{p} + \frac{1}{q} = \frac{(p + q)}{pq}$

$= -\frac{-ba}{ca} = \frac{-b}{c}$

$PQ = \frac{1}{p}.\frac{1}{q} = \frac{1}{pq} = \frac{a}{c}$

$\therefore$ **Required polynomial,**

$x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c} = 0$

$x^2 + \frac{bx}{c} + \frac{a}{c} = 0$

$cx^2 + bx + a$

2. Let $f(x) = x^2 - 3x - m(m + 3)$

Above polynomial can be written as,
$f(x) = x^2 - (m + 3)x + mx - m(m + 3)$
$= x(x - m - 3) + m(x - m - 3)$
$= (x - m - 3)(x + m)$

To find the zeroes of $f(x)$, put $f(x) = 0$
$(x - m - 3)(x + m) = 0$
$x - m - 3 = 0$ or $x + m = 0$
$\therefore x = m + 3$ or $x = -m$

Required zeroes are $(m + 3)$ and $-m$.

3. We have, $3x^2 + 4x - 4$

Splitting the middle term, we get
$3x^2 + 6x - 2x - 4$
Taking the common factors out, we get
$3x(x + 2) -2(x + 2)$
On grouping, we get, $(x + 2)(3x - 2)$
So the zeroes are:
$x + 2 = 0 \quad \Rightarrow x = -2$
$3x - 2 = 0 \quad \Rightarrow 3x = 2$
$\therefore x = \frac{2}{3}$

Therefore, zeroes are $\left(\frac{2}{3}\right)$ and -2.

Verification:

Sum of the zeroes = $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

$\therefore \alpha + \beta = \frac{-b}{a}$

$-2 + \left(\frac{2}{3}\right) = -\frac{(4)}{3} = -\frac{4}{3}$

Product of the zeroes = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$\therefore \alpha\beta = \frac{c}{a}$

Product of the zeroes = $(-2)\left(\frac{2}{3}\right) = \frac{-4}{3}$

4. $f(x) = x^2 - 8x + k = 40.$

Let the zeroes of polynomial $f(x)$ be α and β.

Here, $\alpha + \beta = \frac{-b}{a} = 8$ and $\alpha\beta = \frac{c}{a} = k$

Given that, $\alpha^2 + \beta^2 = 40$

$(\alpha + \beta)^2 - 2\alpha\beta = 40$

$64 - 2k = 40 \quad \therefore k = 12$

Hence value of k is 12.

5. Let the original variable = x

ATQ, expression is = $4x - 12 + x^2$

$\Rightarrow x^2 + 4x - 12$

$\Rightarrow x^2 + 6x - 2x - 12$

$\Rightarrow (x^2 + 6x) + (-2x - 12)$

$\Rightarrow x(x + 6) - 2(x + 6)$

$\Rightarrow (x - 2)(x + 6) = 0$

So, expression as a product of two factors are $(x - 2)$ and $(x + 6)$.

Yes, Rishi was right as above factors have a difference of 8.

6. **Given.** sum of zeroes $(\alpha + \beta) = -1$,

Product of zeroes = -20

Let α & β be the zeroes of the polynomial.

$\therefore p(x) = x^2 - (\text{sum})x + \text{product}$

$= x^2 - (-1)x + (-20)$

$p(x) = x^2 + x - 20$

$= x^2 + 5x - 4x - 20$

$= x(x + 5) - 4(x + 5) = (x - 4)(x + 5)$

$\therefore x = 4 \quad \text{or} \quad x = -5$

Hence, the required polynomial is $p(x) = x^2 + x - 20$ and zeroes are 4 & -5.

7. We have, $p(x) = ax^2 - 4x + 3$

(*i*) Let one zero of the polynomial $(\alpha) = m$ and other zero of the polynomial, $(\beta) = 3m$

Now, Sum of zeroes = $\frac{-b}{a}$

$\alpha + \beta = \frac{-(-4)}{a}$

$m + 3m = \frac{4}{a}$

$\Rightarrow 4m = \frac{4}{a} \quad \Rightarrow m = \frac{4}{a} \times \frac{1}{4}$

$\therefore m = \frac{1}{a}$ ...[A

Product of zeroes $(\alpha\beta) = \frac{c}{a}$

$m \,.\, 3m = \frac{3}{a}$

$\Rightarrow 3m^2 = \frac{3}{a}$

$\Rightarrow 3\left(\frac{1}{a}\right)^2 = \frac{3}{a}$ ...[From (A)

$\Rightarrow 3\left(\frac{1}{a^2}\right) = \frac{3}{a} \quad \Rightarrow \frac{3}{a^2} = \frac{3}{a}$

$\Rightarrow \frac{1}{a} = 1 \quad \therefore a = 1$

(*ii*) Since a is positive, therefore the graph of $p(x)$ is an open upward parabola or open upward like 'U' shaped.

8. $p(x) = x^3 + 3x^2 - 4x + c$

(*i*) Let the values of zeroes of $p(x)$ be $= (-\alpha)$, α and β.

$\alpha + \beta + \gamma = \frac{-b}{a}$

Sum of zeroes = -3

$-\alpha + \alpha + \beta = -3$

$\beta = -3.$

Sum of product of zeroes taken two at a time.

$\alpha\beta - \beta\gamma + \gamma\alpha = \frac{c}{a}$

$-\alpha^2 - \alpha\beta + \beta\alpha = -4$

$-\alpha^2 - \alpha(-3) + (-3)\alpha = -4$

$\Rightarrow -\alpha^2 + 3\alpha - 3\alpha = -4$

$\Rightarrow -\alpha^2 = -4$

$\Rightarrow \alpha^2 = 4$

$\Rightarrow \alpha = \pm\sqrt{4}$

$\therefore \alpha = \pm 2$

So, zeroes of $P(x)$ are -2, + 2 and -3.

(*ii*) Product of zeroes $(-\alpha^2\beta) = -c$

$-\alpha^2\beta = -c$

$(2)^2(-3) = c$

$c = -12$

Long Answers

1. We have, α and β are the zeroes of polynomial $x^2 + px + q$.

Relationship between Zeroes are :

Sum of Zeroes $= \frac{-b}{a} = \frac{-p}{1} = -p$

Product of Zeroes $= \frac{c}{a} = \frac{q}{1} = q$

Now, We have to find the value of $\left(\frac{\alpha}{\beta}+2\right)\left(\frac{\beta}{\alpha}+2\right)$.

$= \frac{\alpha}{\beta}\left(\frac{\beta}{\alpha}+2\right) + 2\left(\frac{\beta}{\alpha}+2\right)$

$= 1 + \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} + 4$

$= 5 + 2\left[\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right] = 5 + 2\left[\frac{\alpha^2+\beta^2}{\alpha\beta}\right]$

$= 5 + 2\left[\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$

$= 5 + 2\left[\frac{(-p)^2 - 2(q)}{q}\right]$

$= 5 + \frac{2p^2 - 4q}{q} = \frac{2p^2+q}{q}$

Therefore, Required value of

$\left(\frac{\alpha}{\beta}+2\right)\left(\frac{\beta}{\alpha}+2\right)$ is $\frac{2p^2+q}{q}$.

2. $f(x) = 4x^2 - 8kx + 8x - 9$

Let one zero be α

and then other will be $= -\alpha$.

Now, Sum of zeroes $= \alpha + \beta$

$\Rightarrow \alpha - \alpha = 0 = \frac{(8k-8)}{4}$

$\Rightarrow k = 1$

$\therefore$ Zeroes of $1(x)^2 + 3(1)x + 2$

$= x^2 + 3x + 2$

$= x^2 + x + 2x + 2$

$= x(x+1) + 2(x+1)$

$= (x+1)(x+2)$

$\therefore$ Zeroes $= x = -1$ and -2

Hence, the answer is -1 and -2.

3. α, β roots of $f(x) = kx^2 + 4x + 4$

Given $\alpha^2 + \beta^2 = 24$

We know, $\alpha + \beta = \frac{b}{a} = \frac{-4}{k}$

and $\alpha\beta = \frac{c}{a} = \frac{4}{k}$

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\left(\frac{-4}{k}\right)^2 = 24 + 2\left(\frac{4}{k}\right)$

$\left(\frac{16}{k^2}\right) = 24 + 2\left(\frac{4}{k}\right)$

$\Rightarrow \frac{16}{k^2} = 24 + \frac{8}{k}$

$\Rightarrow \frac{16}{k^2} = \frac{24k+8}{k}$

$\Rightarrow 16 = 24k^2 + 8k$

$\Rightarrow 3k^2 + k - 2 = 0$

$\Rightarrow 0 = 3k^2 + 3k - 2k - 2$

$\Rightarrow 0 = 3k(k+1) - 2(k+1)$

$\Rightarrow 0 = (k+1)(3k-2)$

$\therefore$ **$k = -1$ and $\frac{2}{3}$**

4. (*i*) We have, $g(x) = px^2 + qx + 152$

Product of zeroes $(\alpha\beta) = \frac{c}{a} = \frac{152}{p}$

Now, 152 as prime factors

$= 2^3 \times 19$

ATQ, zeroes of $g(x)$ are distinct prime numbers.

So, Zeroes of $g(x)$ are 2 and 19

Product of zeroes $(\alpha\beta) = \frac{c}{a}$

$\Rightarrow 2 \times 19 = \frac{152}{p}$

$\therefore p = \frac{152}{38} = 4$ ...(A)

(ii) Sum of zeroes, $(\alpha + \beta) = \frac{-b}{a}$

$\Rightarrow 2 + 19 = \frac{-q}{p}$

$\Rightarrow 21 = \frac{-q}{4}$ ...[From (A)

$\therefore$ **$q = -84$**

Case Based Answers

1. (a) Since the graph of the polynomial intersect the x-axis at $x = \frac{1}{2}, \frac{-7}{2}$ therefore required zeroes of the polynomial are $\frac{1}{2}$ and $\frac{-7}{2}$.

(b) (i) $\frac{1}{2}$ and $\left(\frac{-7}{2}\right)$ are the zeroes of the polynomial.

So, at $x = \frac{1}{2}, \frac{-7}{2}$ the value of the polynomial will be 0.

Required polynomial is

$p(x) = -4x^2 - 12x + 7$

Or

(ii) We have,

$p(x) = -4x^2 - 12x + 7$

$p(3) = -4(3)^2 - 12(3) + 7$

$= -36 - 36 + 7$

$=$ **-65**

(c) Here $f(x) = x^2 + 2x - 8$ and α, β are its zeroes.

$\alpha + \beta = -2$ and $\alpha\beta = -8$

Now, $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= [(-2)^2 - 2(-8)]^2 - 2(-8)^2$

$= [4 + 16]^2 - 2(-8)^2$

$= (20)^2 - 2(64)$

$= 400 - 128 =$ **272**

2. (a) Initially, at $t = 0$,

Annie's height is 48 ft.

So, at $t = 0$, h should be equal to 48.

$h(0) = -16(0)^2 + 8(0) + k = 48.$

So, $k = 48$

(b) (i) When Annie touches the pool, her height = 0 feet

i.e., $-16t^2 + 8t + 48 = 0$ ...[above water level

$2t^2 - t - 6 = 0$

$2t^2 - 4t + 3t - 6 = 0$

$2t(t - 2) + 3(t - 2) = 0$

$(2t + 3)(t - 2) = 0$

i.e., $t = 2$ or $t = \frac{-3}{2}$

Since time cannot be negative, so $t = 2$ sec.

Or

(ii) $t = -1$ and $t = 2$ are the two zeroes of the polynomial $p(t)$.

Then

$p(t) = k(t + 1)(t - 2) = k(t + 1)(t - 2)$

When $t = 0$ (initially) $h_1 = 48$ft

$p(0) = k(0^2 - 0 - 2) = 48$

i.e., $-2k = 48$ $\Rightarrow k = -24$

So the required polynomial

$= -24(t^2 - t - 2)$

$= -24t^2 + 24t + 48.$

(c) Sum of zeroes as 1 and the product as -6 is given by

$q(t) = k(t^2 -$ (sum of zeroes)t + (product of zeroes)

$= k(t^2 - 1t + (-6))$ ...(i)

When $t = 0$; $q(0) = 48$ ft.

$q(0) = k(0^2 - 1(0) - 6) = 48$

i.e., $-6k = 48$ *or* $k = -8$

Putting $k = -8$ in equation (i),

$\therefore$ Reqd. polynomial $= -8(t^2 - 1t + (-6)$

$= -8t^2 + 8t + 48$

3. (a) Given that $x^2 - 2 = 0$

by factorization method,

$\Rightarrow (x + \sqrt{2})(x - \sqrt{2}) = 0$

$\Rightarrow x = \pm\sqrt{2}$

(b) (i) $\alpha^2\beta + \beta^2\alpha = \alpha\alpha\beta + \beta\beta\alpha$

Now, $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, substitute these values into the expression:

$$\alpha\alpha\beta + \beta\beta\alpha = \alpha\left(\frac{-b}{a}\right) + \beta\left(\frac{-b}{a}\right)$$

Factor out $\frac{-b}{a}$:

$$\alpha\left(\frac{-b}{a}\right) + \beta\left(\frac{-b}{a}\right) = \left(\frac{-b}{a}\right)(\alpha + \beta)$$

Now, use Vieta's formula for the sum of the roots:

$$\alpha + \beta = \frac{-b}{a}$$

So,

$$\left(\frac{-b}{a}\right)(\alpha + \beta) = \left(\frac{-b}{a}\right)\left(\frac{-b}{a}\right)$$

$$= \left(\frac{-b}{a}\right)^2 = \frac{b^2}{a^2}$$

$\therefore$ **$\alpha^2\beta + \beta^2\alpha$ is equal to $\frac{b^2}{a^2}$.**

Or

(ii) Here, $a = 3;\ b = -2;\ c = 5k$

$$p(x) = 3x^2 - 2x + 5k$$

Given zeroes are α and $\frac{1}{\alpha}$

and Product of zeroes $= \alpha \cdot \frac{1}{\alpha}$

$$\Rightarrow \frac{c}{a} = 1$$

$$\Rightarrow \frac{(5k)}{3} = 1 \quad \Rightarrow k = \frac{3}{5}$$

(c) Sum of the zeroes $= \alpha$

Product of the zeroes $= \frac{-1}{\alpha}$

Then the quadratic polynomial

$= x^2 -$ (Sum of the zeroes) x + (Product of the zeroes)

$$= x^2 - \alpha x - \frac{1}{\alpha}$$

or $x^2 - \alpha x - \frac{1}{\alpha}$ is the required quadratic polynomial.

(DAY 13 SWAHA)

*****

7 Triangles

What did CBSE ask last year?

MCQs & A/R	2 Questions ($2 \times 1 = 2$ Marks)
Subjective	No Very Short Question
	1 Short Question ($1 \times 3 = 3$ Marks)
	1 Long Question ($1 \times 5 = 5$ Marks)
Case Based	No Case Based Question

Note: All the above typology of questions include 'Competency based Questions' labelled as COMPETENCY

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Introduction and similarity of triangles

- If two figures are congruent, then they are similar but the reverse is not true.

  Congruent ⇒ Similar, But, Similar ≠ Congruent

  Two triangles are similar, if

  (i) Their corresponding angles are equal, and

  (ii) Their corresponding sides are in the same ratio (or proportional).

- ***Thales' Theorem or Basic proportionality theory***

  Theorem 1: State and prove Thales' Theorem.

  Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: In $\triangle ABC$, $DE \parallel BC$.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw $EM \perp AD$ & $DN \perp AE$. Join B to E & C to D.

Proof: In $\triangle ADE$ and $\triangle BDE$, $\frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB}$

...(i) ...[Area of $\triangle$ = ½ × base × corresponding altitude

In $\triangle ADE$ and $\triangle CDE$, $\frac{ar(\triangle ADE)}{ar(\triangle CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC}$ ...(ii)

$\because$ $DE \parallel BC$ ...[Given

$\therefore$ $ar(\triangle BDE) = ar(\triangle CDE)$ ...(iii)

...[$\because$ $\triangle$s on the same base and between same parallel sides are equal in area

From (i), (ii) and (iii), $\frac{AD}{DB} = \frac{AE}{EC}$ (Hence proved)

- ***CRITERION FOR SIMILARITY OF TRIANGLES***

  Two triangles are similar if either of the following three criterions are satisfied:

  AAA Similarity Criterion. If two triangles are equiangular, then they are similar.

  Corollary(AA similarity). If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

  SSS Similarity Criterion. If the corresponding sides of two triangles are proportional, then they are similar.

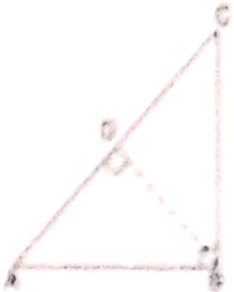
  SAS Similarity Criterion. If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

Results in Similar Triangles based on Similarity Criterion:

(i) Ratio of corresponding sides = Ratio of corresponding perimeters
(ii) Ratio of corresponding sides = Ratio of corresponding medians
(iii) Ratio of corresponding sides = Ratio of corresponding altitudes
(iv) Ratio of corresponding sides = Ratio of corresponding angle bisector segments

Pythagoras' Theorem

❑ **Theorem 2:** State and prove Pythagoras' Theorem.



Statement: Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: ΔABC, is a right triangle right-angled at B.

To prove: $AB^2 + BC^2 = AC^2$

Const.: Draw $BD \perp AC$

Proof: In ΔABC and ΔADB,

$\angle A = \angle A$ ...[common

$\angle ABC = \angle ADB$ ...[each 90°

$\therefore \Delta ABC \sim \Delta ADB$ ...[AA Similarity

$\therefore \dfrac{AB}{AD} = \dfrac{AC}{AB}$ ...[Sides are proportionals

$\Rightarrow AB^2 = AC.AD$ ...(i)

Now in ΔABC and ΔBDC

$\angle C = \angle C$ ...[common

$\angle ABC = \angle BDC$ ...[each 90°

$\therefore \Delta ABC \sim \Delta BDC$ ...[AA Similarity

$\therefore \dfrac{BC}{DC} = \dfrac{AC}{BC}$ ...[Sides are proportional

$BC^2 = AC.DC$ ...(ii)

On adding (i) and (ii), we get

$AB^2 + BC^2 = AC.AD + AC.DC$

$\Rightarrow AB^2 + BC^2 = AC.(AD + DC)$

$AB^2 + BC^2 = AC.AC$

$\therefore$ **$AB^2 + BC^2 = AC^2$ (Hence Proved)**

Note: *Simple formula based questions may appear from this theorem.*

OBJECTIVE QUESTIONS

(DAY 14)

Multiple Choice Questions

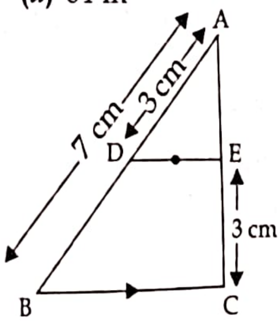
Q.1. If $\Delta PQR \sim \Delta ABC$; PQ = 6 cm, AB = 8 cm and the perimeter of ΔABC is 36 cm, then the perimeter of ΔPQR is

COMPETENCY

(*a*) 20.25 cm (*b*) 27 cm

(*c*) 48 cm (*d*) 64 m

Q.2. In the given figure, DE || BC if AD = 3 cm, AB = 7 cm and EC = 3 cm, then the length of AE is



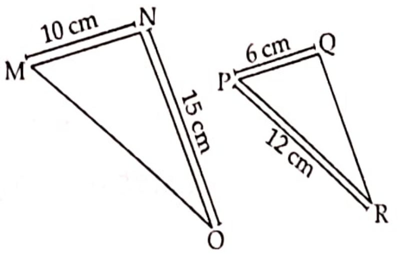
COMPETENCY

(*a*) 2 cm (*b*) 2.25 cm

(*c*) 3.5 cm (*d*) 4 cm

Q.3. Shown below are two triangles ΔMNO and ΔPQR. Dimensions of their two sides are marked in the figure.

COMPETENCY

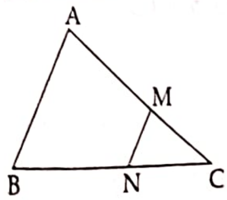


What should be the value of QR if ΔMNO is similar to ΔPQR?

(*a*) 9 cm (*b*) 11 cm

(*c*) 15 cm (*d*) 25 cm

Q.4. In figure, MN || AB, BC = 7.5 cm, AM = 4 cm and MC = 2 cm. Find the length BN.



COMPETENCY

(*a*) 4 cm (*b*) 7 cm

(*c*) 5 cm (*d*) 8 cm

Q.5. In ΔABC, PQ || BC. If PB = 6 cm, AP = 4 cm, AQ = 8 cm find the length of AC.

COMPETENCY

(*a*) 12 cm (*b*) 20 cm

(*c*) 6 cm (*d*) 14 cm

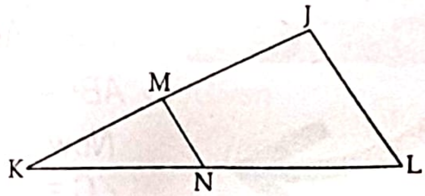
> *FREE ADVICE: Since PQ is parallel to BC, we can use the Thales' theorem. Thales' theorem states that if a line is parallel to one side of a triangle and intersects the other two sides, it divides those sides proportionally.*

Q.6. D and E are the midpoints of side AB and AC of a ΔABC, respectively and BC = 6 cm. If DE || BC, then the length (in cm) of DE is:

(*a*) 2.5 (*b*) 3

(*c*) 5 (*d*) 6

Q.7. In the following figure, MN is drawn such that M and N are mid-points on JK and KL, respectively.



Which of these criteria be used to prove that ΔJKL is similar to ΔMKN?

COMPETENCY

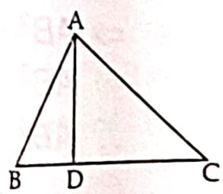
(*a*) SSS similarity criterion

(*b*) SAS similarity criterion

(*c*) AAA similarity criterion

(*d*) All of the similarity criteria can be used.

Q.8. In $\angle BAC = 90^\circ$ and $AD \perp BC$, then



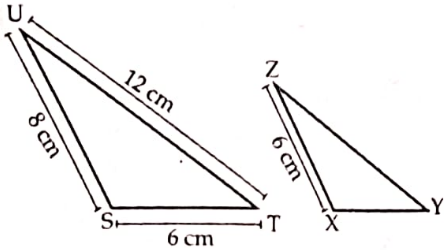
(*a*) $BD.\ CD = BC^2$

(*b*) $AB.\ AC = BC^2$

(*c*) $BD.\ CD = AD^2$

(*d*) $AB.\ AC = AD^2$

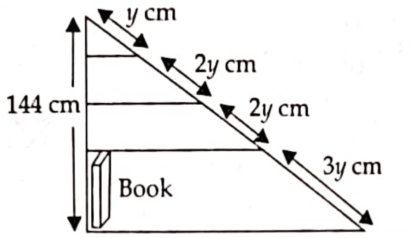
Q.9. In the figures given below, ΔSTU and ΔXYZ are similar. **COMPETENCY**



What is the perimeter of ΔXYZ?

(*a*) 19.5 cm (*b*) 20 cm
(*c*) 26 cm (*d*) 34.67 cm

Q.10. Leela has a triangular cabinet that fits under his staircase. There are four parallel shelves as shown below.

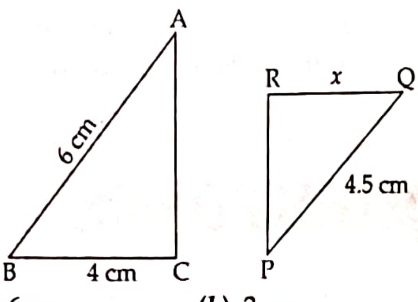


(Note: The figure is not to scale)

The total height of the cabinet is 144 cm. What is the maximum height of a book that can stand upright on the bottom-most shelf? *[CBSE 2024*

(*a*) 18 m (*b*) 36 m
(*c*) 12 cm (*d*) 54 cm

Q.11. In the given Fig., $\Delta ABC \sim \Delta QPR$. If AB = 6 cm, BC = 4 cm, QP = 4.5 cm and RQ = x then the value of x is **COMPETENCY**



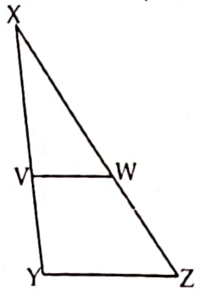
(*a*) 3.6 cm (*b*) 3 cm
(*c*) 10 cm (*d*) 3.2 cm

Q.12. If the area of two similar triangles are 9 cm^2 and 16 cm^2 respectively, then ratio of the corresponding sides are **COMPETENCY**

(*a*) 3 : 4 (*b*) 4 : 3
(*c*) 2 : 3 (*d*) 4 : 5

FREE ADVICE: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. In other words, if you have two similar triangles with corresponding sides in the ratio a:b, then the ratio of their areas is $a^2 : b^2$.

Q.13. In the ΔXYZ given below, VW || YZ. VY = 6 cm, XY = 14 cm, XW = 12 cm.



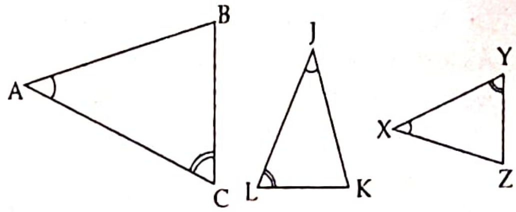
What is the length of XZ? **COMPETENCY**

(*a*) 14 cm (*b*) 21 cm
(*c*) 26 cm (*d*) 28 cm

Q.14. In a rhombus, if $d_1 = 16$ cm, $d_2 = 12$ cm, then the length of the side of the rhombus is

(*a*) 8 cm (*b*) 9 cm
(*c*) 10 cm (*d*) 12 cm

Q.15. Equal angles have been marked in the triangles below. *[CBSE 2024*



Which of these is NOT always true?

(*a*) $\Delta ABC \sim \Delta JKL$
(*b*) $\Delta ABC \sim \Delta XYZ$
(*c*) $\Delta ABC \sim \Delta XZY$
(*d*) All three triangles are similar.

Q.16. If ΔABC and ΔDEF are two triangles and $\frac{AB}{DE} = \frac{BC}{FD}$, then the two triangles are similar if *[NCERT Exemplar*

(*a*) $\angle A = \angle F$ (*b*) $\angle B = \angle D$
(*c*) $\angle A = \angle D$ (*d*) $\angle B = \angle E$

FREE ADVICE: If ABC and DEF are two triangles and $\frac{AB}{DE} = \frac{BC}{FD}$, then the two triangles are similar if $\angle B = \angle D$.

Q.17. In triangles ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, the two triangles are COMPETENCY

(*a*) congruent but not similar.

(*b*) similar but not congruent.

(*c*) neither congruent nor similar.

(*d*) congruent as well as similar.

FREE ADVICE: For triangles to be congruent, the ratio of sides must be 1. But here, its 3. Therefore, triangles are similar but not congruent.

Q.18. It is given that $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR} = \frac{1}{4}$ then, $\frac{ar\,\Delta PRQ}{ar\,\Delta ABC}$ is equal to

[NCERT Exemplar

(*a*) 16 : 1 (*b*) 4

(*c*) $\frac{1}{16}$ (*d*) $\frac{1}{4}$

Q.19. If in two triangles, ΔABC and ΔDEF, $\frac{AB}{DF} = \frac{BC}{FE} = \frac{CA}{ED}$ then

(*a*) $\Delta CAB \sim \Delta FDE$ (*b*) $\Delta ABC \sim \Delta EDF$

(*c*) $\Delta CBA \sim \Delta FDE$ (*d*) $\Delta BCA \sim \Delta FDE$

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

(*b*) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*

(*c*) *(A) is true but (R) is false.*

(*d*) *(A) is false but (R) is true.*

Q.1. Assertion: If in a ΔABC, a line DE $\parallel$ BC intersects AB in D and AC in E, then $\left(\frac{AB}{AD}\right) = \left(\frac{AC}{AE}\right)$. COMPETENCY

Reason: If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

Q.2. Assertion: A right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Reason: If in a triangle, square of one side in equal to the sum of the square of the other two sides, then the angle opposite the first side in 60°.

Q.3. Assertion: The sides of two similar triangles are in the ratio 2 : 5, then the areas of these triangles are in the ratio 4 : 25. COMPETENCY

Reason: The ratio of the areas of two similar triangles is equal to the square of the ratio of their sides.

Q.4. Assertion: If ΔABC and ΔPQR are congruent triangles, then they are also similar triangles. COMPETENCY

Reason: All congruent triangles are similar but the similar triangles need not be congruent.

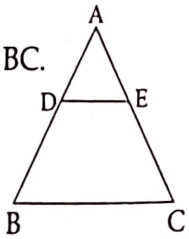
Q.5. Assertion: In the ΔABC, AB = 24 cm, in BC = 10 cm and AC = 26 cm, then ΔABC is a right angle triangle.

Reason: If in two triangles, their corresponding angles are equal, then the triangles are similar.

SUBJECTIVE QUESTIONS

Very Short Answer Questions

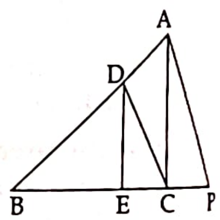
Q.1. In the given Fig., ABC is a triangle in which DE $\parallel$ BC. If AD = x, DB = $x - 2$, AE = $x + 2$ and EC = $x - 1$, then find the value of x.



[CBSE 2023

Q.2. In Fig. DE, AC and DC $\parallel$ AP. Prove that $\left(\frac{BE}{EC}\right) = \left(\frac{BC}{CP}\right)$

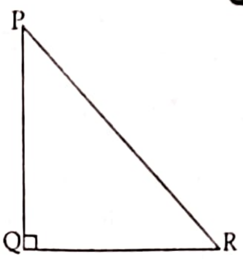
[CBSE 2020



FREE ADVICE: Write the reason and the theorem which you are using for proving a particular result.

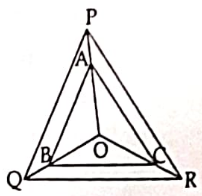
Q.3. R and S are points on the sides DE and EF respectively of ΔDEF such that ER = 5 cm, RD = 2.5 cm, SD = 1.5 cm and FS = 3.5 cm. Find whether RS || DF or not. **COMPETENCY**

Q.4. Shown below is an isosceles right-angled ΔPQR. The area of ΔPQR is 18 cm^2. **COMPETENCY**

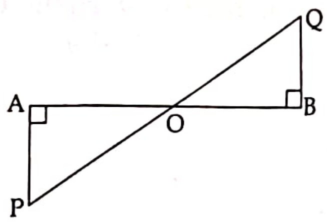


Find the length of PR. Show your work.

Q.5. In A, B and C are points on OP, OQ and QR respectively such that AB || PQ and AC || PR. Show that BC || QR. **COMPETENCY**

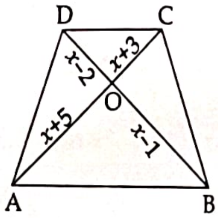


Q.6. In the given figure, if, $\angle A = 90°$ $\angle B = 90°$, OB = 4.5 cm, OA = 6 cm AP = 4 cm, find QB. **COMPETENCY**



Q.7. ΔPQR and ΔSQR are two triangles with common hypotenuse QR if PR and SQ intersect at M such that PM = 3 cm, MR = 6 cm, SM = 4 cm, find the length of MQ. *[NCERT Exemplar*

Q.8. In the given figure, if AB || DC, find the value of x. **COMPETENCY**



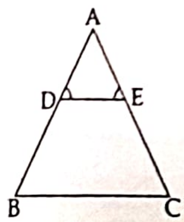
(DAY 15)

Short Answer Questions

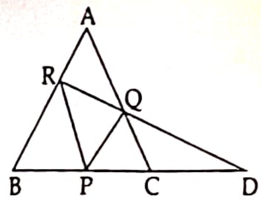
Q.1. Two right triangles ΔABC and ΔDBC are drawn on the same hypotenuse BC and on the same sides of BC. If AC and DB intersect at P, prove that AP × PC = BP × PD. **COMPETENCY**

Q.2. In the given fig. $\angle D = \angle E$ & $\frac{AD}{BD} = \frac{AE}{EC}$.

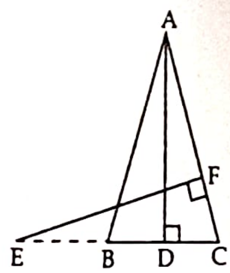
Prove that ΔBAC is an isosceles triangle. *[CBSE 2020*



Q.3. In the given figure PQ || BA and PR || CA. If PD = 12 cm then find BD × CD. **COMPETENCY**

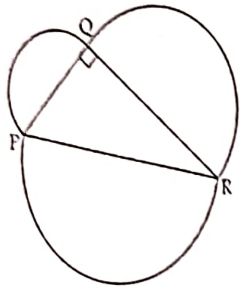


Q.4. In given figure, E is a point on side CB produced of an isosceles triangle ΔABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that $\Delta ABD \sim \Delta ECF$. *[CBSE 2023*



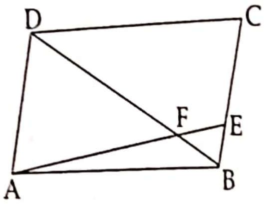
> *FREE ADVICE: Choose the two triangles wisely, for proving similarly according to the sides given in question.*

Q.5. ΔPQR is a right-angled triangle as shown in the figure. There are 3 semicircles with diameters as sides of ΔPQR. All length measurements are in cm. *[CBSE 2024*

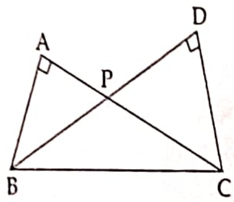


Show that the sum of the areas of semicircles with diameters PQ and QR is equal to the area of semicircle with diameter PR. COMPETENCY

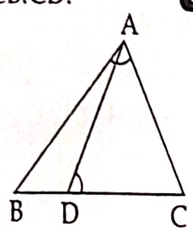
Q.6. In the figure, ABCD is a parallelogram and E divides BC in the ratio 1 : 3. DB and AE intersect at F then DF = 4BF and AF = 4EF. COMPETENCY



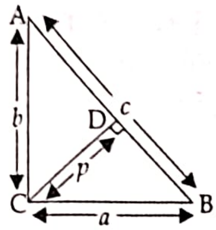
Q.7. In given figure, two triangles ΔBAC and ΔBDC, right angled at A and D respectively, are drawn on the same base BC. If AC and DB intersect at P, prove that $AP \times PC = DP \times PB$ COMPETENCY



Q.8. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$. COMPETENCY



Q.9. ΔABC is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite $\angle A$, $\angle B$ and $\angle C$ respectively, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.



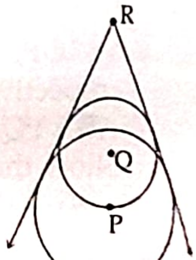
COMPETENCY

(DAY 16)

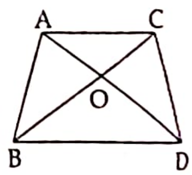
Long Answer Questions

Q.1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. *[CBSE 2023]*

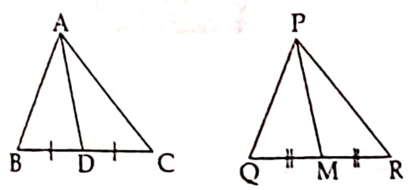
Q.2. In the given figure, P, Q and R are collinear. P and Q are centres of the two circles. P lies on the circumference of the circle with centre Q. R is 10 cm from Q and 15 cm from P. Both circles have 2 common tangents from point R. **Find the radius of circle P. Draw a rough figure and show your steps.** *[CBSE 2024]*



Q.3. In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$. *[CBSE 2023]*

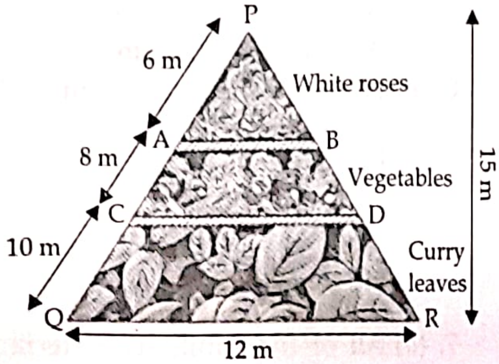


Q.4. If AD and PM are medians of triangles ΔABC and ΔPQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. *[CBSE 2023]*



CASE BASED QUESTIONS

Q.1. In the backyard of house, Saumya has some empty space in the shape of a ΔPQR. She decided to make it a garden. So, she divided the whole space into three parts by making boundaries AB and CD using bricks to grow flowers and vegetables where AB || CD || QR as shown in figure.



Based on the above information answer the following question.

(*a*) What is the length of AB?

(*b*) (*i*) What is the length of CD?

Or

(*ii*) What is that area of whole empty land?

(*c*) What is the area of ΔPCD?

Q.2. Class teacher draws the shape of quadrilateral on board. Ankit observes the shape and explore son his notebook in different ways as shown below.

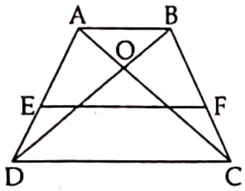


Figure 1

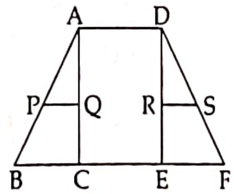


Figure 2

Based on the information, answer the following questions.

(*a*) In Fig. 1, if AB || CD, and DO = $3x - 19$, OB = $x - 5$, OC = $x - 3$ **and AO = 3 then the value of x can be ________.**

(*b*) (*i*) In Fig. 1, if OD = $3x - 1$, OB = $5x - 3$, OC = $2x + 1$ and AO = $6x - 5$, then value of x is ________.

Or

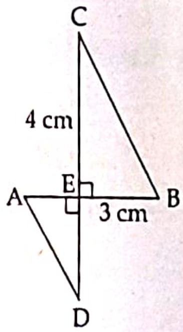
(*ii*) In Fig. 2, in ΔABC, if PQ || BC and AP = 2.4 cm, AQ = 2 cm, QC = 5 cm and BC = 6 cm then AB + PQ is equal to ________.

COMPETENCY

(*c*) In Fig. 2, in ΔDEF, if RS || EF, DR = $4x - 3$, DS = $8x - 7$, ER = $3x - 1$ and FS = $5x - 3$ then the value of x is ________.

COMPETENCY

Q.3. Ankita wants to make a toran for Diwali using some pieces of cardboard. She cuts some cardboard pieces as shown below. If perimeter of ΔADE and ΔBCE are in the ratio 2 : 3, then answer the following questions.



(*a*) What is the Length of BC?

(*b*) What is the Length of AD?

(*c*) (*i*) What is the Length of ED?

Or

(*ii*) What is the Length of AE?

COMPETENCY

* * * * *

ANSWERS

Multiple Choice Answers

1. (*b*) Let perimeter of ΔPQR be x cm.
Since triangles are similar,

$\therefore \frac{PQ}{AB} = \frac{\text{Perimeter of } \Delta PQR}{\text{Perimeter of } \Delta ABC}$

$\Rightarrow \frac{6}{8} = \frac{x}{36}$

$\Rightarrow x = 6 \times \frac{36}{8}$

$\Rightarrow x = 27$

$\therefore$ **Perimeter of ΔPQR = 27 cm**

2. (*b*) In ΔABC
$DE \parallel BC$

$\frac{AD}{DB} = \frac{AE}{EC}$ [By Thales' Theorem

$\Rightarrow \frac{3}{4} = \frac{AE}{3}$ ...[$\because DB = AB - AD = 7cm - 3cm = 4cm$

$AE \times 4$ cm $= 3$ cm $\times 3$ cm

$\therefore$ **AE = 2.25 cm**

3. (*a*) As $\Delta MNO \sim \Delta PQR$

$\Rightarrow \frac{MN}{PQ} = \frac{NO}{QR}$ $\Rightarrow \frac{10}{6} = \frac{15}{QR}$

$\Rightarrow 10 \times QR = 6 \times 15$

$QR = \frac{6 \times 15}{10}$

$\therefore$ QR = 9 cm

4. (*c*) It is given that in ΔABC, $MN \parallel AB$.
And, BC = 7.5 cm, AM = 4 cm and MC = 2 cm.
So, by Thales Theorem,

$\Rightarrow \frac{AC}{MA} = \frac{BC}{BN}$

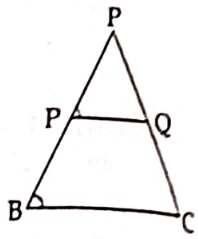
$\Rightarrow \frac{(CM + MA)}{MA} = \frac{BC}{BN}$

$\Rightarrow \frac{(2+4)}{MA} = \frac{BC}{BN}$

$\Rightarrow \frac{6}{4} = \frac{7.5}{BN}$ $\Rightarrow BN = \frac{7.5 \times 4}{6}$

$\Rightarrow BN = \frac{(75 \times 4)}{60} = \frac{300}{60} = 5$ cm

5. (*b*) In ΔABC, $PQ \parallel BC$

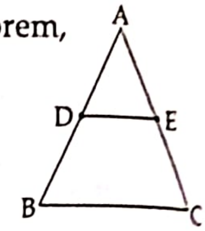


$\frac{AP}{PB} = \frac{AQ}{QC}$

$= \frac{4}{6} = \frac{8}{QC}$

QC = 12 cm
Now, AC = AQ + QC
$\therefore$ **AC = 8 + 12 = 20 cm**

6. (*b*) From mid-point theorem,



$DE = \frac{1}{2} BC$

$\Rightarrow DE = \frac{1}{2} \times 6 = 3$

$\therefore$ **DE = 3 cm**

7. (*d*) All of the similarity criteria can be used.

8. (*c*) $\angle ADB = \angle ADC = 90°$
$\angle DBA = \angle DAC$ **...[each equal to 90°**
$\therefore \Delta ADB \sim \Delta ADC$
...[by AAA similarity criterion

$\therefore \frac{BD}{AD} = \frac{AD}{CD}$

$\therefore$ **$BD.CD = AD^2$**

9. (*a*) In $\Delta STU \sim \Delta XYZ$

$\frac{ST}{XY} = \frac{TU}{YZ} = \frac{SU}{XZ}$

$\Rightarrow \frac{ST}{XY} = \frac{SU}{XZ}$ $\Rightarrow \frac{6}{XY} = \frac{8}{6}$

$8XY = 36$ $\Rightarrow XY = \frac{36}{8}$

XY = 4.5 cm

and, $\frac{TU}{YZ} = \frac{SU}{XZ}$ $\Rightarrow \frac{12}{YZ} = \frac{8}{6}$

$\Rightarrow 8YZ = 12 \times 6$

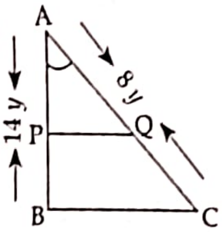
$\Rightarrow YZ = \frac{12 \times 6}{8}$

$\Rightarrow$ YZ = 9 cm

$\therefore$ **Perimeter of ΔXYZ** = XY + YZ + ZX
= 4.5 + 9 + 6
= **19.5 cm**

10. (*d*) In ΔABC

PQ is parallel to BC



$\therefore \frac{PB}{AB} = \frac{QC}{AC}$

$\Rightarrow \frac{PB}{144} = \frac{QC}{AC}$

$\therefore$ **PB** $= \frac{3y}{8y} \times 144 =$ **54 cm**

11. (*b*) $\Delta ABC \sim \Delta PQR$

AB = 6 cm, BC = 4 cm, PQ = 4.5 cm,

QR = x cm

$\frac{AB}{PQ} = \frac{BC}{QR}$ ... [corresponding sides of similar triagles are proportional

$\Rightarrow \frac{6}{4.5} = \frac{4}{x}$

$\Rightarrow 6x = 4.5 \times 4 \quad \Rightarrow x = \frac{4.5 \times 4}{6}$

$\Rightarrow x = \frac{4.5 \times 2}{3} = 1.5 \times 2$

$\therefore$ $\boldsymbol{x = 3}$ **cm**

12. (*a*) The ratio of areas of two similar triangles having areas A_1 and A_2 respectively is given by

$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 \quad \Rightarrow \frac{9}{16} = \left(\frac{s_1}{s_2}\right)^2$

Taking square roots on both sides,

we get, $\left(\frac{s_1}{s_2}\right) = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Hence, ratio of sides = 3 : 4

13. (*b*) XV = 14 – 6 = 8 cm

In ΔXYZ, VW || YZ

By using BPT,

$\Rightarrow \frac{XV}{VY} = \frac{XW}{WZ} \quad \Rightarrow \frac{8}{6} = \frac{12}{WZ}$

$\Rightarrow 8\,WZ = 12 \times 6$

$\Rightarrow WZ = \frac{12 \times 6}{8} = 9$ cm

$\therefore$ **Length of XZ**= XW + WZ

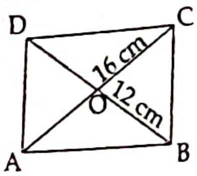
= 12 + 9 = **21 cm**

14. (*c*) **Given.** d_1 = AC = 16 cm

and d_2 = BD = 12 cm

$\therefore$ AO = OC = 8 cm

BO = OD = 6 cm



By using Pythagoras theorem,

In ΔAOB,

$\Rightarrow AB^2 = AO^2 + BO^2$

$\Rightarrow AB^2 = (8)^2 + (6)^2$

$\Rightarrow AB^2 = 64 + 36$

$\Rightarrow AB^2 = 100$

$\therefore$ **AB = 10 cm**

15. (*b*) $\Delta ABC \sim \Delta XYZ$

16. (*b*) By SSS criterion,

$\frac{AB}{DE} = \frac{BC}{DF} = \frac{AC}{EF}$

We know that similar triangles have congruent corresponding angles and the corresponding sides are in proportion.

So, $\angle A = \angle E$

$\angle B = \angle D$

$\angle C = \angle F$

17. (*b*) AAA criterion states that if two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angle will also be equal.

As $\angle B = \angle E$ and $\angle F = \angle C$, the third angle will also be equal.

i.e. $\angle D = \angle A$

Given. AB = 3 DE Since, AB ≠ DE

Therefore, the ΔABC and ΔDEF are similar but not congruent.

18. (*a*) As we know, $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{BC^2}{QR^2}$

$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \left(\frac{BC}{QR}\right)^2$

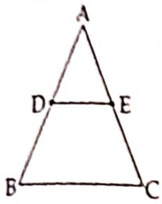
$\Rightarrow \frac{(\text{Area of } \Delta ABC)}{(\text{Area of } \Delta PQR)} = \left(\frac{1}{4}\right)^2$

$\therefore \frac{(\textbf{Area of } \boldsymbol{\Delta PQR})}{(\textbf{Area of } \boldsymbol{\Delta ABC})} = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$

19. (*a*) As we know that according to the basic proportionality theorem (BPT), two triangles ABC and PQR are said to be similar if all the corresponding sides of the two triangles are in the same proportion.

Assertion Reason Answers

1. (*a*) *Explanation.*
Since DE || BC



$\therefore$ *by Thales Theorem*

$$\therefore \frac{AD}{DB}=\frac{AE}{EC} \Rightarrow \frac{DB}{AD}=\frac{EC}{AE}$$

$$\Rightarrow 1+\frac{DB}{AD}=1+\frac{EC}{AE}$$

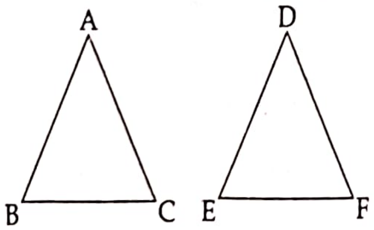
$$\Rightarrow \frac{AD+DB}{AD}=\frac{EC+AE}{AE}$$

$$\Rightarrow \frac{AB}{DB}=\frac{AC}{AE}$$

In this case, AB and AD are corresponding sides of the triangles, and AC and AE are also corresponding sides. This proves the assertion. So, both the assertion and the reason are true, and the reason correctly explains the assertion.

2. (*c*) ***Explanation.*** *Assertion is true as it follows Pythagoras theorem.*
Now in Reason it is given that it follows Pythagoras theorem, so it is a right angled triangle. Now the angle opposite to the first side is 90°. So Reason is not correct.

3. (*a*) ***Explanation.*** *According to question,*



$$\frac{BC}{EF}=\frac{2}{5}$$

As we know that,
ratio of square of corresponding sides = ratio of area of triangle

$$\frac{ar\,(\Delta ABC)}{ar\,(\Delta DFF)}=\frac{BC^2}{EF^2}=\frac{2^2}{5^2}=\frac{4}{25}=4:25$$

4. (*a*) *Explanation. If ΔABC and ΔPQR are congruent triangles, then they are also similar triangles. All congruent triangles are similar but the similar triangles need not be congruent.*

5. (*b*) $AB^2 + BC^2 = AC^2$

$\Rightarrow (24)^2 + (10)^2 = AC^2$

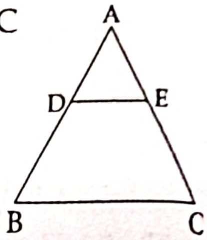
$\Rightarrow 576 + 100 = AC^2 \quad \Rightarrow 676 = AC^2$

$\therefore AC = \sqrt{676} = 26$

Hence, ΔABC is a right angled triangle. Also, two triangles are similar if their corresponding sides are equal.
Assertion and Reason both are true but R is not the correct explanation of A.

Very Short Answers

1. **Given:** In ΔABC, DE || BC



$$\therefore \frac{AD}{DB}=\frac{AE}{EC}$$

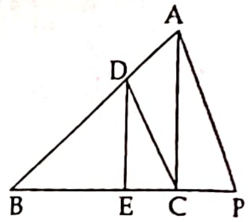
...[By basic proportionally theorem

$$\Rightarrow \frac{x}{x-2}=\frac{x+2}{x-1}$$

$\Rightarrow x(x-1) = (x+2)(x-2)$

$\Rightarrow x^2 - x = x^2 - 4 \qquad \therefore \boldsymbol{x = 4}$

2. In ΔBPA, we have
DC || AP **...[Given**
Therefore, by basic proportionality theorem, we have



$$\frac{BC}{CP}=\frac{BD}{DA} \qquad ...(i)$$

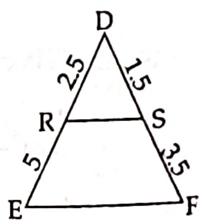
In ΔBCA, we have DE || AC
Therefore, by basic proportionality theorem, we have

$$\frac{BE}{EC}=\frac{BD}{DA} \qquad ...(ii)$$

From (*i*) and (*ii*), we get

$$\frac{BC}{CP}=\frac{BE}{EC} \Rightarrow \boldsymbol{\frac{BE}{EC}=\frac{BC}{CP}}$$ **(Hence proved)**

3. In a triangle ΔDEF, R and S are two points on the sides DE and EF respectively.



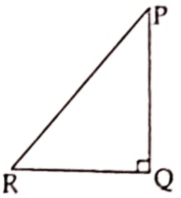
ER = 5 cm, RD = 2.5 cm, FS = 3.5 and SD = 1.5 cm.

$\therefore \frac{ER}{RD} = \frac{5}{2.5} = \frac{2}{1}$ and $\frac{FS}{SD} = \frac{3.5}{1.5} = \frac{7}{3}$

$\because \frac{ER}{RD} \neq \frac{FS}{SD}$

$\therefore$ **RS is not parallel to EF.**

4. As, ΔPQR is an isosceles right-angled triangle.



Let PQ = QR = m

Now, Area of ΔPQR

$= \frac{1}{2} \times$ Base $\times$ Height

$\Rightarrow 18\ cm^2 = \frac{1}{2} \times QR \times PQ$

$\Rightarrow 18\ cm^2 = \frac{1}{2} \times m \times m = \frac{1}{2}\ m^2$

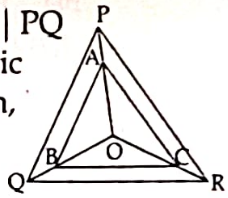
$\Rightarrow \frac{1}{2} \times m^2 = 18$

$\Rightarrow m^2 = 36 \quad \Rightarrow m = 6$

Now the length of PR = $\sqrt{m^2+m^2}$

$= \sqrt{2m^2} = \sqrt{2(36)} =$ **$6\sqrt{2}$ cm**

5. In ΔOPQ, we have AB || PQ



Therefore, by using basic proportionality theorem, we have

$\frac{AP}{OA} = \frac{BQ}{OB}$ ...(*i*)

AC || PR

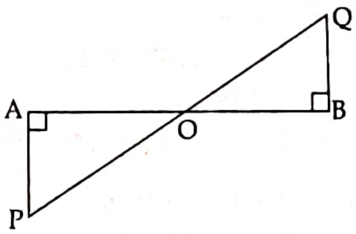
Therefore, by using basic proportionality theorem, we have

$\frac{CR}{OC} = \frac{AP}{OA}$ ...(*ii*)

Comparing (*i*) & (*ii*), we get $\frac{BQ}{OB} = \frac{CR}{OC}$

Therefore, by using converse of basic proportionality theorem, we get BC || QR. **(Hence Proved)**

6. In ΔPAO and ΔQBO



$\angle A = \angle B = 90°$

$\angle POA = \angle QOB$

...[Vertically Opposite Angles

In $\Delta PAO \sim \Delta QBO$

$\therefore \Delta AOP$ and ΔQOB are similar Δs

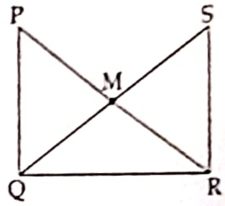
$\therefore \frac{OB}{OA} = \frac{QB}{AP}$

...[in similar Δs the ratio of corresponding sides are constants

$\Rightarrow QB = AP \times \frac{OB}{OA}$

$\therefore QB = 4 \times \frac{4.5}{6} = 4 \times \frac{3}{4} =$ **3 cm**

7.



$\angle QPM = \angle MSR$ ...[Both are 90°

$\angle PMQ = \angle SMR$

...[vertically opposite angles

By AA property,

$\Delta PQM \sim \Delta SRM$

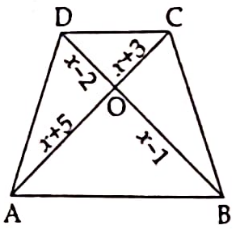
Using the property of similar figures:

$\Rightarrow \frac{PM}{SM} = \frac{MQ}{MR} \qquad \Rightarrow \frac{3}{4} = \frac{MQ}{6}$

$\Rightarrow 4MQ = 3 \times 6 \quad \Rightarrow 4MQ = 18$

$\therefore$ **MQ = $\frac{18}{4}$ = 4.5 cm**

8.



Here $\Delta AOB \sim \Delta COD$

$\therefore \angle CDO = \angle OBA$

and $\angle DCO = \angle OAB$ ...[Alternate interior angles are equal in || lines

$\angle DOC = \angle AOB$

...[vertical opposite angles are equal

So its similar by (Angle Angle Angle) criteria.

So $\Delta AOB \sim \Delta COD$

$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$

From similarity property

$\Rightarrow \frac{x+5}{x+3} = \frac{x-1}{x-2}$

$\Rightarrow (x+5)(x-2) = (x-1)(x+3)$

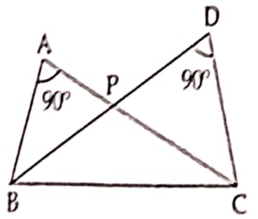
$\Rightarrow x^2 - 2x + 5x - 10 = x^2 - x + 3x - 3$

$\Rightarrow 3x - 10 = 2x - 3$

$\Rightarrow 3x - 2x = -3 + 10 \qquad \therefore x = 7$

Short Answers

1. **Given.** Two right triangles ΔABC & ΔDBC and they are drawn on the same hypotenuse BC & on the same sides BC.
   To prove: $AP \times PC = BP \times PD$ as both are right angle triangles,
   By using Pythagorean's theorem,



In ΔCDP we get, $CD^2 = CP^2 - DP^2$

$CD^2 = (CA + AP)^2 - (DB + BP)^2$

...[$CP = CA + AP$, $DP = DB + BP$)

$\Rightarrow CD^2 = CA^2 + AP^2 + 2CA.AP - [DB^2 + BP^2 + 2DB.BP]$

...[$\because (a+b)^2 = a^2 + 2ab + b^2$

$\Rightarrow CD^2 = CA^2 + AP^2 + 2CA.AP - DB^2 - BP^2 - 2BD.BP$

$\Rightarrow CD^2 + DB^2 = CA^2 + AP^2 - BP^2 + 2CA.AP - 2DB.BP$

$\Rightarrow CB^2 = CA^2 - (BP^2 - AP^2) + 2CA\ AP - 2DB.BP$

$\Rightarrow CB^2 = CA^2 - AB^2 + 2CA.AP - 2DB.BP$

$\Rightarrow CB^2 - CA^2 + AB^2 = 2CA.AP - 2DB.BP$

$\Rightarrow AB^2 + AB^2 = 2CA.AP - 2DB.BP$

$\Rightarrow 2AB^2 = 2[CA.AP - DB.BP]$

$\Rightarrow AB^2 = (PC - AP)AP - (DP - BP)BP$

$\Rightarrow AB^2 = AP \times PC - AP^2 - DP \times BP + BP^2$

$\Rightarrow AB^2 = AP.PC - DP.BP + AB^2$

$0 = AP \times PC - DP \times BP$

$\therefore$ $AP \times PC = DP \times BP$ **(Hence Proved)**

2. **Given.** $\angle D = \angle E$,

$\frac{AD}{DB} = \frac{AE}{EC}$

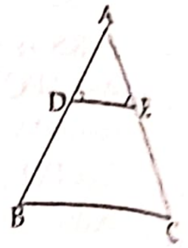
In triangle ADE, $\angle D = \angle E$

$\Rightarrow AD = AE$

Now,

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \quad \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$

$\Rightarrow \frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$



$\Rightarrow \frac{DB+AD}{AD} = \frac{EC+AE}{AE}$

$\Rightarrow \frac{BA}{AD} = \frac{CA}{AE}$

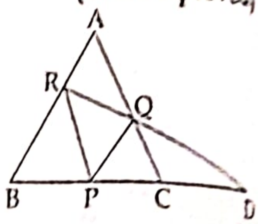
$\Rightarrow \frac{BA}{AE} = \frac{CA}{AE}$ ...[From (i

$\Rightarrow BA = CA$

$\therefore$ **ΔBAC is an isosceles triangle**

(Hence proved)

3. **Given.** $PQ \parallel BA$



$\Rightarrow PQ \parallel BR$

Now, In ΔBDR,

$\Rightarrow PQ \parallel BR$

By BPT,

$\Rightarrow \frac{PD}{BD} = \frac{DQ}{DR}$ ...(1)

$PR \parallel CA$ ...[Given

$\Rightarrow PR \parallel CQ$

Now,

In ΔPRD, $PR \parallel CQ$

By BPT $\frac{CP}{PD} = \frac{DQ}{DR}$ ...(2)

From (1) and (2), we get

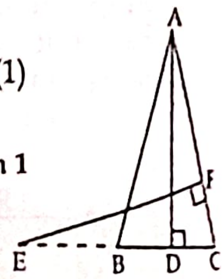
$\frac{PD}{BD} = \frac{CD}{PD}$

$\Rightarrow BD \times CD = (PD)^2$

$\Rightarrow BD \times CD = (12)^2$

$\therefore$ **$BD \times CD = 144\ cm^2$**

4. **Given.** $AB = AC$



$\therefore \angle B = \angle C$ ...(1)

In ΔABD and ΔECF

$\angle B = \angle C$ ...[From 1

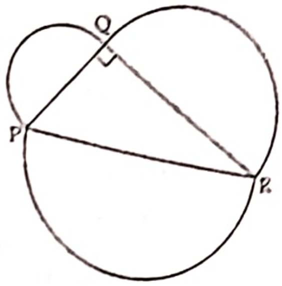
$\angle ADB = \angle EFC = 90°$

...[$\because AD \perp BC$ and $EF \perp AC$,]

$\because$ By AA Criterion of Similarity,

$\therefore \Delta ABD \sim \Delta ECF$ **(Hence Proved)**

5. If diameter = PQ



Area of semi-circle

$= \pi \frac{(PQ)^2}{8} cm^2$

Now, if diameter $= QR$

Area of semi-cirlce

$= \pi \frac{(QR)^2}{8} cm^2$

If diameter = PR

Area of semi-cirlce $= \pi \frac{(PR)^2}{8} cm^2$

To find the sum of area of semi-circles with diameter PQ and QR

$$\Rightarrow \frac{\pi(PQ)^2}{8} + \frac{\pi(QR)^2}{8} = \pi\left(\frac{PQ^2 + QR^2}{8}\right) \quad ...(i)$$

Now, in ΔPQR by using pythagoras theoram,

$(PQ)^2 + (QR)^2 = (PR)^2$

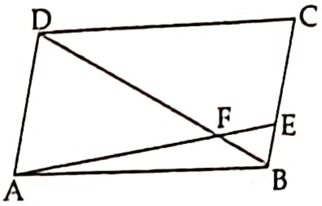
Putting value in equation (i),

$$\Rightarrow \frac{\pi(PQ)^2}{8} + \frac{\pi(QR)^2}{8} = \frac{\pi(PR)^2}{8} cm^2$$

$$\frac{\pi(PQ)^2}{8} + \frac{\pi(QR)^2}{8} = \frac{\pi(PR)^2}{8} cm^2$$

Hence, concludes that the sum of the areas of semicircles with diameters PQ and QR is equal to the area of semicircle with diameter PR. **(Hence Proved)**

6.



It is given that ABCD is a parallelogram and E divides BC in the ratio 1 : 3. DB and AE intersect at F.

In ΔFAD and ΔFEB,

$\angle FAD = \angle FEB$ **...[Alternate interior angles**

$\angle FDA = \angle FBE$ **...[Alternate interior angles**

So by AA similarity criterion,

$\Delta FAD \sim \Delta FEB$

We know that the corresponding sides of similar triangles have the same ratios, then

$$\frac{AF}{EF} = \frac{AD}{BE} = \frac{DF}{BF}$$

As opposite sides of parallelogram are equal, $AD = BC$

then,

$$\frac{AF}{EF} = \frac{BC}{BE} = \frac{DF}{BF} \quad ...(1)$$

As $BE : EC = 1 : 3$,

$\Rightarrow \frac{BE}{EC} = \frac{1}{3} \quad \Rightarrow EC = 3BE$

So, $BC = BE + CE$

$\Rightarrow BC = BE + 3BE$

$\Rightarrow BC = 4BE$

Putting the value of BC in (1).

$$\frac{AF}{EF} = \frac{4BE}{BE} = \frac{DF}{BF}$$

$$\Rightarrow \frac{AF}{EF} = \frac{4}{1} = \frac{DF}{BF} \quad ...(2)$$

From (2) we get,

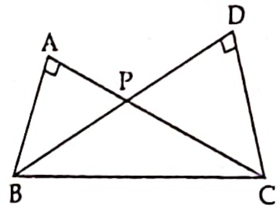
$\frac{4}{1} = \frac{DF}{BF} \quad \Rightarrow DF = 4BF$

And,

$\frac{AF}{EF} = \frac{4}{1} \quad \Rightarrow AF = 4EF$

Therefore, it is true that ABCD is a parallelogram and E divides BC in the ratio 1 : 3. DB and AE intersect at F then DF = 4BF and AF = 4FE.

7.



In ΔAPB & ΔDPC,

$\angle A = \angle D$ **...[90° each**

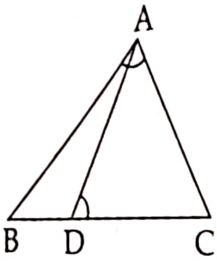
$\angle APB = \angle DPC$ **...[vertically opposite angles**

$\Rightarrow \Delta APB \sim \Delta DPC$ **...[By AA Similarity Criterion**

$\Rightarrow \frac{AP}{DP} = \frac{PB}{PC}$ **...[Cooresponding sides of similar triangles are proportional**

$\therefore AP \times PC = DP \times PB$ (Hence proved)

8. In ΔADC and ΔBAC



$\angle ADC = \angle BAC$ ...[Given

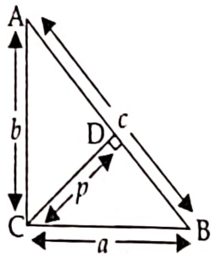
$\angle C = \angle C$ ...[Common

$\Delta ADC \sim BAC$ ...[By AA Criterion of Similarity,

$\Rightarrow \frac{BA}{AD} = \frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$ $\Rightarrow CA^2 = CB.CD$

9. In ΔACB and ΔCDB



$\Rightarrow \Delta ACB = \angle CDB$ ...[90° each

$\angle B = \angle B$ ...[common

$\Delta ACB \sim \Delta CDB$ ...[by AA Similarity

$\frac{b}{p} = \frac{c}{a}$ $\Rightarrow \frac{1}{p} = \frac{c}{ab}$

On Squaring on both sides,

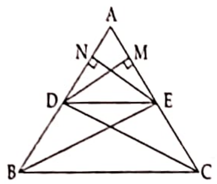
$\frac{1}{p^2} = \frac{c^2}{a^2.b^2}$ $\Rightarrow \frac{1}{p^2} = \frac{a^2+b^2}{a^2.b^2}$ ...[$c^2 = a^2 + b^2$

...[By Pythagoras theorem

$\therefore \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (Hence Proved)

Long Answers

1. In ΔABC, $DE \parallel BC$ and intersects AB and AC in points D and E respectively.



To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD. Draw perpendicular lines DM and EN to the sides AC and AB of ΔABC respectively as shown here.

Proof: Consider a triangle ABC as shown. In ΔADE and ΔDEB

$ar(\Delta ADE) = \frac{1}{2} \times AD \times EN$

$ar(\Delta DEB) = \frac{1}{2} \times BD \times EN$

Thus, $\frac{ar(\Delta ADE)}{ar(\Delta DEB)} = \frac{AD}{DB}$ ...(i)

Similarly, consider the ΔADE, with base AE and height DM.

Area of $\Delta ADE = \frac{1}{2} \times AE \times DM$

Area of $\Delta CDE = \frac{1}{2} \times CE \times DM$

$\frac{ar(ADE)}{ar(DEC)} = \frac{AE}{EC}$ ...(ii)

Here, ΔBDE and ΔCDE have the same base DE and lie in between the parallel lines DE and BC. *i.e.* ($DC \parallel BC$)

So, their areas are equal.

$ar(\Delta DEB) = ar(\Delta DEC)$ ...(iii)

$\frac{ar(\Delta ADE)}{ar(\Delta DEB)} = ar\frac{(\Delta ADE)}{(\Delta DEC)}$ ...[From (i), (ii) and (iii)

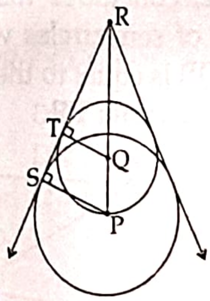
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

2. In ΔPSR and ΔQTR

$\angle PRS = \angle QRT$ ....[Common

$\angle PSR = \angle QTR = 90°$

...[(90° each) as radius is perpendicular to tangent at the point of contact



$\Delta PSR \sim \Delta QTR$ ...[By AA

Radius of circle Q, PQ = PR – RQ

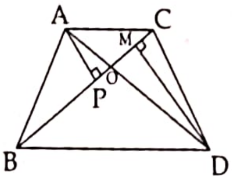
= 15 – 10 = 5 cm

$\Rightarrow \frac{PS}{QT} = \frac{PR}{QR}$ ...[By CPST

$\Rightarrow \frac{PS}{5} = \frac{15}{10}$ $\Rightarrow \frac{15 \times 5}{10} = \frac{15}{2} = 7.5$ cm

3. ΔABC and ΔDBC are two triangles on the same base BC. If AD intersects BC at O.

To Prove: $\frac{ar\,\Delta ABC}{ar\,\Delta DBC} = \frac{AO}{DO}$



Construction:
Draw AP ⊥ BC and
and DM ⊥ BC.

Area of Δ = $\frac{1}{2}$ × base × height

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\left(\frac{1}{2}\times BC\times AP\right)}{\left(\frac{1}{2}\times BC\times DM\right)} = \frac{AP}{DM}$$

In ΔAPO and ΔDMO,

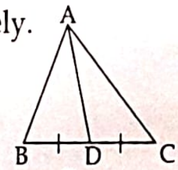
ΔAPO = ΔDMO = 90° ...[construction

∠AOP = ∠DOM ...[Vertically opposite angles

∴ ΔAPO ~ ΔDMO ...[AAA Similarity criterion

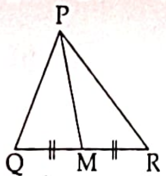
$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$ **(Hence Proved)**

4. Consider the triangles ΔABC and ΔPQR, AD and PM being the mediums from vertex A and P respectively.



Given: ΔABC ~ ΔPQR

To prove: $\frac{AB}{PQ} = \frac{AD}{PM}$



It is given that

ΔABC ~ ΔPQR

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

...[From the side-ratio property of similar triangles

⇒ ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ...(A)

BC = 2BD; QR = 2 QM

...[P, M being the mid points of BC *q* QR respectively

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AC}{PR}$$

$$\Rightarrow \frac{PQ}{AB} = \frac{QM}{BD} = \frac{PR}{AC} \quad ...(i)$$

Now in ΔABD and ΔPQM

$$\frac{PQ}{AB} = \frac{QM}{BD}$$ ...[from (*i*)

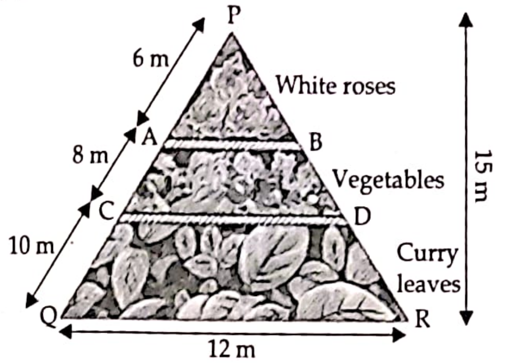
∠B = ∠Q ...[from (A)

⇒ ΔABD ~ ΔPQM ...[By SAS property of similar triangle

From the side property of similar triangle $\frac{AB}{PQ} = \frac{AD}{PM}$

Case Based Answers

1. (*a*)



In ΔPAB and ΔPQR,

∠P = ∠P ...[Common

∠A = ∠Q ...[Corresponding angles

By AA similarity criterion,

ΔPAB ~ ΔPQR

$$\Rightarrow \frac{AB}{QR} = \frac{PA}{PQ}$$

$$\Rightarrow \frac{AB}{12} = \frac{6}{24}$$ ∴ **AB = 3 m**

(*b*) (*i*) In ΔPCD and ΔPQR

$$\frac{PC}{PQ} = \frac{CD}{QR}$$

$$\Rightarrow \frac{14}{24} = \frac{CD}{12}$$ ∴ **CD = 7 m**

Or

(*ii*) Area of whole empty ΔPQR

= $\frac{1}{2}$ × base × height

= $\frac{1}{2}$ × 12 × 15 = **90 m²**

(*c*) Since, ΔPCD ~ ΔPQR

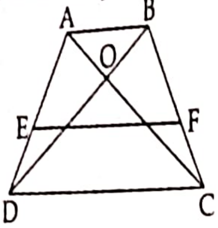
$$\frac{ar(\Delta PCD)}{ar(\Delta PQR)} = \frac{PC}{QR}$$

$$\Rightarrow \frac{ar(\Delta PCD)}{ar(\Delta PQR)} = \left(\frac{14}{24}\right)^2$$

$$\therefore \mathbf{ar(\Delta PCD)} = \frac{90\times 196}{144} = \frac{245}{2} = \mathbf{122.5\ m^2}$$

2. (a) Since, $\Delta AOB \sim \Delta COD$

...[By AA similarity criterion



$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{3}{(x-3)} = \frac{x-5}{3x-19}$$

$\Rightarrow 3(3x - 19) = (x - 5)(x - 3)$

$\Rightarrow 9x - 57 = x^2 - 3x - 5x + 15$

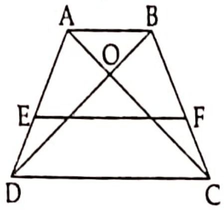
$\Rightarrow x^2 - 17x + 72 = 0$

$\Rightarrow (x - 8)(x - 9) = 0$

$\therefore$ **x = 8 or 9 cm**

(b) (i) Since, $\Delta AOB \sim \Delta COD$

...[By AA similarity criterion



$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{6x-5}{2x+1} = \frac{5x-3}{3x-1}$$

$\Rightarrow 18x^2 - 6x - 15x + 5 = 10x^2 + 5x - 6x - 3$

$\Rightarrow 8x^2 - 20x + 8 = 0$

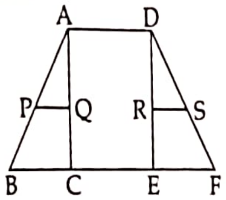
$\Rightarrow 2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0;\ 2x - 1 = 0, x = 2$

$\therefore$ **$x = \frac{1}{2}$ (rejected) or x = 2**

Or

(ii) Since, $\Delta APQ \sim \Delta ABC$



$$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

$$\Rightarrow \frac{2.4}{AB} = \frac{2}{5} = \frac{PQ}{6}$$

$$AB = \frac{2.4 \times 5}{2} = 6 \text{ cm}$$

$$\text{and } PQ = \frac{2 \times 6}{5} = 2.4 \text{ cm}$$

$\therefore$ **AB + PQ = 6 + 2.4 = 8.4 cm**

(c) Since, $\Delta DRS \sim \Delta DEF$

...[By AA similarity criterion

$$\therefore \frac{DE}{DR} = \frac{DF}{DS} \quad \Rightarrow \frac{DE}{DR} - 1 = \frac{DF}{DS} - 1$$

$$\Rightarrow \frac{DE - DR}{DR} = \frac{DF - DS}{DS}$$

$$\frac{DR}{ER} = \frac{DS}{FS}$$

$$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$\Rightarrow 20x^2 - 12x - 15x + 9$
$\quad = 24x^2 - 8x - 21x + 7$

$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow (2x^2 - 2x) + (x - 1) = 0$

$\Rightarrow 2x(x - 1) + 1(x - 1) = 0$

$\therefore (2x + 1)(x - 1) = 0$

$\therefore$ **x = 1 satisfies this equation.**

3. (a) Using Pythagorean Theorem

$EB^2 + EC^2 = BC^2$

$\Rightarrow 3^2 + 4^2 = BC^2$

$\Rightarrow 5^2 = BC^2$

$\therefore$ **BC = 5**

(b) As, $\frac{BE}{AE} = \frac{CE}{DE}$ ...[Ratio of corresponding sides of similar triangles is equal

$$\frac{AD}{BC} = \frac{2}{3}$$

$$\Rightarrow \frac{AD}{5} = \frac{2}{3} \Rightarrow \textbf{AD} = \frac{10}{3}\textbf{cm}$$

(c) (i) As, $\frac{ED}{CE} = \frac{2}{3}$

$$\Rightarrow \frac{ED}{4} = \frac{2}{3} \quad \therefore \textbf{ED} = \frac{8}{3}\textbf{cm}$$

Or (ii) $\frac{AE}{BE} = \frac{2}{3}$

$$\Rightarrow AE = \frac{2}{3} BE \Rightarrow AE = \sqrt{AD^2 - DE^2}$$

$$\Rightarrow AE = \left(\frac{2}{3}\right)\sqrt{BC^2 - CE^2}$$

$$\Rightarrow AE = \left(\frac{2}{3}\right)\sqrt{25 - 9}$$

$$\Rightarrow AE = \frac{2}{3} \times 4 \quad \therefore \textbf{AE} = \frac{8}{3}\textbf{ cm}$$

(DAY 16 SWAHA)

* * * * *

DAY 17

"Congratulate yourself on completing the half way of your 33 days journey. Share your experience with others via video review on 'Amazon' , 'FlipKart' , and 'Instagram'—

@padhle.akshay."

— Akshay Bhaiya

8 Introduction to Trigonometry

What did CBSE ask last year?

MCQs & A/R	**2 Questions (1 × 2 = 2 Marks)**
Subjective	2 Very Short Questions (2 × 2 = 4 Marks)
	1 Short Question (1 × 3 = 3 Marks)
	No Long Questions
Case Based	No Case Base Question

Note: *All the above typology of questions include 'Competency based Questions' labelled as* **COMPETENCY**.

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Introduction to Trigonometry

- **Trigonometric Ratios:** Ratios of sides of angled triangle are called trigonometric ratios.
- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of an angle can be easily determined.

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Trigonometric Identities

An equation involving trigonometric ratio of angle(s) is called a trigonometric identity, if it is true for all values of the angles involved. These are:

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$ • $\cot\theta = \frac{\cos\theta}{\sin\theta}$
- $\sin^2\theta + \cos^2\theta = 1$ $\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ $\Rightarrow \cos^2\theta = 1 - \sin^2\theta$
- $\mathrm{cosec}^2\theta - \cot^2\theta = 1$ $\Rightarrow \mathrm{cosec}^2\theta = 1 + \cot^2\theta$ $\Rightarrow \cot^2\theta = \mathrm{cosec}^2\theta - 1$
- $\sec^2\theta - \tan^2\theta = 1$ $\Rightarrow \sec^2\theta = 1 + \tan^2\theta$ $\Rightarrow \tan^2\theta = \sec^2\theta - 1$
- $\sin\theta\ \mathrm{cosec}\,\theta = 1$ • $\cos\theta\sec\theta = 1$ • $\tan\theta\cot\theta = 1$

(Note: Very short & short questions to prove identities based on only the first identity will be asked)

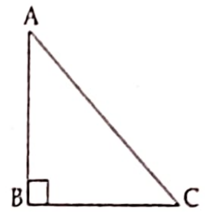
❑ 't-RATIOS' of Complementary Angles

If ΔABC is a right-angled triangle, right-angled at B, then

$$\angle A + \angle C = 90^\circ$$

or $$\angle C = (90^\circ - \angle A)$$

...[∵ $\angle A + \angle B + \angle C = 180^\circ$ angle-sum-property of a Δ]



Thus, $\angle A$ and $\angle C$ are known as complementary angles and are related by the following relationships:

$\sin(90^\circ - A) = \cos A$; $\quad$ $\operatorname{cosec}(90^\circ - A) = \sec A$

$\cos(90^\circ - A) = \sin A$; $\quad$ $\sec(90^\circ - A) = \operatorname{cosec} A$

$\tan(90^\circ - A) = \cot A$; $\quad$ $\cot(90^\circ - A) = \tan A$

DUCTION TO
NOMETRY

In a right triangle ABC, right-angled at B. Once we have identified the sides, we can define six t-Ratios with respect to the sides.

Case I	*Case II*
(*i*) **sin A** $= \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$	(*i*) **sine C** $= \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$
(*ii*) **cosine A** $= \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$	(*ii*) **cosine C** $= \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$
(*iii*) **tangent A** $= \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AB}$	(*iii*) **tangent C** $= \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{BC}$
(*iv*) **cosecant A** $= \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{AC}{BC}$	(*iv*) **cosecant C** $= \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{AC}{AB}$
(*v*) **secant A** $= \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AC}{AB}$	(*v*) **secant C** $= \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AC}{BC}$
(*vi*) **cotangent A** $= \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{AB}{BC}$	(*vi*) **cotangent C** $= \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{BC}{AB}$

(DAY 18)

Multiple Choice Questions

Q.1. In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. The value of tan C is: [CBSE 2023

(a) $\frac{12}{7}$ (b) $\frac{24}{7}$ (c) $\frac{20}{7}$ (d) $\frac{7}{24}$

Q.2. If $\tan\theta = \frac{x}{y}$ then $\cos\theta$ is equal to **COMPETENCY**

(a) $\frac{x}{\sqrt{(x^2+y^2)}}$ (b) $\frac{y}{\sqrt{(x^2+y^2)}}$

(c) $\frac{x}{\sqrt{(x^2-y^2)}}$ (d) $\frac{y}{\sqrt{(x^2-y^2)}}$

Q.3. If θ is an acute angle such that $\cos\theta = \frac{3}{5}$, then $\frac{\sin\theta\tan\theta - 1}{2\tan^2\theta}$ [CBSE 2024

(a) $\frac{16}{25}$ (b) $\frac{1}{36}$

(c) $\frac{3}{160}$ (d) $\frac{160}{3}$

Q.4. In a right-angled triangle, there is an acute angle p such that $\tan P = \frac{12}{5}$.

What is the value of sec (90° – P)?

(a) $\frac{5}{13}$ (b) $\frac{5}{12}$ (c) $\frac{12}{13}$ (d) $\frac{13}{12}$

Q.5. What is the minimum value of sin A, $0 \le A \le 90°$. **COMPETENCY**

(a) –1 (b) 0

(c) 1 (d) $\frac{1}{2}$

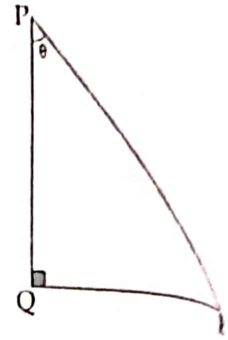
Q.6. If $\cot\theta = \frac{7}{8}$, then what is the value of:

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$. [CBSE 2020

(a) $\frac{36}{48}$ (b) $\frac{49}{64}$ (c) $\frac{49}{81}$ (d) $\frac{36}{81}$

Q.7. Given below is ΔPQR, right-angled at Q.

What is the value of $\tan\theta$?



(a) $\frac{PQ}{QR}$

(b) $\frac{QR}{PQ}$

(c) $\frac{PQ}{RP}$

(d) $\frac{QR}{RP}$

Q.8. If $\cos X = \frac{a}{b}$, then sin X is equal to: **COMPETENCY**

(a) $\frac{(b^2-a^2)}{b}$ (b) $\frac{(b-a)}{b}$

(c) $\frac{\sqrt{(b^2-a^2)}}{b}$ (d) $\frac{\sqrt{(b-a)}}{b}$

FREE ADVICE: Just recall an identity out of the given t-ratios i.e., here 'sin' and 'cos' remind us about ($\sin^2\theta + \cos^2\theta = 1$) which helps in solving this question.

Q.9. The value of

(sin 30° + cos 30°) – (sin 60° + cos 60°)

is [NCERT EXEMPLAR

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Q.10. What is the minimum value of $\cos\theta$, $0 \le \theta \le 90°$? **COMPETENCY**

(a) –1 (b) 0 (c) 1 (d) $\frac{1}{2}$

FREE ADVICE: Remember this:
Min value (sin θ) = 0 at 0°
Min value (cos θ) = 0 at 90°
Max value (sin θ) = 1 at 90°
Max value (cos θ) = 1 at 0°

Q.11. If θ is an acute angle of a right angled triangle, then which of the following equations is not true? [CBSE 2023

(a) $\sin\theta \cot\theta = \cos\theta$

(b) $\cos\theta \tan\theta = \sin\theta$

(c) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

(d) $\tan^2\theta - \sec^2\theta = 1$

Q.12. **sec θ when expressed in terms of cot θ, is equal to:** COMPETENCY

(a) $\frac{(1+\cot^2\theta)}{(\cot\theta)}$ (b) $\sqrt{(1+\cot^2\theta)}$

(c) $\frac{\sqrt{(1+\cot^2\theta)}}{(\cot\theta)}$ (d) $\frac{\sqrt{(1-\cot^2\theta)}}{(\cot\theta)}$

Q.13. **$(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$ is equal to:** [CBSE 2023

(a) –1 (b) 1 (c) 2 (d) 0

Q.14. **$(\cos^4 A - \sin^4 A)$ on simplification gives** COMPETENCY

(a) $2\sin^2 A - 1$ (b) $2\sin^2 A + 1$

(c) $2\cos^2 A + 1$ (d) $2\cos^2 A - 1$

Q.15. **If $\sec\theta + \tan\theta = p$, then $\tan\theta$ equals.** COMPETENCY

(a) $\frac{(p^2+1)}{(2p)}$ (b) $\frac{(p^2-1)}{(2p)}$

(c) $\frac{(p^2-1)}{(p^2+1)}$ (d) $\frac{(p^2+1)}{(p^2-1)}$

Q.16. **If $\cot 81° = \tan\theta$, what is the value of $\sec 5\theta$? (Note: $0° \le 5\theta \le 90°$)**

(a) $\frac{1}{\sqrt{2}}$ (b) 1

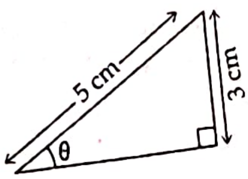
(c) $\sqrt{2}$ (d) 5θ will always be greater than 90°

Q.17. **What is the value of**

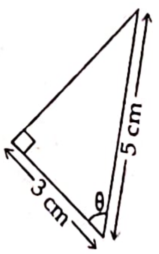
$$\frac{1}{1+\cot^2\theta} + \frac{1}{1+\tan^2\theta} = ?$$ COMPETENCY

(a) $\frac{1}{2}$ (b) 1 (c) 0 (d) $\frac{1}{\sqrt{2}}$

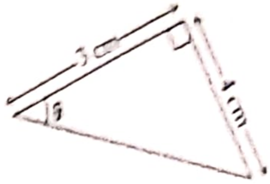
Q.18. **Which of these triangles have $\sin\theta = \frac{4}{5}$?** [CBSE 2024



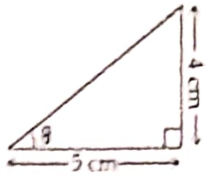
P



Q



R



S

(a) 1 only Q & R

(b) 2 only Q & S

(c) 3 only Q, R & S

(d) All – P, Q, R & S

Q.19. Given that, $\sin A + \sin^2 A = 1$, then the value of $\cos^2 A + \cos^4 A$ is [CBSE 2020

(a) $\frac{1}{2}$ (b) 1 (c) 0 (d) $\frac{1}{\sqrt{2}}$

Q.20. Which of these is equal to $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$?

(a) $\sec\theta + \tan\theta$ (b) $\sec\theta - \tan\theta$

(c) $\sec^2\theta + \tan^2\theta$ (d) $(\sec\theta + \tan\theta)^2$

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(a) *Both A and R are true and R is the correct explanation of A.*

(b) *Both A and R are true but R is not the correct explanation of A.*

(c) *A is true and R is false.*

(d) *A is false, but R is true.*

Q.1. **Assertion:** In a right ΔABC, right angled at B, if $\tan A = 1$, then $2\sin A \cdot \cos A = 1$.

Reason: cosec A is the abbreviation used for cosecant of angle A. COMPETENCY

Q.2. **Assertion:** The value of

$\tan 20° = \frac{\tan 60°}{3} = \frac{\sqrt{3}}{3}$.

Reason: For an acute angle θ,

$\tan\left(\frac{1}{3}\theta\right) = \frac{1}{3}\tan\theta$.

Q.3. **Assertion:** ΔABC, right-angled at B, AB = 24 cm, BC = 7 cm. The value of tan C is $\frac{24}{7}$. COMPETENCY

Reason: $\tan C = \frac{\textit{Opposite side}}{\textit{Adjacent side}}$

Q.4. Assertion: If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$ then the value of $x + y = 3$.
Reason: For any value of θ,
$\sin^2\theta + \cos^2\theta = 1$ **COMPETENCY**

Q.5. Assertion: For $0 < \theta \le 90°$, $\operatorname{cosec}\theta - \cot\theta$ and $\operatorname{cosec}\theta + \cot\theta$ are reciprocal of each other.
Reason: $\operatorname{cosec}^2\theta - \cot^2\theta = 1$. **COMPETENCY**

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. Evaluate:

$$\frac{5}{\cot^2 30°} + \frac{1}{\sin^2 60°} - \cot^2 45° + 2\sin^2 90°$$

COMPETENCY

Q.2. What is the value of

$\sin^2\theta + \sec^2\theta + \cos^2\theta - \tan^2\theta$ where θ is an acute angle? Show your work.

Q.3. If $\sin\theta - \cos\theta = 0$, then find the value of $\sin^4\theta + \cos^4\theta$. [CBSE 2024

Q.4. If $\tan\theta = \frac{3}{4}$, find the value of $\frac{(1-\cos^2\theta)}{(1+\cos^2\theta)}$

[CBSE 2024

Q.5. In ΔABC, $AC = 25$ cm and $\sin C = \frac{4}{5}$.
Find the length of BC. Show your work.

Q.6. Prove: $\frac{\sin\theta}{(1+\cos\theta)} + \frac{(1+\cos\theta)}{\sin\theta} = 2\operatorname{cosec}\theta$.

[NCERT EXEMPLAR

Short Answer Questions

Q.1. Prove that:

$$\frac{(\sin A - 2\sin^3 A)}{(2\cos^3 A - \cos A)} = \tan A$$ [CBSE 2023

Q.2. Prove that: [CBSE 2024
$\sec A(1 - \sin A)(\sec A + \tan A) = 1$

Q.3. Prove that:

$$\frac{(1+\sec A)}{\sec A} = \frac{\sin^2 A}{(1-\cos A)}$$ **COMPETENCY**

Q.4. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then prove that $q(p^2 - 1) = 2p$.

[CBSE 2024

Q.5. Prove the identity $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$.

COMPETENCY

Q.6. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$ [CBSE 2020

Q.7. Prove that:

$$\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = \operatorname{cosec}\theta - \cot\theta$$

Q.8. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$.
[CBSE 2020

(DAY 19)

Long Answer Questions

Q.1. If $3\cot A = 4$, check whether $\frac{(1-\tan^2 A)}{(1+\tan^2 A)} = \cos^2 A - \sin^2 A$ or not. [NCERT

Q.2. Prove that:

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \frac{1}{(1 - 2\cos^2 A)}$$

[CBSE 2019

Q.3. Prove:

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$$

Q.4. If $\sec A + \tan A = m$, show that $\frac{m^2 - 1}{m^2 + 1} = \sin A$. **COMPETENCY**

Q.5. Solve the following:

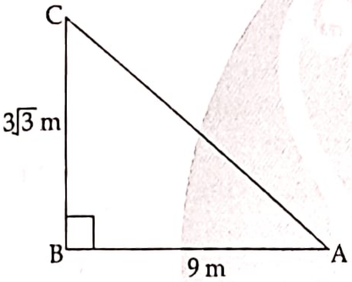
(*i*) Given $\tan A = \frac{5}{12}$, find $\sin A$, $\cos A$, $\cot A$, $\sec A$, $\operatorname{cosec} A$.

(*ii*) Given $4\cos^2 A + 8\sin^2 A = 5$, show that $\cot A = \sqrt{3}$.

Show your work.

CASE BASED QUESTIONS

Q.1. Three friends – Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right angled triangle such that AB = 9 m, BC = $3\sqrt{3}$ m and B = 90°.



Based on the above information, answer the following questions.

(a) What is the measure of $\angle A$.

(b) (i) What is the measure of $\angle C$.

COMPETENCY

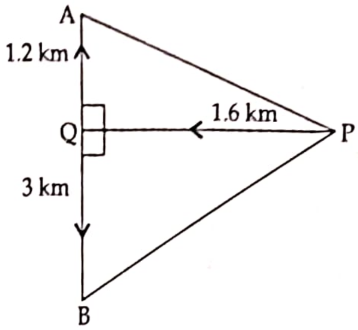
Or

(ii) What is the length of AC.

(c) What is the value of cos 2A.

COMPETENCY

Q.2. Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400 km/hr and 500 km/hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes.



Based on the above information, answer the following questions.

(a) Find tan p; if $\angle APQ = 0$.

COMPETENCY

(b) (i) Find cot B.

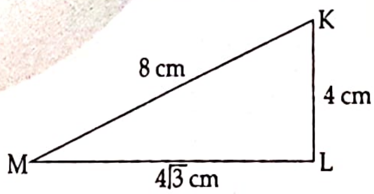
Or

(ii) Find tan A.

(c) Find sec A.

Q.3. Ritu's daughter is feeling so hungry and so thought to eat something. She looked into the fridge and found some bread pieces.

She decided to make a sandwich. She cut the piece of bread diagonally and found that it forms a right angled triangle with 4 cm, $4\sqrt{3}$ cm and 8 cm.



Based on the above information, answer the following questions.

(a) What is the value of $\angle M$?

(b) (i) The value of $\angle K$.

Or

(ii) Find the value of cot M.

COMPETENCY

(c) What is the value of $\frac{(\tan^2 45 - 1)}{(\tan^2 45 + 1)}$?

COMPETENCY

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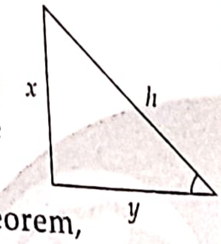
Multiple Choice Answers

1. (*b*) AB = 24 cm and BC = 7 cm

$$\tan C = \frac{\textit{Opposite side}}{\textit{Adjacent side}}$$

$$\therefore \tan C = \frac{24}{7}$$

2. (*b*) $\tan\theta = \frac{x}{y}$



So the perpendicular sides of the triangle are x and y

By Pythagoras' theorem,

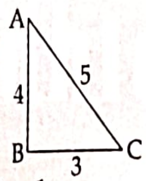
$(\text{Hypotenuse})^2 = x^2 + y^2$

Hypotenuse (H) $= \sqrt{x^2 + y^2}$

$$\therefore \cos\theta = \frac{y}{H} = \frac{y}{\sqrt{x^2+y^2}}$$

3. (*c*) $\cos\theta = \frac{3}{5}$ ...[given

$AB = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$



$$\therefore \sin\theta = \frac{AB}{AC} = \frac{4}{5}$$

$$\therefore \tan\theta = \frac{AB}{BC} = \frac{4}{3}$$

Now, A.T.Q, $\frac{\sin\theta\tan\theta - 1}{2\tan^2\theta}$

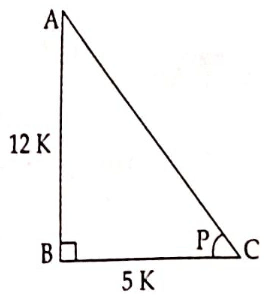
$$= \frac{\frac{4}{5}\times\frac{4}{3} - 1}{2\times\frac{4}{3}\times\frac{4}{3}}$$

$$= \frac{\frac{16}{15} - 1}{\frac{32}{9}} = \frac{\frac{1}{15}}{\frac{32}{9}} = \frac{1}{15}\times\frac{9}{32} = \frac{3}{160}$$

4. (*d*) **Given.** $\tan P = \frac{12}{5} = \frac{P}{B}$

AB = 12 K;

BC = 5 K



By pythagoras theorem,

$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$

$\Rightarrow (AC)^2 = (12\text{ K})^2 + (5\text{ K})^2$

$\Rightarrow (AC)^2 = 144\text{ K}^2 + 25\text{ K}^2$

$\Rightarrow (AC)^2 = 169\text{ K}^2 \quad \therefore AC = 13\text{ K}$

$$\sec(90^\circ - P) = \text{cosec } P = \frac{H}{P} = \frac{13\text{ K}}{12\text{ K}}$$

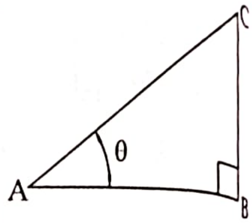
$$\therefore \textbf{cosec } P = \frac{13}{12}$$

5. (*b*) We can simply find the minimum value of sin A by using the trigonometric properties of sin A.

sin 0° = 0 (minimum value).

sin 90° = 1 (maximum value)

6. (*b*) Let AB = $7k$ and BC = $8k$, where k is a positive integer.



By applying Pythagoras theorem in ΔABC, We get

$AC^2 = AB^2 + BC^2 = (7k)^2 + (8k)^2$

$= 49k^2 + 64k^2 = 113k^2$

$AC = \sqrt{113k^2} = \sqrt{113}\,k$

Therefore,

$$\sin\theta = \frac{\textit{side opposite to }\theta}{\textit{hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{8k}{\sqrt{113}\,k} = \frac{8}{\sqrt{113}}$$

Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta} \quad [\therefore (a+b)(a-b) = (a^2-b^2)]$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{\left(1-\frac{64}{113}\right)}{\left(1-\frac{49}{113}\right)}$$

$$= \frac{\left(\frac{49}{113}\right)}{\left(\frac{64}{113}\right)} = \frac{49}{64}$$

7. (*b*) $\tan\theta = \frac{P}{B} = \frac{QR}{PQ}$

8. (*c*) $\cos X = \frac{a}{b}$ ...(*i*)

By trigonometry identities, we know that:

$\sin^2 X + \cos^2 X = 1$

$\sin^2 X = 1 - \cos^2 X = 1 - \left(\frac{a}{b}\right)^2$ ...[From(*i*)

$\therefore$ **$\sin X = \frac{\sqrt{(b^2 - a^2)}}{b}$**

9. (*a*) Using the trigonometric identities,

$\sin 30 = \frac{1}{2}$; $\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$; $\cos 60° = \frac{1}{2}$

$(\sin 30° + \cos 30°)$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{(1+\sqrt{3})}{2}$

$(\sin 60° + \cos 60°)$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{(\sqrt{3}+1)}{2}$

Now, $(\sin 30° + \cos 30°) - (\sin 60° + \cos 60°)$

$= \left(\frac{(1+\sqrt{3})}{2}\right) - \left(\frac{(\sqrt{3}+1)}{2}\right)$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} - \frac{1}{2} = \mathbf{0}$

10. (*b*) In the context of angles between 0 and 90 degrees, the cosine function is at its minimum value of 1 when θ is 0 degrees, which corresponds to the base case of a right triangle with a 90-degree angle. As the angle θ increases from 0 to 90 degrees, the value of cos θ decreases from 1 to 0.

11. (*d*) $\tan^2 \theta - \sec^2 \theta = 1$

12. (*c*) We know that, $\sec \theta = \frac{1}{\cos \theta}$

We can convert it into

$= \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta}$

$= \frac{1}{\cot \theta} \cdot \operatorname{cosec} \theta$ ...[$\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$

$= \frac{\sqrt{(1 + \cot^2 \theta)}}{\cot \theta}$

13. (*b*) $\sec^2 \theta - \tan^2 \theta = 1$

$\operatorname{cosec}^2\theta - \cot^2 \theta = 1$

L.H.S. $= (\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1)$

$= \tan^2 \theta \times \cot^2 \theta = \mathbf{1}$

14. (*d*) $\cos^4 A - \sin^4 A$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A) . 1$

$= \cos^2 A - (1 - \cos^2 A) = \mathbf{2\cos^2 A - 1}$.

15. (*b*) $\frac{(p^2 - 1)}{(2p)}$

16. (*c*) **Given.** $\cot 81° = \tan \theta$

$\cot(90° - 9°) = \tan \theta$

$\tan 9° = \tan \theta$ $\therefore \theta = 9°$

The value of $\sec 5\theta = \sec 5 \times 9°$

$= \sec 45° = \frac{1}{\cos 45°} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

17. (*b*) We know that,

$1 + \tan^2 \theta = \sec^2 \theta$

and $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$\therefore \frac{1}{1+\tan^2 \theta} + \frac{1}{1+\cot^2 \theta}$

$= \frac{1}{\sec^2 \theta} + \frac{1}{\operatorname{cosec}^2\theta}$

$= \cos^2 \theta + \sin^2 \theta = \mathbf{1}$

18. (*a*) First we calculate we know sides

In Fig P, side $= \sqrt{5^2 - 3^2}$

$= \sqrt{25 - 9} = \sqrt{16} = \mathbf{4}$

In Fig Q, side $= \sqrt{5^2 - 3^2}$

$= \sqrt{25 - 9} = \sqrt{16} = \mathbf{4}$

In Fig R, side $= \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16} = \sqrt{25} = \mathbf{5}$

In Fig S, side $= \sqrt{5^2 + 4^2}$

$= \sqrt{25 + 16} = \mathbf{\sqrt{41}}$

$\therefore$ Option of only Q and R correct

$\therefore \sin \theta = \frac{P}{H} = \frac{4}{5}$

19. (*b*) **Given.** $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A$

$\Rightarrow \sin A = \cos^2 A$

Now, $\cos^2 A + \cos^4 A$

$= \cos^2 A[1 + \cos^2 A]$

$= \sin A[1 + \sin A]$

$= \sin A + \sin^2 A = 1$

Hence, the answer is 1.

20. (*a*) We have, $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

$$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1)^2 - \sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \mathbf{\sec\theta + \tan\theta}$$

Assertion Reason Answers

1. (*b*) Both (A) and (R) are true but (R) is not the correct explanation of (A).

Explanation: *We have, $\tan A = 1$*

$\frac{\sin A}{\cos A} = 1$

$\Rightarrow \sin A = \cos A \quad \Rightarrow \sin A - \cos A = 0$

Squaring both sides, we get

$(\sin A - \cos A)^2 = 0$

$\Rightarrow \sin^2 A + \cos^2 A - 2 \sin A \cos A = 0$

$\Rightarrow 1 - 2 \sin A \cos A = 0$

$\therefore \mathbf{2 \sin A \cos A = 1}$

2. (*d*) (A) is false, but (R) is true.

3. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: *In a right-angled triangle ΔABC, right-angled at B, if AB = 24 cm and BC = 7 cm, you can find the value of tan C as follows:*

$\tan C = \frac{\text{Opposite side}}{\text{Adjacent side}}$

In this case, the side opposite angle C is AB, which is 24 cm, and the side adjacent to angle C is BC, which is 7 cm.

So, $\tan C = \frac{AB}{BC} = \frac{24}{7}$.

4. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation:

$x + y = 2\sin^2\theta + 2\cos^2\theta + 1$

$= 2(\sin^2\theta + \cos^2\theta) + 1$

$= 2 + 1 = 3 \quad ...[\because \sin^2\theta + \cos^2\theta = 1]$

5. (*d*) (A) is false, but (R) is true.

Explanation:

Here, $\text{cosec}^2\theta - \cot^2\theta = 1$

therefore R is True.

Very Short Answers

1. **Given.** $\frac{5}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cot^2 45^\circ - 2\sin^2 90^\circ$

$$= \frac{5}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - 1^2 - 2$$

$$= \frac{5}{3} + \frac{4}{3} + 1 = \frac{(5+4+3)}{3} = \frac{12}{3} = \mathbf{4}$$

2. We have,

$\sin^2\theta + \sec^2\theta + \cos^2\theta - \tan^2\theta$

$= (\sin^2\theta + \cos^2\theta) + (\sec^2\theta - \tan^2\theta)$

$= 1 + 1 = \mathbf{2}$

$...[\sin^2\theta + \cos^2\theta = 1, \sec^2\theta - \tan^2\theta = 1]$

3. From the given equation, We get

$\cos\theta = \sin\theta$

Hence, $\theta = 45^\circ$

Therefore, $\sin^4\theta + \cos^4\theta$

$= \sin^4 45^\circ + \cos^4 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \mathbf{\frac{1}{2}}$$

4. **Given.** $\tan\theta = \frac{3}{4}$

$\Rightarrow \tan^2\theta = \frac{9}{16} \quad \Rightarrow \sec^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\Rightarrow \cos^2\theta = \frac{16}{25} \quad \Rightarrow \cos\theta = \frac{4}{5}$

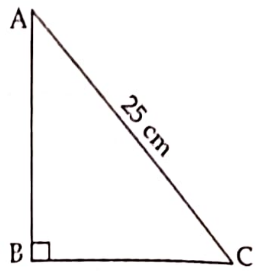
Now, $$\frac{(1-\cos^2\theta)}{(1+\cos^2\theta)} = \left[\frac{1-\left(\frac{4}{5}\right)^2}{1+\left(\frac{4}{5}\right)^2}\right]$$

$...[\because \cos\theta = \frac{4}{5}]$

$$= \frac{\left(\frac{(25-16)}{25}\right)}{\left(\frac{(25+16)}{25}\right)} = \mathbf{\frac{9}{41}}$$

5. We have,

$AC = 25$ cm



$\sin C = \frac{4}{5} = \frac{P}{H}$

$\Rightarrow \quad \frac{AB}{AC} = \frac{4}{5}$

$\Rightarrow \quad \frac{AB}{25} = \frac{4}{5}$

$\therefore \quad AB = \frac{25 \times 4}{5} = 20$ cm

In ΔABC, by using Pythagoras theorem,

$(H)^2 = (B)^2 + (P)^2$

$(AC)^2 = (BC)^2 + (AB)^2$

$\Rightarrow (25)^2 = (BC)^2 + (20)^2$

$\Rightarrow (BC)^2 = 625 - 400 = 225$

$\therefore$ **BC** $= \sqrt{225} = 15$ **cm**

So, length of BC = 15 cm.

6. LHS $\frac{\sin\theta}{(1+\cos\theta)} + \frac{(1+\cos\theta)}{\sin\theta}$

$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \frac{[\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta]}{[\sin\theta(1+\cos\theta)]}$

As, $\sin^2\theta + \cos^2\theta = 1$

$= \frac{[1+1+2\cos\theta]}{[\sin\theta(1+\cos\theta)]}$

$= \frac{[2+2\cos\theta]}{[\sin\theta(1+\cos\theta)]}$

$= \frac{2[1+\cos\theta]}{[\sin\theta(1+\cos\theta)]} = \frac{2}{\sin\theta}$

$= 2\,\text{cosec}\,\theta$ ...[**As** $\frac{1}{\sin\theta} = \text{cosec}\,\theta$

= RHS

Therefore, it is proved.

Short Answers

1. We have, $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$

$= \frac{\sin A}{\cos A} \cdot \frac{\cos 2A}{\cos 2A} =$ **tan A**

2. **Given.**

L.H.S $\sec A(1 - \sin A)(\sec A + \tan A)$

Here, $\sec A = \frac{1}{\cos A}$ and $\tan A = \frac{\sin A}{\cos A}$

$\Rightarrow \frac{1}{\cos A} \times (1 - \sin A) \times \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$\Rightarrow \frac{1}{\cos^2 A} \times (1 - \sin A) \times (1 + \sin A)$

$\Rightarrow \frac{1}{\cos^2 A}(1 - \sin^2 A)$

$\Rightarrow \frac{1 - \sin^2 A}{\cos^2 A} \quad \Rightarrow \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$

$\Rightarrow \sec^2 A - \tan^2 A = 1 =$ R.H.S.

3. L.H.S. $\left(\frac{1+\sec A}{\sec A}\right) = \frac{\left(1 + \frac{1}{\cos A}\right)}{\left(\frac{1}{\cos A}\right)}$

$= \frac{\left(\frac{\cos A + 1}{\cos A}\right)}{\left(\frac{1}{\cos A}\right)} = \frac{\cos A + 1}{\cos A} \times \frac{\cos A}{1}$

$= 1 + \cos A$

$= 1 + \cos A \times \frac{(1 - \cos A)}{(1 - \cos A)}$

$= \frac{(1^2 - \cos^2 A)}{(1 - \cos A)}$

$= \frac{1 - \cos^2 A}{1 - \cos A} = \frac{\sin^2 A}{1 - \cos A} =$ **R.H.S.**

4. We have, $q(p^2 - 1)$

$= (\sec\theta + \text{cosec}\,\theta)[(\sin\theta + \cos\theta)^2 - 1]$

$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$

$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)(2\sin\theta\cos\theta)$

$= 2(\sin\theta + \cos\theta) = 2p =$ R.H.S.

5. L.H.S. $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$

$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= \left(\frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta)}{\sin\theta\cos\theta}\right)$

$= (\sin\theta + \cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)$

$$= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$

$= \sec\theta + \operatorname{cosec}\theta$ = R.H.S. (Hence proved)

6. $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 2\sin\theta\cos\theta = 2$

$\Rightarrow \sin\theta\cos\theta = 1$

$\Rightarrow \sin\theta\cos\theta = \sin^2\theta + \cos^2\theta$

$$\Rightarrow 1 = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$\Rightarrow \tan\theta + \cot\theta = 1$

7. $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} = \operatorname{cosec}\theta - \cot\theta$

L.H.S.

$$\sqrt{\frac{(\sec\theta - 1)}{(\sec\theta + 1)} \times \frac{(\sec\theta - 1)}{(\sec\theta - 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\sec^2\theta - 1}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{(\sec\theta + 1)(\sec\theta - 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\sec^2\theta - 1}} = \sqrt{\left(\frac{\sec\theta - 1}{\tan\theta}\right)^2}$$

$$= \frac{\sec\theta - 1}{\tan\theta} = \frac{\sec\theta}{\tan\theta} - \frac{1}{\tan\theta}$$

$$= \left(\frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}\right) - \frac{1}{\tan\theta}$$

$\operatorname{cosec}\theta - \cot\theta$ = R.H.S.

8. L.H.S. $\tan^4\theta + \tan^2\theta$

$\tan^2\theta\,(1 + \tan^2\theta)$

$(\sec^2\theta - 1)(\sec^2\theta)$ $[\because \tan^2\theta + 1 = \sec^2\theta$

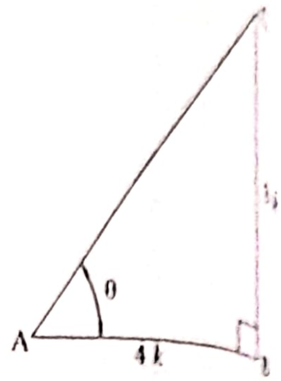
$\sec^4\theta - \sec^2\theta$ (Hence Proved)

Long Answers

1. We use the basic concepts of trigonometric ratios like cot, tan, cos, and sin to solve the question.

3 cot A = 4

Let ΔABC be a right-angled triangle where angle B is a right angle.



cot A = $\dfrac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A}$

$$= \frac{AB}{BC} = \frac{4}{3}$$

Let AB = $4k$ and BC = $3k$

...where [k is a positive integer]

By applying the Pythagoras theorem in ΔABC, we get,

$AC^2 = AB^2 + BC^2$

$= (4k)^2 + (3k)^2 = 16k^2 + 9k^2 = 25k^2$

$AC = \sqrt{25}\,k^2 = 5k$

Therefore,

$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\text{L.H.S} = \frac{(1 - \tan^2 A)}{(1 + \tan^2 A)}$$

$$= \frac{\left[1 - \left(\frac{3}{4}\right)^2\right]}{1 + \left(\frac{3}{4}\right)^2} = \frac{\left(\frac{16 - 9}{16}\right)}{\left(\frac{16 + 9}{16}\right)} = \frac{16 - 9}{16 + 9} = \frac{7}{25}$$

R.H.S = $\cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{16 - 9}{25} = \frac{7}{25}$$

Therefore, $\dfrac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$,

(Hence Proved)

2. L.H.S. $\dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A}$

We know,

$\tan A = \dfrac{\sin A}{\cos A}$;

$$\text{cosec}(A) = \frac{1}{\sin A}$$

and $$\sec(A) = \frac{1}{\cos A}$$

Now, A.T.Q

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\sin^2 A} - \frac{1}{\cos^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A \sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} \times \frac{\cos^2 A}{\sin^2 A - \cos^2 A} + \frac{1}{\sin^2 A} \times \frac{\cos^2 A \sin^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{(1 - \cos^2 A) - \cos^2 A}$$

$$\text{...Using} \left[\begin{array}{l} \because \sin^2 A + \cos^2 A = 1 \\ \text{and } \sin^2 A = 1 - \cos^2 A \end{array} \right.$$

$$= \frac{1}{1 - 2\cos^2 A} = \text{R.H.S.}$$

3. We have, $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1 + \sin\theta}{\cos\theta}$

L.H.S.

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$\Rightarrow \frac{\tan\theta + \sec\theta - \{\sec^2\theta - \tan^2\theta\}}{1 + \tan\theta - \sec\theta}$$

$$\text{...}[\because 1 + \tan^2\theta = \sec^2\theta$$

$$\Rightarrow \frac{(\tan\theta + \sec\theta) - \{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)\}}{1 + \tan\theta - \sec\theta}$$

$$\Rightarrow \frac{(\tan\theta + \sec\theta)\{1 - \sec\theta + \tan\theta\}}{(1 + \tan\theta - \sec\theta)}$$

$$\Rightarrow \tan\theta + \sec\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \frac{1 + \sin\theta}{\cos\theta} = \text{R.H.S. (Hence proved)}$$

4. L.H.S. $\dfrac{m^2 - 1}{m^2 + 1}$

$$= \frac{(\sec A + \tan A)^2 - 1}{(\sec A + \tan A)^2 + 1}$$

$$= \frac{[\sec^2 A + \tan^2 A + 2\sec A \tan A - 1]}{[\sec^2 A + \tan^2 A + 2\sec A \tan A + 1]}$$

$$= \frac{[(\sec^2 A - 1) + \tan^2 A + 2\sec A \tan A]}{[\sec^2 A + (\tan^2 A + 1) + 2\sec A \tan A]}$$

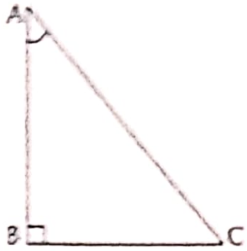
$$= \frac{[2\tan^2 A + 2\sec A \tan A]}{[2\sec^2 A + 2\sec A \tan A]}$$

$$= \frac{2\tan A}{2\sec A}$$

$$= \frac{\sin A}{\cos A \times \cos A} = \sin A = \text{RHS}$$

(Hence proved)

5. (*i*) $\tan A = \dfrac{5}{12} = \dfrac{P}{B}$



$P = 5\,K$

$B = 12\,K$

In ΔABC,

By using pythagoras theorem,

$$(H)^2 = (B)^2 + (P)^2$$

$$\Rightarrow (H)^2 = (5K)^2 + (12K)^2$$

$$\Rightarrow (H)^2 = 25K^2 + 144\,K^2$$

$$\Rightarrow (H)^2 = 169K^2$$

$$\Rightarrow H = \sqrt{169K^2} \quad \therefore H = 13K$$

Now, $\sin A = \dfrac{P}{H} = \dfrac{5K}{13K} = \dfrac{5}{13}$

$\cos A = \dfrac{B}{H} = \dfrac{12K}{13K} = \dfrac{12}{13}$

$\sec A = \dfrac{1}{\cos A} = \dfrac{13}{12}$

$\text{cosec } A = \dfrac{1}{\sin A} = \dfrac{13}{5}$

$\cot A = \dfrac{\cos A}{\sin A} = \dfrac{12}{5}$

(*ii*) We have, $4\cos^2 A + 8\sin^2 A = 5$

$$\Rightarrow 4\cos^2 + 8(1 - \cos^2 A) = 5$$

$$\Rightarrow 4\cos^2 A + 8 - 8\cos^2 A = 5$$

$$\Rightarrow -4\cos^2 A = -3$$

$$\Rightarrow \cos^2 A = \frac{3}{4}$$

$$\Rightarrow \cos A = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

Now, $\sin^2 A = 1 - \cos^2 A$

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

$$\sin A = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\text{Now, } \cot A = \frac{\cos A}{\sin A}$$

$$\therefore \cot A = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3} \textbf{ (Hence Proved)}$$

Case Based Answers

1. (*a*) We have, $AB = 9$ m $BC = 3(\sqrt{3})$ m
In triangle ABC we have

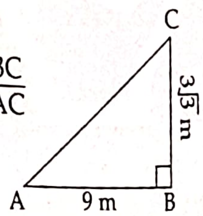
$$\tan A = \frac{BC}{AB} = \frac{(3\sqrt{(3)})}{9} = \frac{1}{(\sqrt{3})}$$

$\tan A = \tan 30°$ $\therefore$ $\angle A = 30°$

(*b*) (*i*) Similarly, $\tan C = \dfrac{AB}{BC} = \dfrac{9}{(3\sqrt{3})}$

$\Rightarrow \tan C = \sqrt{(3)}$
$\Rightarrow \tan C = \tan 60°$
$\therefore \angle C = 60°$

Or



(*ii*) Since, $\sin A = \dfrac{BC}{AC}$

$\Rightarrow \sin 30° = \dfrac{BC}{AC}$

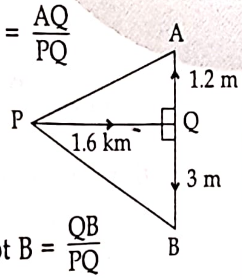
$\Rightarrow \dfrac{1}{2} = \dfrac{3\sqrt{3}}{AC}$ $\Rightarrow$ **$AC = 6\sqrt{3}$ m**

(*c*) $\angle A = 30°$
therefore, $\cos 2A = \cos 2(30)°$

$= \cos 60° = \dfrac{1}{2}$

2. (*a*) In ΔAPQ, $\tan P = \dfrac{AQ}{PQ}$

$\tan P = \dfrac{1.2}{1.6} = \dfrac{3}{4}$



(*b*) (*i*) In ΔPBQ, $\cot B = \dfrac{QB}{PQ}$

$\therefore \cot B = \dfrac{3}{1.6} = \dfrac{15}{8}$

Or

(*ii*) In ΔAPQ,

$$\tan A = \frac{PQ}{AQ} = \frac{1.6}{1.2} = \frac{4}{3}$$

(*c*) We have, $\tan^2 A + 1 = \sec^2 A$

$$\sec A = \sqrt{\left[\left(\frac{4}{3}\right)^2 + 1\right]}$$

$$= \sqrt{\left(\frac{16}{9} + 1\right)} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

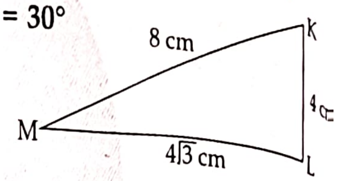
3. (*a*) $\tan M = \dfrac{KL}{LM}$

$\Rightarrow \tan M = \dfrac{4}{4\sqrt{3}}$

$\Rightarrow \tan M = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \tan M = \tan 30°$

$\therefore$ **$M = 30°$**



(*b*) (*i*) $\tan K = \dfrac{LM}{KL}$

$\Rightarrow \tan K = \dfrac{4\sqrt{3}}{4}$

$\Rightarrow \tan K = \sqrt{3}$

$\Rightarrow \tan K = \tan 60°$

$\therefore$ **$K = 60°$**

Or

(*ii*) $\cot M = \cot 30° = \sqrt{3}$

(*c*) $\dfrac{(\tan^2 45° - 1)}{(\tan^2 45° + 1)}$

$= \dfrac{(1-1)}{(1+1)}$

$= \dfrac{0}{2} = 0$

(DAY 19 SWAHA)

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9 Some Applications of Trigonometry

What did CBSE ask last year?

MCQs & A/R	1 Question (1 × 1 = 1 Mark)
Subjective	**No Very Short Question Asked**
	1 Short Question (1 × 3 = 3 Marks)
	1 Long Question (1 × 5 = 5 Marks)
Case Based	1 Case Based Question (1 + 1 + 2 = 4 Marks)

Note: All the above typology of questions include 'Competency based Questions' labelled as **COMPETENCY**.

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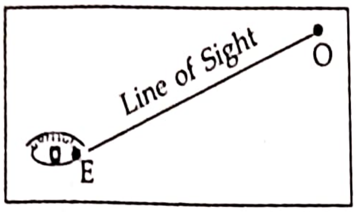
Height and Distance

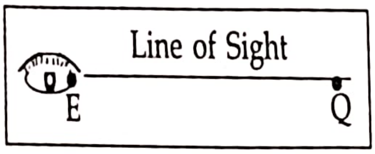
❑ Line of Sight

When an observer looks from a point E (eye) at an object O then the straight line EO between the eye E and the object O is called the line of sight.



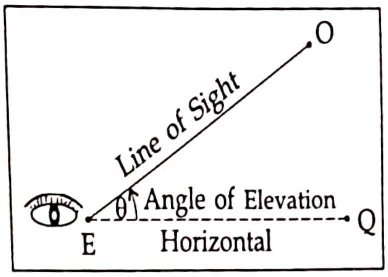
❑ Horizontal

an observer looks from a point E (eye) to another point Q which is horizontal to E, then the straight line, EQ between E and Q is called the horizontal line.



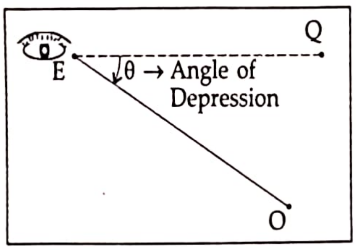
❑ Angle of Elevation

When the eye is below the object, then the observer has to look up from the point E to the object O. The measure of this rotation (angle θ) from the horizontal line is called the angle of elevation.



❑ Angle of Depression

When the eye is above the object, then the observer has to look down from the point E to the object. The horizontal line is now parallel to the ground. The measure of this rotation (angle θ) from the horizontal line is called the angle of depression.



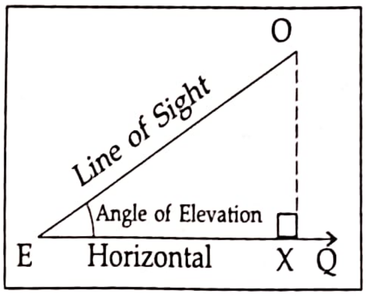
❑ **How to convert above figure into right triangle.**

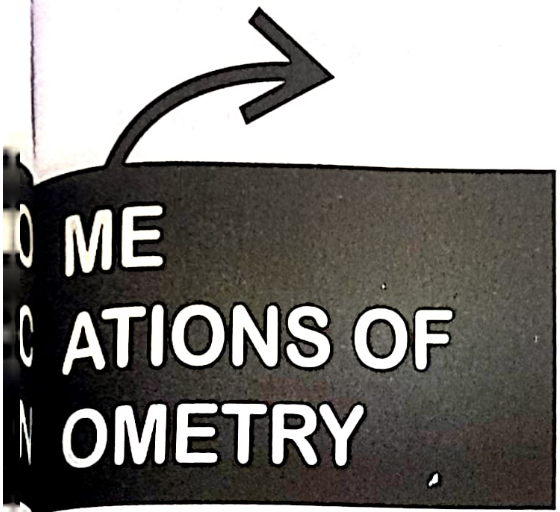
Case I: Angle of Elevation is known

Draw OX perpendicular to EQ Now $\angle OXE = 90^\circ$

$\angle OXE$ is a rt. D,

$$\ldots\text{where}\begin{cases} OE = \text{Hypotenuse} \\ OX = \text{Opposite side (Perpendicular)} \\ EX = \text{Adjacent side (Base)} \end{cases}$$





Case II: Angle of Depression is known

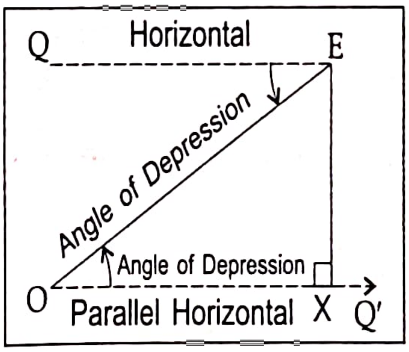
(i) Draw OQ′ parallel to EQ

(ii) Draw perpendicular EX on OQ′

(iii) Now $\angle QEO = \angle EOX$ = Interior alternate angles

$\therefore$ ΔEXO is a rt. Δ.

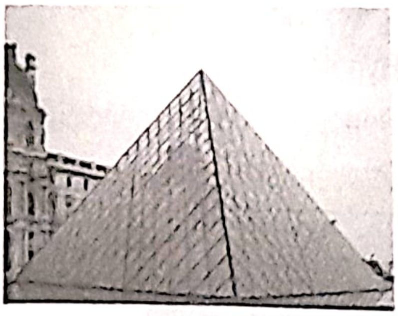
...where [EO = Hypotenuse, OX = Adjacent side (base), EX = Opposite side (Perpendicular)



(DAY 20)

Multiple Choice Questions

Q.1. Shown here is the Louvre Museum, the world's most visited museum. In the shape of a square pyramid, it is 21.6 m high and has a base of edge 34 m.



Which of these angles is closest to the angle of elevation of the top of the museum from the mid-point of its 34 m edge? COMPETENCY

(a) 30° (b) 50° (c) 60° (d) 75°

Q.2. From a point on the ground, 20 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is 60°. What is the height of the tower? COMPETENCY

(a) $10\sqrt{3}$ m (b) $3\sqrt{3}$ m

(c) $4\sqrt{3}$ m (d) $20\sqrt{3}$ m

Q.3. The angle of elevation of the top of a tower from point A on the ground is 30°. The tower is 50 m high.
Approximately how far is point A from the foot of the tower?

(Note: Take $\sqrt{3}$ as 1.73.)

(a) 28.90 m (b) 50 m

(c) 86.50 m (d) 100 m

Q.4. The shadow of a tower is equal to its height at 10:45 a.m. The sun's altitude is: COMPETENCY

(a) 30° (b) 45° (c) 60° (d) 90°

Q.5. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower is 30°. The distance of the car from the base of the tower (in *m*) is: COMPETENCY

(a) $25\sqrt{3}$ (b) $50\sqrt{3}$

(c) $75\sqrt{3}$ (d) 150

Q.6. Two men standing on opposite sides of a flag staff measure the angles of the top of the flagstaff as 30° and 60°. If the height of the flagstaff is 18 m the distance between the men is : COMPETENCY

(a) 24 (b) $24\sqrt{3}$ (c) $\frac{24}{\sqrt{3}}$ (d) 31.2

Q.7. A kite is tied to a point on the ground. The length of the string between the kite and the point on the ground is 80 m. The string makes an angle θ with the ground such that tan θ = 60°.
What is the height of the kite above the ground? [CBSE 2024

(a) $2\sqrt{3}$ (b) 2 m (c) $40\sqrt{3}$ (d) $80\sqrt{3}$

Q.8. A plane is observed to be approaching the airport. It is at a distance of 12 km from the point of observation and makes an angle of elevation of 60°. The height above the ground of the plane is COMPETENCY

(a) $6\sqrt{3}$ m (b) $4\sqrt{3}$ km

(c) $3\sqrt{3}$ km (d) $2\sqrt{3}$ km

Q.9. From the top of a hill, it is observed that the angle of depression of the top of a tree and its foot are 45° and 60° respectively. The height of the tree is 20 m.
What is the height of the hill?
(Note: The base of the hill and the tree are on the same level.) COMPETENCY

(a) $10(\sqrt{3}+1)$ m

(b) $20+20(\sqrt{3}+1)$ m

(c) $20+10(\sqrt{3}+1)$m

(d) $20-10(\sqrt{3}+1)$ m

Q.10. The upper part of a tree broken over by the wind makes an angle of 30° with the ground, and the distance from the root to the point where the top of the tree meets the ground is 15 m. The present height of the tree is COMPETENCY

(a) 15 (b) $\frac{10}{\sqrt{3}}$

(c) 20 (d) None of these

Q.11. The angles of elevation of the top of a tower from two points at a distance of 4 m and 12 m from the base of the tower and in the same straight line with it are complementary. What is the height of the tower?

(a) 10 m (b) 8 m (c) 5 m (d) 12 m

Q.12. If the length of the shadow of a tree is decreasing then the angle of elevation is: COMPETENCY

(a) Increasing

(b) Decreasing

(c) Remains the same

(d) None of the above

Q.13. The angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level is called: [NCERT EXEMPLAR

(a) Angle of elevation

(b) Angle of depression

(c) No such angle is formed

(d) None of the above

Q.14. If a tower 6m high casts a shadow of $2\sqrt{3}$ m long on the ground, then the sun's elevation is:

(a) 60° (b) 45° (c) 30° (d) 90°

Q.15. The angle of elevation of the top of a building 30 m high from the foot of another building in the same plane is 60°, and also the angle of elevation of the top of the second tower from the foot of the first tower is 30°, then the distance between the two buildings is: COMPETENCY

(a) $10\sqrt{3}$ m (b) $15\sqrt{3}$ m

(c) $12\sqrt{3}$ m (d) 36 m

Q.16. A 10 m tall pole casts a shadow of 15 m when the sun is at a certain inclination. At the same time, a nearby building casts a shadow of 25 m.
How tall is the building? COMPETENCY

(a) 16.67 m (b) 20 m

(c) 37.5 m

(d) Cannot be determined with the given information.

Q.17. The angle of depression of an object on the ground, from the top of a 25 m high tower is 30°. The distance of the object from the base of the tower is COMPETENCY

(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m

(c) $75\sqrt{3}$ m (d) 50 m

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

(b) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*

(c) *(A) is true and (R) is false.*

(d) *(A) is false, but (R) is true.*

Q.1. **Assertion:** If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°.
Reason: According to Pythagoras theorem, $h^2 = l^2 + b^2$,
where h = hypotenuse, l = length and b = base.

Q.2. **Assertion:** The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Reason: Trigonometric ratios are used to find height or length of an object or distance between two objects. **COMPETENCY**

Q.3. Assertion: The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level.

Reason: The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal level when it is below the horizontal level. **COMPETENCY**

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. Find the length of the shadow on the ground of a pole of height 18 m when the angle of elevation θ of the sun is such that $\tan \theta = \frac{6}{7}$. [CBSE 2023

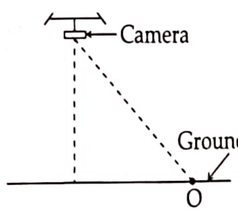
Q.2. A straight highway leads to the foot of a tower. A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60°, which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. (use $\sqrt{3} = 1.73$) [CBSE 2023

Q.3. If the ratio of height of the tower and the length of its shadow is $\sqrt{3} : 1$. What is the angle of elevation?

Q.4. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun? **COMPETENCY**

Q.5. AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of the pole. If AD = 2.54 m, find the length of the ladder. (use $\sqrt{3} = 1.73$) **COMPETENCY**

Q.6. A drone camera is used by a photographer for shooting videos. On some day, it is focussing at point O on the ground as shown in the figure.



The camera is at a perpendicular height of 6 m from the ground and at a distance of $4\sqrt{3}$ m from point O.

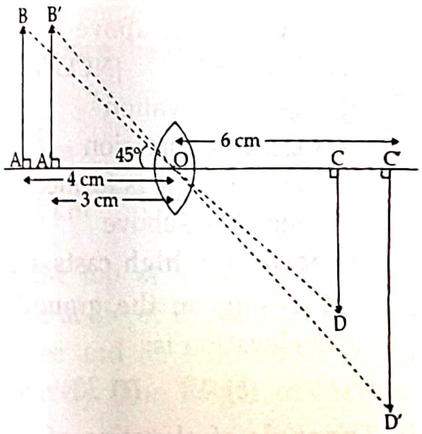
Find the angle of depression of point O from the camera. Show your steps.

(DAY 21)

Short Answer Questions

Q.1. The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill. **COMPETENCY**

Q.2. An inverted image CD is formed on the other side when an object AB is positioned in front of a convex lens at a distance of 4 cm from point O. Similarly when the same object is placed at a distance of 3 cm, it forms an image C'D' as depicted in the diagram.



(i) Find the height of the object AB.

(ii) Find the height of the image C'D'.

Show your work. [CBSE 2024

Q.3. Two men on either side of a 75 m high building and in line with base of the building, observe the angles of elevation of the top of the building as 30° and 60° respectively. Find the distance between the two men. **COMPETENCY**

Q.4. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string *[NCERT*

Q.5. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building. **COMPETENCY**

Q.6. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building *[CBSE 2020,*

Q.7. On a straight line passing through the foot of a tower, two points C and D are at distance of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower. **COMPETENCY**

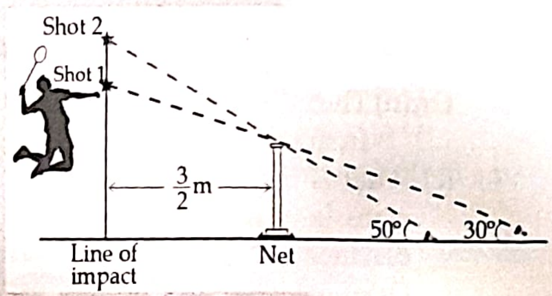
(DAY 22)

Long Answer Questions

Q.1. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles *[NCERT*

Q.2. A girl on a ship standing on a wooden platform, which is 50 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the platform and the height of the hill. *[CBSE 2021*

Q.3. A smash shot in badminton is when the shuttlecock travels in a straight line just above the nets. The line of impact and the net are perpendicular to the horizontal ground.



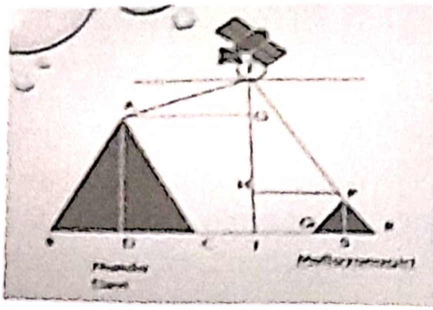
How much higher is shot 2 than shot 1? Draw a rough diagram and show your work. *[CBSE 2024*
(**Note:** Take sin 30° as 0.5, cos 30° as 0.9, sin 50° as 0.8 and cos 50° as 0.6.)

Q.4. Amit standing on a horizontal plane, finds a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, if the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposites sides of the birds. Find the distance of the birds from Deepak. *[CBSE 2019*

CASE BASED QUESTIONS

Q.1. A Satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka, them being Nanda Devi (height 7,816 m) and Mullayanagiri (height 1,930 m). The angles of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are 30° and 60°

respectively. If the distance between the peaks of two mountains is 1973 km, and the satellite is vertically above the mid-point of the distance between the two mountains.



On the basis of above information, answer the following questions.

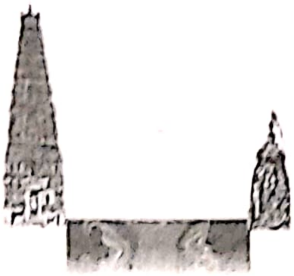
(a) The distance of the satellite from the top of Nanda Devi is ______.

(b) (i) The distance of the satellite from the top of Mullayanagiri is ______. **COMPETENCY**

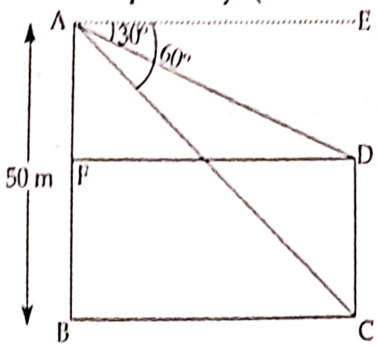
Or, (ii) The distance of the satellite from the sea level is ______.

(c) What is the angle of elevation if a man is standing at point C, a distance of 7816 m from Nanda Devi? **COMPETENCY**

Q.2. There are two temples on each bank of a river. One temple is 50 m high. A man, who is standing



on the top of 50 m high temple, observed from the top that angle of depression of the top and foot of other temple are 30° and 60° respectively. (Take $\sqrt{3} = 1.73$)



... above information, answer the following questions.

(a) What is the Width of the river?

(b) (i) What is the Measure of $\angle ADF$?

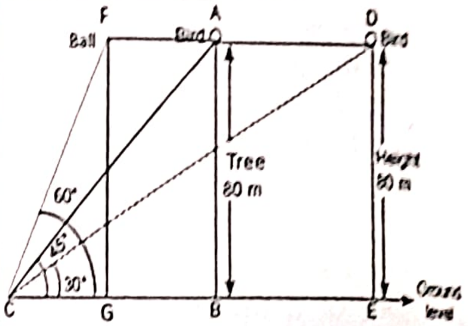
Or

(ii) What is the Measure of $\angle ACB$?

(c) What is the Height of the other temple? **COMPETENCY**

Q.3. One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on a nearby tree of height 80m. He observed a bird on the tree at an angle of elevation of 45°.

When a sixer was hit, a ball flew through the tree frightening the bird to fly away. In 2 seconds, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.



On the basis of above information, answer the following questions.

(a) At what distance from the foot of the tree was he observing the bird sitting on the tree?

(b) (i) How far did the bird fly in the mentioned time? [CBSE 2024]

Or

(ii) After hitting the tree, how far did the ball travel in the sky when Kaushik saw the ball?

(c) What is the speed of the bird in m/min if it had flown $20(\sqrt{3}+1)$ m? [CBSE 2024]

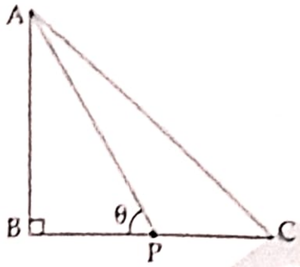
*****

ANSWERS

Multiple Choice Answers

1. (*b*) Height (h) = 21.6 m

Base length (b) = 34 m



As, P is mid-point of BC

$\Rightarrow$ BP = PC = $\frac{34}{2}$ = 17 m

In ΔABP,

$(AP)^2 = (AB)^2 + (BP)^2$

$(AP)^2 = (21.6)^2 + (17)^2$

$(AP)^2 = 466.56 + 289$

AP = 755.56 = 27.5 m

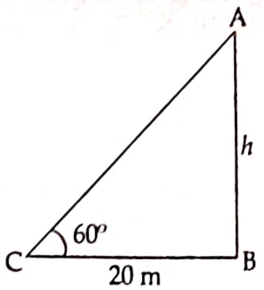
In ΔABP,

$\tan\theta = \frac{AB}{BP} = \frac{21.6}{17}$

$\Rightarrow \tan\theta = 1.27$

$\therefore$ **$\theta = 50^\circ$ (approx.), closest to angle of elevation of top of museum.**

2. (*d*) *Explanation.*



In ΔABC,

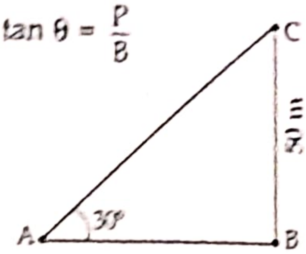
$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$\Rightarrow \tan 60^\circ = \frac{AB}{BC}$ ...[$\because \tan 60^\circ = \sqrt{3}$

$\Rightarrow h = 20\sqrt{3}$ m

Therefore, the height of the tower is $20\sqrt{3}$ m.

3. (*c*) As we know, $\tan\theta = \frac{P}{B}$



$\Rightarrow \tan 30^\circ = \frac{BC}{AB}$

$\Rightarrow \tan 30^\circ = \frac{50}{AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{AB}$

$\therefore$ **AB = $50\sqrt{3}$ = 50(1.73) = 86.50 m**

4. (*b*) *Explanation.*

Suppose θ be altitude of the sun.

$\tan\theta$ is given by:

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan\theta = \frac{\text{height of tower}}{\text{shadow of tower}}$

As height = shadow at 10:45 am,

$\tan\theta = 1$

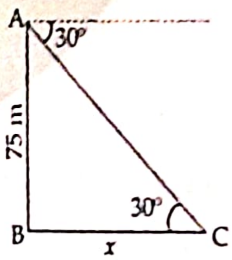
Taking tan inverse, both sides

$\tan^{-1}\tan\theta = \tan^{-1} 1$

$\Rightarrow \theta = 45^\circ$

Hence, altitude of the sun is 45°.

5. (c) *Explanation.*



In right angle ΔABC the $\angle$ACB = 30° (Angle of depression of a car) and the tower is 75 m high.

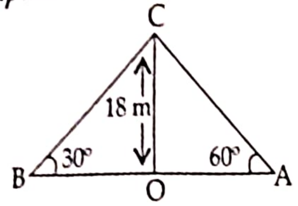
Let the distance of car from ground is x m.

Then $\tan 30^\circ = \frac{AB}{BC} = \frac{75}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x}$

$\Rightarrow x = 75\sqrt{3}$ m

6. (*b*) *Explanation.*



Let A and B be the points where the two men standing respectively.
Now let angle of elevation of the top of the flag-staff from A be 60° and that of, from the point B be 30°.
Then from ΔAOC

$$\tan 60^\circ = \frac{OC}{OA}$$

$$\Rightarrow \tan 60^\circ = \frac{18}{OA}$$

$$\Rightarrow OA = 6\sqrt{3} \quad ...(1)$$

And, from ΔOBC

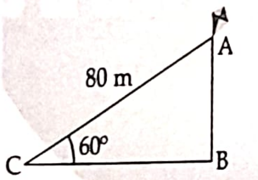
$$\tan 30^\circ = \frac{OC}{OB}$$

$$\Rightarrow \tan 30^\circ = \frac{18}{OB}$$

$$\Rightarrow OB = 18\sqrt{3} \text{ m} \quad ...(2)$$

Thus, the distance between the two men is $AB = OA + OB = 24\sqrt{3}$ cm.

7. (*c*) $\sin\theta = \frac{P}{H}$



$$\Rightarrow \sin 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AB}{80}$$

$$\Rightarrow AB = \frac{80\sqrt{3}}{2}$$

$$\therefore AB = 40\sqrt{3}$$

8. (*a*) *Explanation.*

Substitute the Values and using

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} = \frac{\text{Height above Ground}}{12}$$

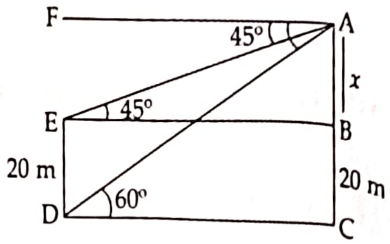
$\Rightarrow 6\sqrt{3}$ = Height above Ground

$\Rightarrow$ Height above Ground = 10.39

(Using $\sqrt{3} = 1.732$)

The height above the ground of the plane is $6\sqrt{3}$ km or about 10.39 km.

9. (*c*) Let AC be height of hill and BC be height of tree and BC = ED



In ΔABE,

$$\tan 45^\circ = \frac{AB}{EB}$$

$$\Rightarrow \tan 45^\circ = \frac{x}{EB}$$

$$\Rightarrow 1 = \frac{x}{EB}$$

$\Rightarrow EB = x$ m

$\therefore EB = DC = x$ cm ...(*i*)

In ΔADC,

$$\tan 60^\circ = \frac{AC}{DC}$$

$$\sqrt{3} = \frac{(20+x)}{x} \quad ...[\text{From } (i)]$$

$$\Rightarrow \sqrt{3}x = 20 + x$$

$$\Rightarrow \sqrt{3}x - x = 20$$

$$\Rightarrow x(\sqrt{3}-1) = 20$$

$$\Rightarrow x = \frac{20\times(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$\Rightarrow x = \frac{20(\sqrt{3}+1)}{3-1}$$

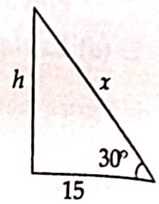
$$\Rightarrow x = \frac{20(\sqrt{3}+1)}{2}$$

$$\therefore x = 10(\sqrt{3}+1)$$

Height of the hill, AC

$= AB + BC = 20 + 10(\sqrt{3}+1)$ m

10. (*d*) None of these



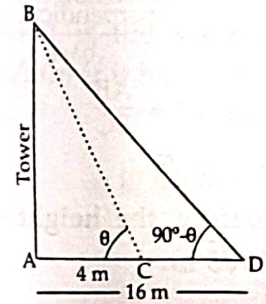
Here, $\tan 30^\circ = \frac{h}{15}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{15}$$

$$\Rightarrow h = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{3} \text{ m}$$

$$\therefore h = 5\sqrt{3}$$

11. (*b*) *Explanation.*



Here AB be the height of the tower.

In ΔACB, $\tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = \frac{AB}{4}$ ...(*i*)

In ΔABD,

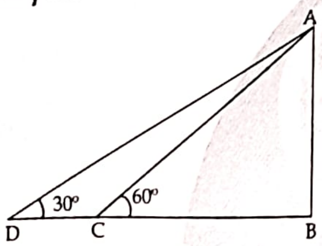
$\tan(90° - \theta) = \frac{AB}{AD}$

$\Rightarrow \cot\theta = \frac{AB}{16}$ or $\tan\theta = \frac{16}{AB}$ ...(*ii*)

Solving (*i*) and (*ii*), $\frac{AB}{4} = \frac{16}{AB}$

$\Rightarrow AB^2 = 64 \quad \therefore$ **AB = 8 m**

12. (*a*) *Explanation.*



As the shadow reaches from point D to C towards the direction of the tree, the angle of elevation *increases* from 30° to 60°.

13. (*a*) *Explanation.*

The angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level is called **angle of elevation.**

14. (*a*) *Explanation.*

We know, θ is the angle of elevation.

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

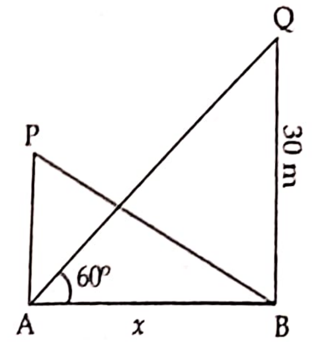
Hence, $\tan\theta = \frac{6}{2\sqrt{3}}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60° \quad \therefore \theta = 60°$

15. (*a*) *Explanation.*

As per the given question the following diagram will be formed. PA being the length of the first building and QB of the second building, AB is the distance between the two buildings.



Let the distance be *x*.

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

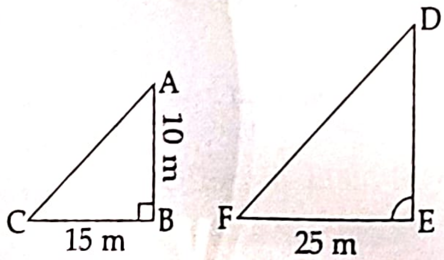
Hence, $\tan 60° = \frac{30}{x}$

$\sqrt{3} = \frac{30}{x}$

$\Rightarrow x = \frac{30}{\sqrt{3}}$

$\therefore$ **$x = 10\sqrt{3}$ m**

16. (*a*)



Let Pole height, AB = 10 m

Pole shadow, BC = 15 m

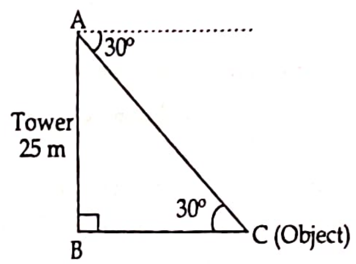
Building shadow, EF = 25 m

Now, $\tan\theta = \frac{DE}{EF}$

$\tan\theta = \frac{10}{15} = \frac{2}{3} \quad \Rightarrow \frac{2}{3} = \frac{DE}{25}$

$\therefore DE = \frac{25 \times 2}{3} = \frac{50}{3}$ **= 16.67 m**

17. (*a*) *Explanation.*



In the right triangle ABC,

We know that,

$\tan\theta$ (angle of elevation)

$$= \frac{\text{height of pole, AB}}{\text{its distance from the point, BC}}$$

Hence, $\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{25}{BC}$

$\therefore BC = 25\sqrt{3}$ m

Therefore, the distance of the object from the base of the tower is $25\sqrt{3}$ m.

Assertion Reason Answers

1. (*b*) *Both (A) and (R) are true and (R) is not the correct explanation of (A)*

   Explanation.

   Then, length of shadow of the pole $= h\ m$

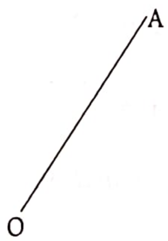
   Now, In ΔABC, $\tan\theta° = \frac{AB}{BC}$

   $\Rightarrow \frac{AB}{BC} = 1$

   $\therefore \theta° = 45$

2. (*b*) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*

   A line drawn from the eye of the observer to the point in the object viewed by the observer is called line of sight.



   Here, OA is line of sight and
   A is the object and O is the eye

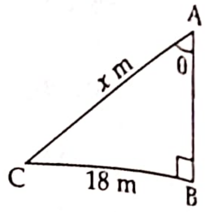
3. (*b*) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*

   The angle of elevation is indeed the angle formed by the line of sight with the horizontal when the object being viewed is above the horizontal level. This angle is measured from the horizontal line upward to the line of sight.

Very Short Answers

1. Let length of shadow $= x$ m

   Then, In ΔABC,



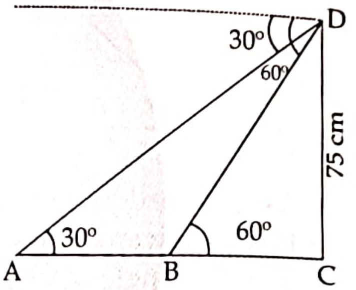
   $\tan\theta = \frac{BC}{AB}$

   $\Rightarrow \frac{6}{7} = \frac{18}{x}$

   $\Rightarrow x = \frac{(18 \times 7)}{6}$

   $\Rightarrow x = 3 \times 7 = 21$

   $\therefore$ **$x = 21$ m**

2. 

   In ΔBCD,

   $\Rightarrow \tan(\theta) = \frac{BC}{CD}$

   $\Rightarrow \tan(30°) = \frac{BC}{CD} \quad \Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{75}$

   $\Rightarrow BC = \frac{75}{\sqrt{3}}$ m ...(i)

   In ΔACD,

   $\Rightarrow \tan(\theta) = \frac{CD}{AC}$

   $\Rightarrow \tan(60°) = \frac{75}{AC} \quad \Rightarrow \sqrt{3} = \frac{75}{AC}$

   $\Rightarrow AC = 75\sqrt{3}$ m ...(ii)

   The distance between the cars is AB.

   $\therefore AB = AC - BC$

   $\Rightarrow AB = 75\sqrt{3} - \frac{75}{\sqrt{3}}$ ...[From (*i*) & (*ii*)]

   $\Rightarrow AB = 75\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$\Rightarrow AB = 75\left(\frac{2}{\sqrt{3}}\right)$

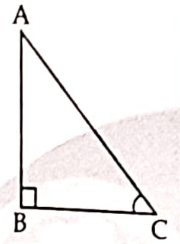
$\Rightarrow AB = 75\left(\frac{2\sqrt{3}}{3}\right)$

$\therefore$ **AB = $50\sqrt{3}$ m**

3. Let the height of tower, AB = h m

The length of shadow, BC = x m

According to given question,



$\Rightarrow \frac{AB}{BC} = \frac{\sqrt{3}}{1}$

$\Rightarrow \frac{h}{x} = \frac{\sqrt{3}}{1}$

We know that,

In ΔABC,

$\Rightarrow \frac{AB}{BC} = \tan\theta$

$\Rightarrow \tan\theta = \frac{h}{x}$

$\Rightarrow \tan\theta = \frac{\sqrt{3}}{1}$

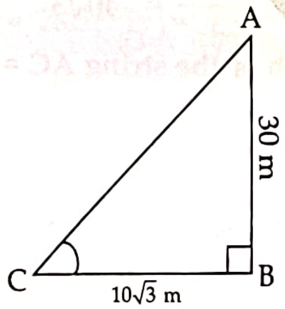
$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60°$

$\therefore \theta = 60°$

Hence, the angle of elevation is 60°.

4.



In ΔABC,

Let AB be the tower and BC be its shadow.

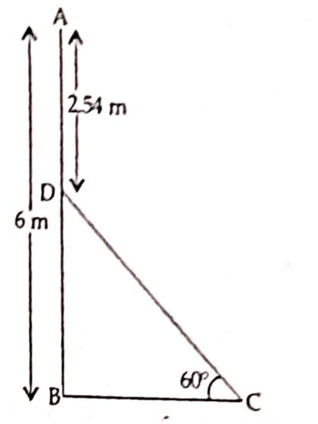
$\therefore \tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60°$

$\Rightarrow$ **θ = 60°**

5.



In ΔBCD, $\sin 60° = \frac{3.46}{CD}$

$\Rightarrow CD = \frac{3.46}{\sin 60°}$

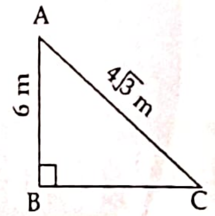
$\Rightarrow CD = 3.46 \times \frac{2}{\sqrt{3}}$

$\Rightarrow CD = 3.46 \times \frac{2}{1.73}$

$\Rightarrow CD = 2 \times 2 = 4$

$\therefore$ **Length of ladder = 4 m**

6. **Using sine ratio**



$\sin d = \frac{AB}{AC} = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

...*where* [*d* is the angle of depression

$\Rightarrow \sin d = \sin 60°$ $\qquad \therefore \angle d = 60°$

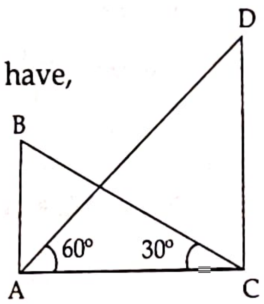
Short Answers

1. Let AB be the tower and CD be the hill.

Then, $\angle ACB = 30°$, $\angle CAD = 60°$ and AB = 50 m.

Let CD = x m

In right ΔBAC, we have,



$\cot 30° = \frac{AC}{AB}$

$\Rightarrow \sqrt{3} = \frac{AC}{50}$

$\Rightarrow AC = 50\sqrt{3}$ m

In right ΔACD, we have,

$$\tan 60^\circ = \frac{CD}{AC}$$

$$\Rightarrow \sqrt{3} = \frac{x}{50\sqrt{3}} \qquad \therefore x = 50 \times 3 = 150 \text{ m}$$

Therefore, the height of the hill is 150 m.

2. (*i*) In ΔAOB,

$$\Rightarrow \tan\theta = \frac{AB}{OA}$$

$$\Rightarrow \frac{AB}{4} = \tan 45^\circ \quad \Rightarrow AB = 4$$

$\therefore$ **Height of the object AB = 4 cm.**

(*ii*) Let $\angle B'OA'$ be θ

Now, In $\Delta B'OA'$,

$$\tan\theta = \frac{A'B'}{A'O} = \frac{4}{3} \qquad \text{...[}A'B' = AB = 4 \text{ cm]}$$

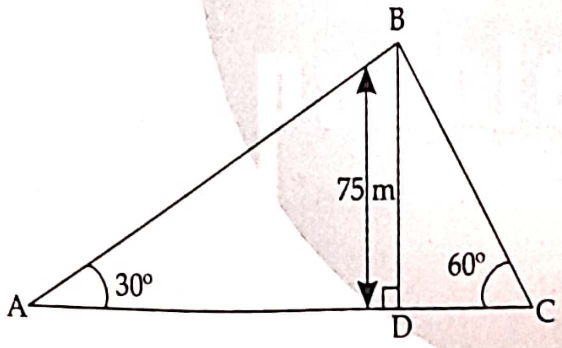
As we know alternate angles are equal

$\angle C'OD' = \theta$

$$\therefore \tan\theta = \frac{4}{3} = \frac{C'D'}{6}$$

$$\Rightarrow C'D' = \frac{24}{3} = 8 \text{ cm}$$

3.



In ΔABD,

$$\tan 30^\circ = \frac{BD}{AD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{AD} \qquad \therefore AD = 75\sqrt{3} \text{ m}$$

In ΔBCD,

$$\tan 60^\circ = \frac{BD}{CD} \qquad \Rightarrow \sqrt{3} = \frac{75}{CD}$$

$$\therefore CD = \frac{75}{\sqrt{3}}$$

Now, $AC = AD + CD$

$$= \frac{75\sqrt{3}}{1} + \frac{75}{\sqrt{3}}$$

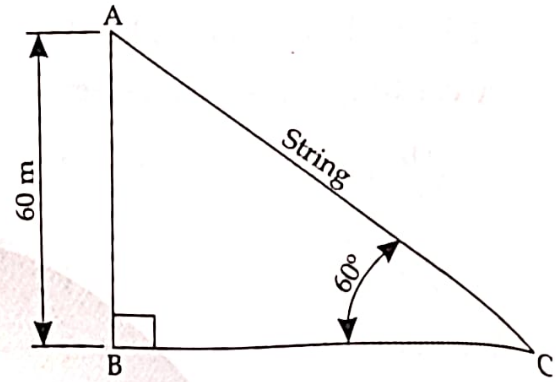
$$= \frac{75 \times 3 + 75}{\sqrt{3}} = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{300\sqrt{3}}{3}$$

$$= 100\sqrt{3} = 100 \times 1.73$$

= 173 m

4.



In ΔABC,

$$\Rightarrow \sin C = \frac{AB}{AC}$$

$$\Rightarrow \sin 60^\circ = \frac{60}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

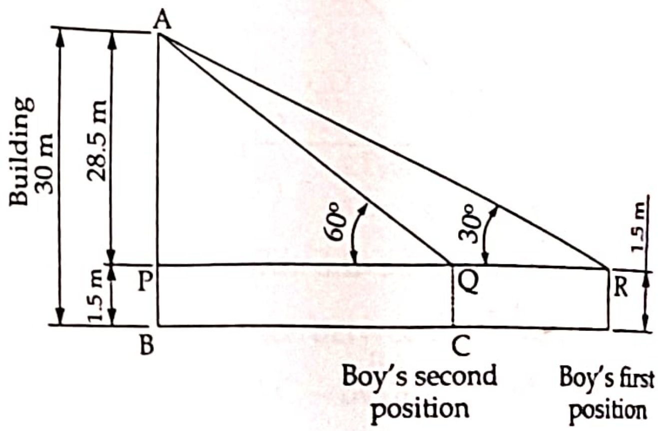
$$\Rightarrow AC = \left(60 \times \frac{2}{\sqrt{3}}\right)$$

$$= \left(\frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}\right)$$

$$= \frac{120\sqrt{3}}{3} = 40\sqrt{3}$$

$\therefore$ **Length of the string AC = $40\sqrt{3}$ m.**

5.



Distance walked towards the building

RQ = PR - PQ

Trigonometric ratio involving AP, PR and $\angle R$ and AP, PQ and $\angle Q$ is $\tan\theta$ ...[Refer the diagram to visualise AP, PR and PQ

In ΔAPR,

$$\tan R = \frac{AP}{PR}$$

$$\tan 30^\circ = \frac{28.5}{PR}$$

$$\frac{1}{\sqrt{3}} = \frac{28.5}{PR}$$

$$PR = 28.5 \times \sqrt{3}\text{ m}$$

In ΔAPQ,

$$\tan Q = \frac{AP}{PQ}$$

$$\tan 60^\circ = \frac{28.5}{PQ}$$

$$\sqrt{3} = \frac{28.5}{PQ}$$

$$PQ = \frac{28.5}{\sqrt{3}}$$

Therefore,

$$PR - PQ = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$$

$$= 28.5\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$$

$$= 28.5\left(\frac{(3-1)}{\sqrt{3}}\right)$$

$$= 28.5\left(\frac{2}{\sqrt{3}}\right) = \frac{57}{\sqrt{3}}$$

$$= \frac{57 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

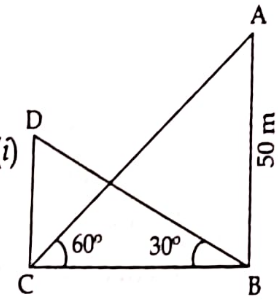
$$= \frac{57\sqrt{3}}{3}$$

$$= 19\sqrt{3}\text{ m}$$

The distance walked by the boy towards the building is $19\sqrt{3}$ m.

6. Let the height of the tower be AB and the height of the building be CD.
Distance between the foot of the tower and the building is BC.
Trigonometric ratio involving sides AB, CD, BC and angles $\angle B$ and $\angle C$ is $\tan\theta$.

In ΔABC,



$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{50}{BC}$$

$$BC = \frac{50}{\sqrt{3}} \quad ....(i)$$

In ΔBCD,

$$\tan 30^\circ = \frac{CD}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{BC}$$

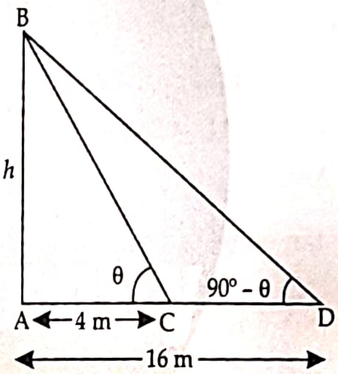
$$\Rightarrow CD = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}} = \frac{50}{3} \quad ...\text{[From (i)}$$

$$\Rightarrow CD = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}}$$

$$\therefore CD = \frac{50}{3}$$

$\therefore$ **Height of the building CD = $\frac{50}{3}$ m.**

7.



Based on the given information, we can draw the diagram shown above.

Given, $\angle C$ and $\angle D$ are complementary.

Hence, if $\angle C = \theta$ then,

$$\angle D = 90^\circ - \theta$$

We know that, $\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$

Hence, in ΔABC and ΔABD,

$$\tan\theta = \frac{h}{4} \quad ...(1)$$

$$\text{and } \tan(90-\theta) = \frac{h}{16}$$

$$\Rightarrow \cot\theta = \frac{h}{16} \quad ...[\because \tan(90-\theta) = \cot\theta$$

$$\Rightarrow \frac{1}{\cot\theta} = \frac{16}{h}$$

$$\Rightarrow \tan\theta = \frac{16}{h} \quad ....(2)$$

Solving (1) and (2), we get,

$\Rightarrow \frac{h}{4} = \frac{16}{h}$

$\Rightarrow h^2 = 64$

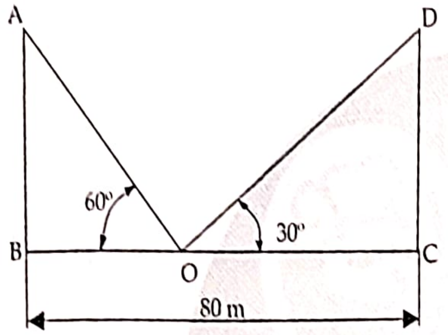
$\therefore h = \pm 8$

But, h cannot be negative.

Hence, h = 8 m

Hence, the height of the tower is 8 m.

Long Answers

1. 

Let the height of the poles be x.

Therefore AB = DC = x

In ΔAOB,

$\therefore \tan 60° = \frac{AB}{BO}$

$\Rightarrow \sqrt{3} = \frac{x}{BO}$

$\Rightarrow BO = \frac{x}{\sqrt{3}}$ ...(i)

In ΔOCD,

$\therefore \tan 30° = \frac{DC}{OC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{(BC - OB)}$

$\Rightarrow \frac{1}{\sqrt{3}} = \left(\frac{x}{80 - \frac{x}{\sqrt{3}}}\right)$ ...[From (i)

$\Rightarrow 80 - \frac{x}{\sqrt{3}} = \sqrt{3}x$

$\Rightarrow \frac{x}{\sqrt{3}} + \sqrt{3}x = 80$

$\Rightarrow x\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) = 80$

$\Rightarrow x\left(\frac{(1+3)}{\sqrt{3}}\right) = 80$

$\Rightarrow x\left(\frac{4}{\sqrt{3}}\right) = 80$

$\Rightarrow x = \frac{80\sqrt{3}}{4}$

$\therefore x = 20\sqrt{3}$

Height of the poles $x = 20\sqrt{3}$ m.

Distance of the point O from the pole AB,

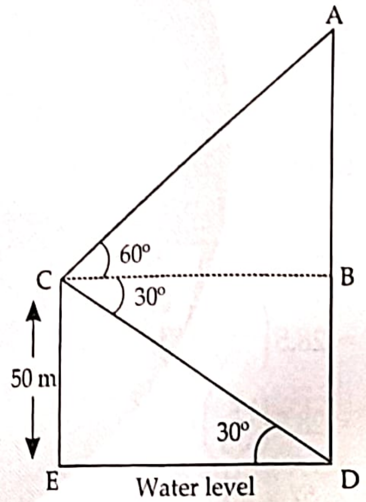
$BO = \frac{x}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20$

Distance of the point O from the pole CD,

$OC = BC - BO$

$= 80 - 20 = 60$

The height of the poles is $20\sqrt{3}$ m and the distance of the point from the poles is 20 m and 60 m respectively.

2. 

Now, in right angled ΔCED,

We have,

$\tan 30° = \frac{50}{ED}$

$ED = \frac{50}{\tan 30°}$

$ED = \left(\frac{50}{\frac{1}{\sqrt{3}}}\right) = 50\sqrt{3}$

Also, in right angled ΔABC,

We have, $\tan 60° = \frac{AB}{BC}$ or $\frac{AB}{ED}$

...[From Fig. BC = ED

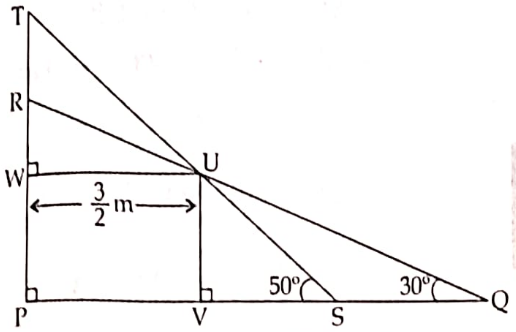
$\sqrt{3} = \frac{AB}{50\sqrt{3}} \quad \Rightarrow AB = 50\sqrt{3}\sqrt{3}$

$\therefore$ AB = 150 m

Now, AD = AB + BD = 150 + 50 = 200 m

Therefore, the distance of the hill from the ship is $50\sqrt{3}$ m and the height of the hill is 200 m.

3.



Given. $\angle WUT = 50°$ and $\angle WUR = 30°$.

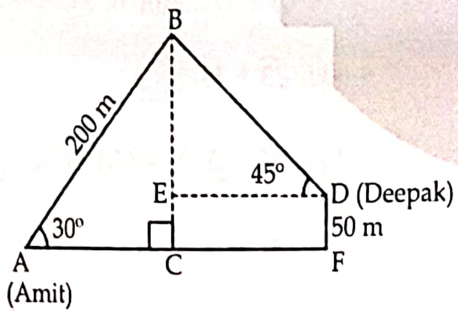
$WU = \frac{3}{2}$ m

In ΔUVS, $WT = \frac{3}{2} \tan 50° = \frac{3}{2} \times \frac{0.8}{0.6} = 2$ m

In ΔWRU, $WR = \frac{3}{2} \tan 30° = \frac{3}{2} \times \frac{0.5}{0.9} = \frac{5}{6}$

So, the height by which shot 2 is higher than shot 1 $= 2 - \frac{5}{6} = \frac{7}{6}$ **m**

4.



From the horizontal plane 'A' be Amit at a distance of AB = 200 m from bird. From the roof DF = 50 m. Deepak is standing at a distance of BD = prime x m from the same bird. $\angle BAC = 30°$ and $\angle BDE = 45°$

In ΔACB, $\sin 30° = \frac{BC}{AB}$

$\frac{1}{2} = \frac{(h + 50)}{200}$

$\Rightarrow \frac{200}{2} = h + 50$

$\Rightarrow h = 100 - 50$

$\therefore h = 50$ m ...(*i*)

In ΔBED, $\sin 45° = \frac{(BE)}{(BD)}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{h}{x}$

$\Rightarrow x = h\sqrt{2}$

$\therefore x = 50\sqrt{2}$ m ...[From (*i*)

Hence, the distance of the bird from Deepak is $= 50\sqrt{2}$ m.

Case Based Answers

1. (*a*) In ΔAFG, $AF = \frac{AG}{\cos 30°}$

$= \frac{\frac{1973}{2}}{\frac{\sqrt{3}}{2}} = \frac{1973}{\sqrt{3}}$ **= 1139.15 km**

(*b*) (*i*) In ΔFPH,

$\cos 60° = \frac{PH}{PF}$

$PF = \frac{PH}{\cos 60°}$

$= \frac{\left(\frac{1973}{2}\right)}{\left(\frac{1}{2}\right)}$ **= 1973 km**

Or

(*ii*) In ΔFAG,

$\tan 30° = \frac{FG}{AG}$

$\Rightarrow FG = \frac{1973}{2\sqrt{3}}$

FI = GF + GI

= 569.6 km + 7.816 km

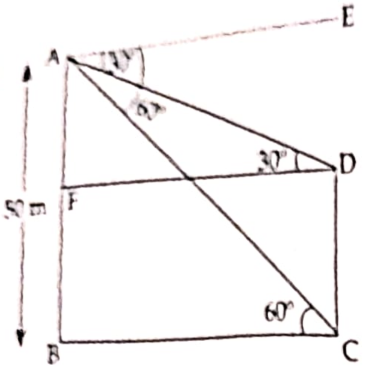
= 577.42 km

(*c*) In ΔACD, $\tan\theta = \frac{P}{B} = \frac{7816}{7816} = 1$

$\Rightarrow \tan\theta = \tan 45°$

$\therefore$ **$\theta = 45°$**

2.



(*a*) In ΔABC,

$\therefore \tan 60° = \dfrac{AB}{BC} \Rightarrow \sqrt{3} = \dfrac{50}{BC}$

$\Rightarrow BC = \dfrac{50}{\sqrt{3}} = 28.90$ m

(*b*) (*i*) Since, $AE \parallel FD$

$\angle EAD = \angle ADF = 30°$

—[Alternate interior angles

Or

(*ii*) Since, $AE \parallel BC$

$\therefore \angle EAC = \angle ACB = 60°$

(*c*) In ΔADF, $\tan 30° = \dfrac{AF}{FD}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{AB - BF}{FD}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{50 - CD}{\left(\dfrac{50}{\sqrt{3}}\right)}$

$\Rightarrow \dfrac{50}{3} = 50 - CD \quad \Rightarrow CD = 50 - \dfrac{50}{3}$

$\therefore CD = \dfrac{150 - 50}{3} = \dfrac{100}{3}$

$= 33.33$ m

3. (*a*) As we know, $\tan 45° = \dfrac{80}{CB}$

$\therefore CB = 80$ m

(*b*) (*i*) $\tan 30° = \dfrac{80}{CE}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{80}{CE} \quad \Rightarrow CE = 80\sqrt{3}$

Distance the bird flew

$= AD = BE = CE - CB$

$= 80\sqrt{3} - 80 = 80(\sqrt{3} - 1)$ m

Or

(*ii*) As we know, $\tan 60° = \dfrac{80}{CG}$

$\Rightarrow \sqrt{3} = \dfrac{80}{CG} \quad \Rightarrow CG = \dfrac{80}{\sqrt{3}}$

Distance the ball travelled after hitting the tree

$= FA = GB = CB - CG$

$\therefore GB = 80 - \dfrac{80}{\sqrt{3}} = 80\left(1 - \dfrac{1}{\sqrt{3}}\right)$ m

(*c*) Speed of the bird

$= \dfrac{\text{distance}}{\text{time taken}} = \dfrac{20(\sqrt{3}+1)\text{m}}{2\,\text{sec}}$

$= \dfrac{20(\sqrt{3}+1)}{2} \times \dfrac{60\,\text{m}}{\text{min}}$

$= 600(\sqrt{3}+1)\ \dfrac{\text{m}}{\text{min}}$

(DAY 22 SWAHA)

*****

10 Coordinate Geometry

What did CBSE ask last year?

MCQs	2 Questions (2 × 1 = 2 Marks)
Subjective	No Very Short Question
	No Short Question
	1 Long Question (1 × 5 = 5 Marks)
Case Based	1 Question (1 + 1 + 2 = 4 Marks)

Note: All the above typology of questions include 'Competency based Questions' labelled as **COMPETENCY**.

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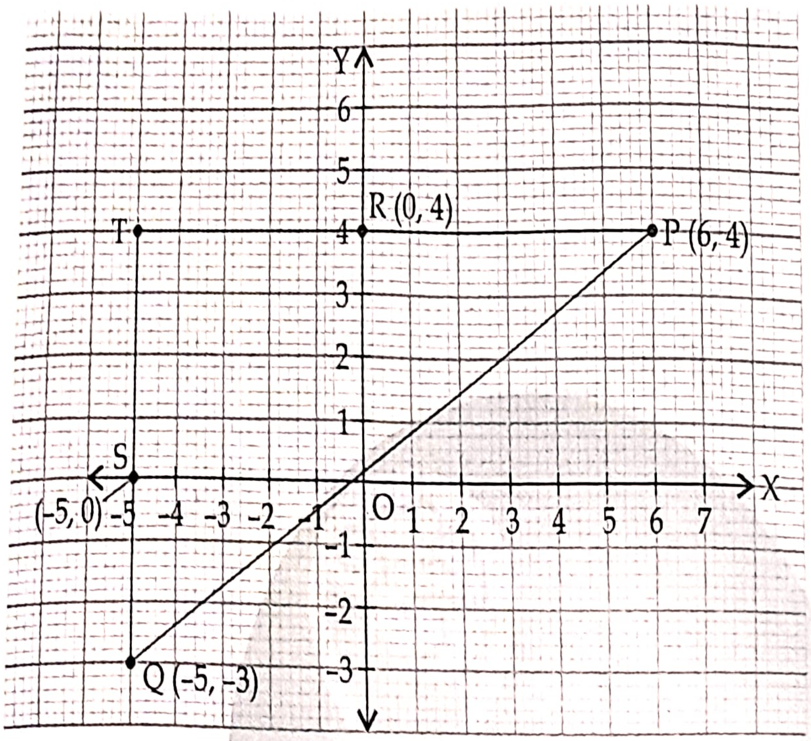
Introduction to the chapter

❑ Graph of linear equations

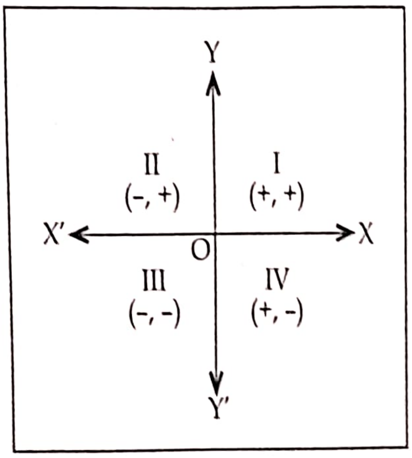


Mid point: $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

Centroid of a triangle: $\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}$

> ***Note:*** *Graph based and formula based questions come from this topic very frequently.*

❑ *Signs of abscissa and ordinate in different quadrants are as given in the diagram:*



- Any point on the X-axis is of the form (x, 0).
- Any point on the Y-axis is of the form (0, y).
- The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

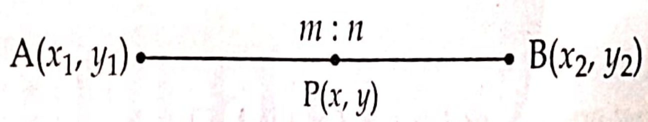
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note: *If O is the origin, the distance of a point P(x, y) from the origin O(0, 0) is given by*

$$OP = \sqrt{x^2 + y^2}$$

Section formula:

The coordinates of the point which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are:



$$P(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

The above formula is section formula. The ratio $m : n$ can also be written as $\frac{m}{n} : 1$ or $k : 1$, The co-ordinates of P can also be written as $P(x, y) = \frac{kx_2 + x_1}{k + 1}, \frac{ky_2 + y_1}{k + 1}$

Note: *Short, long and case based questions come very often from this topic.*

OBJECTIVE QUESTIONS

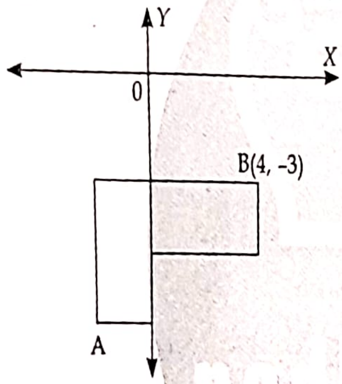
(DAY 23)

Multiple Choice Questions

Q.1. The distance of the point (-6, 8) from x-axis is *[CBSE 2023]*

(*a*) 6 units
(*b*) -6 units
(*c*) 8 units
(*d*) 10 units

Q.2. Shown below are 2 identical rectangles such that their breadth is half their length. *[CBSE 2024]*



What are the coordinates of point A?

(*a*) (4, –5) (*b*) (–4, –6)
(*c*) (–2, –7) (*d*) (–2, –9)

Q.3. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and (2, y), then the value of y is **COMPETENCY**

(*a*) 3 (*b*) –3
(*c*) 7 (*d*) 4

FREE ADVICE: *You can use the formula for the midpoint of a line segment.*

Q.4. AOBC is a rectangle whose three vertices are vertices A(0, 3), O(0, 0) and B(5, 0). The length of its diagonal is *[NCERT Exemplar]*

(*a*) 5 (*b*) 3
(*c*) 4 (*d*) $\sqrt{34}$

Q.5. The base BC of an equilateral ΔABC lies on the y-axis. The co-ordinates of C are (0, -3). If the origin is the mid point of the base BC. What are co-ordinates of A and B? *[CBSE 2022]*

(*a*) $A(3\sqrt{3}, 0)$, B(0, 3)

(*b*) $A(3\sqrt{3}, 0)$, B(3, 0)

(*c*) $A(3\sqrt{3}, 0)$, B(0, –3)

(*d*) $A(-3\sqrt{3}, 0)$, B(3, 0)

FREE ADVICE: *The y-coordinate of point B is the same as the y-coordinate of point C because BC is parallel to the y-axis. So, point B is (0, 3).*

Q.6. If A(4, –2), B(7, –2) and C(7, 9) are the vertices's of a ΔABC, then ΔABC is: **COMPETENCY**

(*a*) equilateral triangle
(*b*) isosceles triangle
(*c*) right angled triangle
(*d*) isosceles right angled triangle

Q.7. If A(3, $\sqrt{3}$), B(0, 0) and C(3, k) are the three vertices of an equilateral triangle ABC, then the value of k is: *[CBSE 2022]*

(*a*) 2 (*b*) –3
(*c*) $\pm\sqrt{3}$ (*d*) $-\sqrt{2}$

Q.8. What is the distance between the points (–1, 3) and (2, –5)?

(*a*) $\sqrt{5}$ (*b*) $\sqrt{55}$
(*c*) $\sqrt{65}$ (*d*) $\sqrt{73}$

Q.9. Three vertices of a parallelogram ABCD are A(1, 4) B(–2, 3) and C(5, 8). The ordinate of the fourth vertex D is: *[CBSE 2024]*

(*a*) 8 (*b*) 9
(*c*) 7 (*d*) 6

FREE ADVICE: *Diagonals of a parallelogram bisect each other. Ordinate of a point is y coordinate of that point Mid point of (a , b) and (c , d) can be calculated as (a + c)/2 , (b + d)/2]*

Q.10. The point of intersection of the line represented by $3x - y = 3$ and y-axis is given by [CBSE 2023

(*a*) (0, –3) (*b*) (0, 3)
(*c*) (2, 0) (*d*) (–2, 0)

FREE ADVICE: *To find the point of intersection of the line represented by the equation $3x - y = 3$ with the y-axis, you need to substitute $x = 0$ into the equation.*

Q.11. The coordinates of the point where the line $2y = 4x + 5$ crosses x-axis is [CBSE 2023

(*a*) $\left(0, -\frac{5}{4}\right)$ (*b*) $\left(0, \frac{5}{2}\right)$

(*c*) $\left(-\frac{5}{4}, 0\right)$ (*d*) $\left(-\frac{5}{2}, 0\right)$

FREE ADVICE: *This point will have a y-coordinate of 0 since it lies on the x-axis.*

Q.12. Point A(–1, y) and B(5, 7) lie on a circle with centre O(2, –3y). The values of y are: [CBSE 2022

(*a*) 1, –7 (*b*) –1, 7
(*c*) 2, 7 (*d*) –2, –7

Q.13. The ratio in which the line $3x + y - 9 = 0$ divides the line segment joining the points (1, 3) and (2, 7) is: [CBSE 2022

(*a*) 3 : 2 (*b*) 2 : 3
(*c*) 3 : 4 (*d*) 4 : 3

FREE ADVICE: *In this case, you want to divide the line segment in a given ratio. Let's say the ratio is m : n, and solve further.*

Q.14. If (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4) and $a - 2b = 18$ the value of k is:

(*a*) 30 (*b*) 22
(*c*) 4 (*d*) 40

Q.15. If the point P(k, 0) divides the line segment joining the points A(2, –2) and B(–7, 4) in the ratio 1 : 2, then the value of k is **COMPETENCY**

(*a*) 1 (*b*) 2
(*c*) –2 (*d*) –1

FREE ADVICE: *The value of k when point P(k, 0) divides the line segment joining points A(2, –2) and B(–7, 4) in the ratio 1 : 2, you can use the section formula.*

Q.16. P(1, 7), Q(–3, 2) and R(6, 1) are the coordinates of the vertices of a triangle. Which of the following types of triangle is ΔPQR?

(*a*) Scalene triangle
(*b*) Equilateral triangle
(*c*) Isosceles right-angled triangle
(*d*) Isosceles acute-angled triangle

Q.17. A line segment is of length 10 units. If the coordinates of its one end are (2, –3) and the abscissa of the other end is 10, then its ordinate is

(*a*) 9, 6 (*b*) 3, –9
(*c*) –3, 9 (*d*) 9, –6

Q.18. If P(a/3, 4) is the mid-point of the segment joining the points Q(–6, 5) and R(–2, 3), then the value of 'a' is **COMPETENCY**

(*a*) 12 (*b*) –6
(*c*) –12 (*d*) –4

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*
(*b*) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*
(*c*) *(A) is true and (R) is false.*
(*d*) *(A) is false, but (R) is true.*

Q.1. **Assertion:** The value of y is 6, for which the distance between the points, P(2, –3) and Q(10, y) is 10.

Reason: Distance between two given points A(x_1, y_1) and B(x_2, y_2)

$$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

Reason: The x-coordinate ... on y-axis is zero. COMPETENCY

Q.3. **Assertion:** The points A (–1, 0), B (3, 1), C (2, 2) and D (–2, 1) are the vertices of a parallelogram.

Reason: The coordinates of the mid-points of both the diagonals AC and BD are $\frac{1}{2}$, 1.

...ate.

Reason: The distance of a point from the y-axis is called its x-coordinate. COMPETENCY

Q.5. **Assertion:** Mid-point of a line segment divides line in the ratio 1 : 1.

Reason: The ratio in which the point (–3, k) divides the line segment joining the points (–5, 4) and (–2, 3) is 1 : 2. COMPETENCY

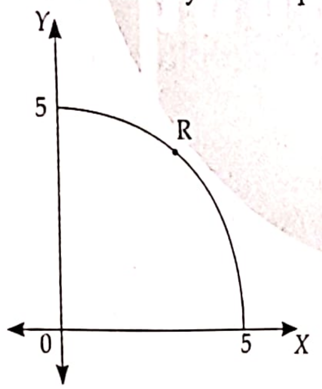
SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. Find the points on the x-axis, each of which is at a distance of 10 units from the point A(11, –8). *[CBSE 2023*

Q.2. If the point (0, 2) is equidistant from the points (3, k) and (k, 5) then find the value of k. COMPETENCY

Q.3. Shown below is a quarter of a circle with centre at (0, 0). An arbitrary point R lies on the boundary of the quadrant.



Write one possible pair of coordinates of point R.

Q.4. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by X-axis? Also find the coordinates of this point on X-axis. COMPETENCY

Q.5. A circle has its centre at the origin. The radius of the circle is 5 units. Does the point (3, –5) lie inside the circle, on its circumference or outside the circle?

Show your work.

Q.6. The mid point of the line segment joining A (2a, 4) and B (–2, 3b) is (1, 2a + 1). Find the values of a and b. COMPETENCY

Q.7. Find the ratio in which (4, m) divides the line segment joining the points A (2, 3) and B (6, –3). Hence find m. *[CBSE 2018*

Q.8. A line intersects the Y-axis and X-axis at the points P and Q respectively. If (2,-5) is the mid-point of PQ, then find the coordinates of P and Q. *[CBSE 2017*

(DAY 24)

Short Answer Questions

Q.1. Show that the points (1, 7),(4, 2),(–1, –1) and (–4, 4) are the vertices of a square.

Q.2. Show that the points A (1, –2), B (3, 6), C(5, 10) and D(3, 2) are the vertices of a parallelogram. COMPETENCY

Q.3. If the point C(–1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B. *[CBSE 2020*

Q.4. The line segment joining the points A (2, 1) and B (5, –8) is trisected at the points P and Q such that is nearer to A. If P also lies on the line given by $2x - y + k = 0$ find the value of k.

COMPETENCY

> *FREE ADVICE: Given line is cutting the line AB at point P, where it trisects, i.e., the ratio is 1 : 2.*

Q.5. Find the ratio in which the x-axis divides the line segment joining the points A(4, 9) and B(3, –5). Show your work.

Q.6. If P $(9a - 2, -b)$ divides line segments joining A $(3a + 1, -3)$ and B $(8a, 5)$ in the ratio 3 : 1, find the values of a and b.

COMPETENCY

Q.7. Points P, Q, R and S divide the line segment joining the points A(1, 2) and B(6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

COMPETENCY

Long Answer Questions

Q.1. If (a, b) is the mid-point of the line segment joining the points A (10, –6) and B $(k, 4)$ and $a - 2b = 18$, find the value of k and the distance AB.

[NCERT EXEMPLAR]

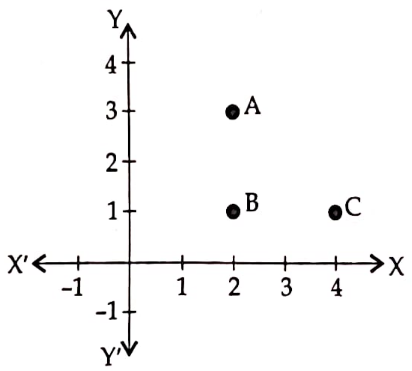
Q.2. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (–1, 4) and (–2, –1) taken in order. *[CBSE 2024]*

> *FREE ADVICE:*
>
> Area of a rhombus $= \frac{1}{2}$
>
> $\times$ *(product of its diagonals)*

Q.3. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points (–2, –5) and (6, 3). Find the coordinates of the point of intersection. **COMPETENCY**

CASE BASED QUESTIONS

Q.1. Akanksha and Aditi are friends living on the same street in Patel Nagar. Aditi's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Aditi's house. Suppose the school is situated at the point O, *i.e.*, the origin, Akanksha's house is at A. Aditi's house is at B and library is at C.



Based on the above information, answer the following questions:

(*a*) How far is Aditi's house from Akanksha's house?

(*b*) (*i*) How far is the library from Akanksha's house?

Or

(*ii*) How far is the library from Aditi's house? **COMPETENCY**

(*c*) How far is the school from Aditi's house than Akanksha's house?

COMPETENCY

Q.2. In an examination hall, students are seated at a distance of 2 m from each other, to maintain the social distance due to CORONA virus pandemic. Let three students sit at points A, B and C whose coordinates are (4, –3), (7, 3) and (8, 5) respectively.

Based on the above information, answer the following questions:

(a) What is the distance between A and C?

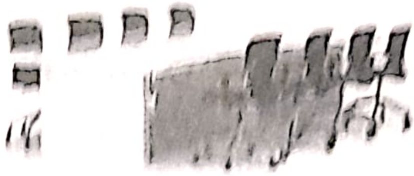
(b) (i) If an invigilator at the point L, lying on the straight line joining B and C such that it divides the distance between them in the ratio of 1 : 2. Find the coordinates of L.

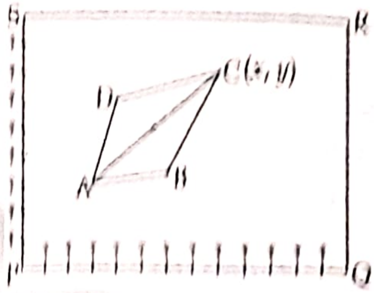
Or

(ii) Find the mid-point of the line segment joining A and C.

(c) Find the ratio in which B divides the line segment joining A and C.

Q.3 An award function of a Multi National Company (MNC) is arranged in a rectangular shaped meeting room PQRS. 30 flower vases placed along two sides of a room, at a distance of 1 m from each other. The employees which are nominated for the position of 'Best Employee of the year' are seated at points A, B, C and D.





If P is considered as origin, then answer the following questions:

(a) What are the coordinates of A?

(b) (i) What is the mid-point of the line segment joining A and C.

Or

(ii) Which coordinate is near to A?

(c) Which point is equidistant from B and D?

[CBSE 2024]

* * * * *

ANSWERS

Multiple Choice Answers

1. (c) Here, $x_1, y_1 = (-6, 0)$; $x_2, y_2 = (-6, 8)$

Distance formula

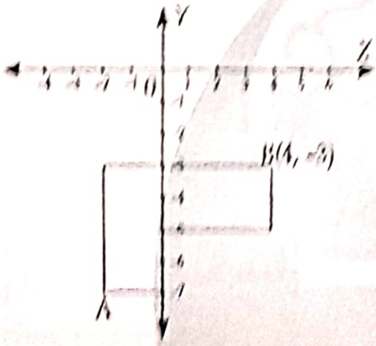
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance from x-axis

$$= \sqrt{[-6 - (-6)]^2 + (8 - 0)^2}$$

$$= \sqrt{[(0)^2 + (8)^2]} = \sqrt{64} = 8 \text{ units}$$

2. (c) Let the coordinate of the given points are A and B



Hence, the coordinates of A = (-2, -7)

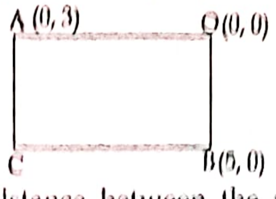
3. (a) Using mid-point formula,

$$\Rightarrow (3, 5) = \left(\frac{(4+2)}{2}, \frac{(7+y)}{2}\right)$$

$$\Rightarrow 5 = \frac{7+y}{2}$$

$$\Rightarrow 10 = 7 + y \qquad \therefore\ y = 3$$

4. (d) A(0, 3) and B(5, 0)



$\because$ Distance between the points(x_1, y_1) and (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

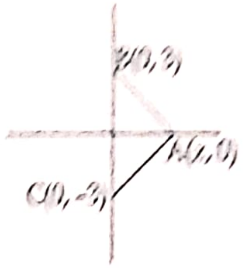
Here, $x_1 = 0$, $y_1 = 3$ and $x_2 = 5$, $y_2 = 0$

$\therefore$ Distance between points (0, 3) and (5, 0).

$$\therefore\ AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{25+9} = \sqrt{34} \text{ units}$$

5. (a) Let the coordinates of point A be (x, 0).



Using distance formula,

$$AB = \sqrt{(0-x)^2 + (3-0)^2} = x^2 + 9$$

$$BC = \sqrt{(0-0)^2 + (-3-3)^2} = 36$$

Also, AB = BC

$\therefore\ \sqrt{x^2+9} = \sqrt{36} \qquad \Rightarrow x^2 + 9 = 36$

$\Rightarrow x^2 = 27 \qquad \Rightarrow x = \pm 3\sqrt{3}$

$\therefore$ **Coordinates of A = $(3\sqrt{3}, 0)$**

Coordinates of B(0, 3)

6. (c) Vertices A(4, –2), B(7, –2) and C(7, 9).

$$AB^2 = (7-4)^2 + (-2+2)^2 = 9$$

$$BC^2 = (7-7)^2 + (9+2)^2 = 121$$

$$AC^2 = (7-4)^2 + (-2-9)^2$$

$$\Rightarrow AC^2 = 9 + 121 = 130$$

$$\Rightarrow AC^2 = AB^2 + BC^2$$

$$130 = 9 + 121$$

It is a right angled triangle.

7. (c) Given. AB = BC

$$\therefore\ \sqrt{(0-3)^2 + (0-\sqrt{3})^2} = \sqrt{(3-0)^2 + (k+0)^2}$$

$$\Rightarrow \sqrt{9+3} = \sqrt{9+k^2}$$

$$\Rightarrow \sqrt{12} = \sqrt{9+k^2}$$

$$\Rightarrow k^2 = 3 \qquad \therefore\ k = \pm\sqrt{3}$$

8. (d) Distance of point AB are (–1, 3) and (2, –5)

$$\therefore\ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

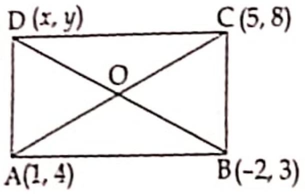
$$= \sqrt{[2-(-1)]^2 + (-5-3)^2}$$

$$= \sqrt{(2+1)^2 + (-8)^2} = \sqrt{(3)^2 + 64}$$

$$= \sqrt{9+64} = \sqrt{73} \text{ units}$$

9. (b) Coordinates of mid-point O,

$\frac{1+5}{2}, \frac{8+4}{2} = 3, 6$



Ordinate of D

$\frac{y+3}{2} = 6$

$\Rightarrow y + 3 = 12$

$\therefore y = 12 - 3 = 9$

10. (a) $3x - y = 3$ ...(i)

For y-axis, $x = 0$

So, $3 \times (0) - y = 3 \quad \Rightarrow y = -3$

Now, put $y = -3$ in equation (i)

$3x - (-3) = 3 \quad \Rightarrow x3x + 3 = 3$

$3x = 0 \quad \therefore x = 0$

So, intersection point is (0, –3).

11. (c) Let point of intersection at x-axis be $(x, 0)$. Thus, value of point y is 0 and Substitute $y = 0$ in equation,

$2y = 4x + 5$

$\Rightarrow 2(0) = 4x + 5 \quad \Rightarrow 4x + 5 = 0$

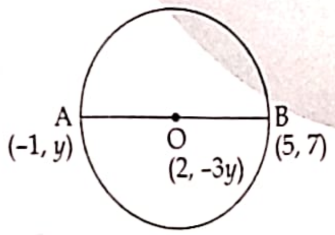
$\Rightarrow 4x = -5 \quad \therefore x = \frac{-5}{4}$

Thus, coordinates of intersection point is $\left(\frac{-5}{4}, 0\right)$.

12. (b) Points A(-1, y) and B(5, 7) lie on a circle with centre O(2, –3y).

Which means OA = OB.

...[Radii of the same cirlce



or $OA^2 = OB^2$

O is mid point of AB. So,

Mid point $(2, -3y) = \left(\frac{5-1}{2}\right), \left(\frac{y+7}{2}\right)$

$(2, -3y) = \left(2, \frac{y+7}{2}\right)$

...[Comparing ordinals

$-3y = \frac{y+7}{2} \quad \Rightarrow -6y = y + 7$

$\Rightarrow -7y = 7$

$\therefore y = -1$

Alternatively,

Using distance formula, we get

$(-1 - 2)^2 + (y - (-3y))^2 = (5 - 2)^2 + (7 - (-3y))^2$

$\Rightarrow 9 + 16y^2 = 9 + (7 + 3y)^2$

$\Rightarrow 16y^2 = 49 + 42y + 9y^2$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow 7(y^2 - 6y - 7) = 0$

$\Rightarrow y^2 - 7y + y - 7 = 0$

$\Rightarrow y(y - 7) + 1(y - 7) = 0$

$\therefore (y + 1)(y - 7) = 0$

Therefore, $y = 7$ or $y = -1$

13. (c) Let the line segment divides the line K : 1.

$(x, y) = \left(\frac{2K+1}{K+1}\right), \left(\frac{7K+3}{K+1}\right)$

On putting the values of x, y in given equation,

$3\left(\frac{2K+1}{K+1}\right) + \left(\frac{7K+3}{K+1}\right) - 9$

$\Rightarrow 6K + 3 + 7K + 3 - 9K - 9 = 0$

$\Rightarrow 4K = 3 \quad \therefore K = \frac{3}{4}$

14. (b) **Given.** $a - 2b = 18$ ...(i)

Here, $(x_1, y_1) = (10, -6)$

and $(x_2, y_2) = (k, 4)$

Mid-point of AB,

$\left[\frac{(10+k)}{2}, \frac{(-6+4)}{2}\right] = (a, b)$

$\left[\frac{(10+k)}{2}, \frac{-2}{2}\right] = (a, b)$

Now, $\frac{(10+k)}{2} = a$

$10 + k = 2a$

$k = 2a - 10$ ...(ii)

Also, $b = \frac{-2}{2} = -1$

Put $b = -1$ in (i),

$a - 2(-1) = 18 \quad \Rightarrow a + 2 = 18$

$a = 18 - 2 = \mathbf{16}$

Put $a = 16$ in (ii),

$k = 2(16) - 10$

$\Rightarrow k = 32 - 10 = 22$

Therefore, the value of k is 22.

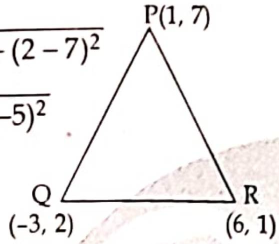
15. (*d*) The section formula states that if a point P(k, 0) divides a line segment AB in the ratio 1 : 2, then the coordinates of point P are given by:

$$k = \left(\frac{1(-7)+2(2)}{1+2}\right) = \frac{-7+4}{3}$$

$$\Rightarrow k = \frac{-3}{3} = -1 \qquad \therefore \mathbf{k = -1}$$

16. (*a*) Scalene triangle

Explanation:



$$PQ = \sqrt{(-3-1)^2+(2-7)^2}$$

$$= \sqrt{(-4)^2+(-5)^2}$$

$$= \sqrt{16+25}$$

$$= \sqrt{41}$$

$$QR = \sqrt{(6+3)^2+(1-2)^2}$$

$$= \sqrt{81+1} = \sqrt{82}$$

$$PR = \sqrt{(6-1)^2+(1-7)^2}$$

$$= \sqrt{25+36} = \sqrt{61}$$

As all sides are unequal so it is scalene triangle.

17. (*b*) Given, one point = (2, –3)

Let, other point = (10, b)

Abscissa is x coordinate.

$$\Rightarrow \sqrt{(10-2)^2+(b+3)^2} = 10$$

$$\Rightarrow \sqrt{8^2+(b^2+9+6b)} = 10$$

$$\Rightarrow 64+b^2+9+6b = 100$$

$$\Rightarrow (b+3)^2 = 36$$

$$\Rightarrow b+3 = 6 \quad \text{or} \quad b+3 = -6$$

$$\therefore \mathbf{b = 3} \quad \text{or} \quad \mathbf{b = -9}$$

18. (*c*) P is mid-point of QR

$$\Rightarrow \frac{a}{3} = \left(\frac{(-6-2)}{2}\right)$$

$$\Rightarrow 2a = -24 \qquad \therefore \mathbf{a = -12}$$

Assertion Reason Answers

1. (*d*) (A) is false, but (R) is true.

Explanation: PQ = 10

$PQ^2 = 100$

$(10-2)^2 + (y+3)^2 = 100$

$(y+3)^2 = 100 - 64 = 36$

$y + 3 = \pm 6$

$y = -3 + 6$ or $y = -3 - 6$

$\therefore \mathbf{y = 3}$ or $\mathbf{y = -9}$

2. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: *The co-ordinates of the point (0, 4) is zero. Point p lies on y-axis.*

3. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: *The distance between two points (x_1, y_1) and (x_2, y_2) is*

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

First let us consider

$$AB = \sqrt{(3+1)^2+(1-0)^2}$$

$$= \sqrt{16+1} = \sqrt{17}$$

$$BC = \sqrt{(2-3)^2+(2-1)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$CD = \sqrt{(-2-2)^2+(1-2)^2}$$

$$= \sqrt{16+1} = \sqrt{17}$$

$$AD = \sqrt{(-2+1)^2+(1-0)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

So, we get

AB = CD and BC = AD

4. (*a*) Both (A) and (R) are true and (R) is not the correct explanation of (A).

Explanation: *The distance of a point from the Y-axis is known as the abscissa and it is the x-coordinate. The distance of a point from the X-axis is known as the ordinate and it is the y-coordinate. So the distance of a point from the Y-axis is called its x-coordinate.*

5. (*c*) (A) is true and (R) is false.

Explanation: *Let (–3, k) divides the line segment joining the points (–5, 4) and (–2, 3) in the ratio m : n.*

$$-3 = \frac{-2m-5n}{m+n}$$

or $-3(m+n) = -2m - 5n$

or $-3m - 3n = -2m - 5n$

or $-3m + 2m = -5n + 3n$

or $-m = -2n$

or $m = 2n$

or $\mathbf{m : n = 2 : 1}$

Very Short Answers

1. Let the points on x-axis be $P(x, 0)$ and $Q(y, 0)$ which are at distance of 10 units from point $A(11, -8)$.

Which implies:

$PA = QA$ or $PA^2 = QA^2$

$(11 - x)^2 + (-8)^2 = (10)^2$

$121 - 22x + x^2 + 64 = 100$

$x^2 - 22x + 85 = 0$

$x^2 - 17x - 5x + 85 = 0$

$x(x - 17) - 5(x - 17) = 0$

$(x - 17)(x - 5) = 0$

Either $(x - 17) = 0$ or $(x - 5) = 0$

$x = 17$ or $x = 5$

So, the points are: (17, 0) and (5, 0).

2. Let's calculate the distances between the points P, A, and B using the distance formula:

$\therefore PA = PB$

$\Rightarrow \sqrt{(3-0)^2 + (k-2)^2} = \sqrt{(k-0)^2 + (5-2)^2}$

$\Rightarrow \sqrt{9 + (k-2)^2} = \sqrt{(k^2 + 9)}$

$\Rightarrow 9 + (k - 2)^2 = k^2 + 9$ ...[Squaring both sides

$\Rightarrow k^2 - 4k + 4 = k^2$ $\Rightarrow -4k + 4 = 0$

$\Rightarrow -4k = -4$ $\Rightarrow k = 1$

Therefore, the value of k that makes $P(0, 2)$ equidistant from $A(3, k)$ and $B(k, 5)$ is 1.

3. The Radius of the quadrant is 5 units, Let the coordinates of point $p(x, y)$. Now, By using distance formula,

$(x - 0)^2 + (y - 0)^2 = 5^2$

$x^2 + y^2 = 25$

One possible pair satisfying above equation is (3, 4).

4. Let the line segment joining the point $(1, -3)$ and $(4, 5)$ is divided by x-axis is $k : 1$ and the point of intersection be $(x, 0)$.

Section formula, $\left(\dfrac{m.x_2 + n.x_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$

Using section formula, we get

$\Rightarrow \dfrac{my_2 + my_1}{m + n} = 0$ $\Rightarrow \dfrac{k(5) + 1(-3)}{k+1} = 0$

$\Rightarrow 5k - 3 = 0$ $\therefore k = \dfrac{3}{5}$

The x-axis divides the line segment joining the point $(1, -3)$ and $(4, 5)$ is $3 : 5$.

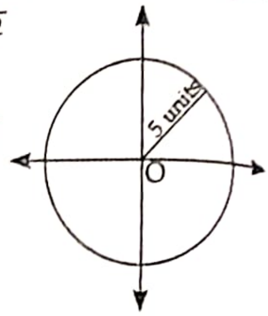
Using section formula, we get

$\dfrac{my_2 + ny_1}{(m+n)} = \dfrac{3(4) + 5(1)}{3+5} = \dfrac{17}{8}$

Therefore the point of intersection is $\left(\dfrac{17}{8}, 0\right)$.

5. Distance of point $(3, -5)$ from the origin

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$= \sqrt{(3 - 0)^2 + (-5 - 0)^2}$

$= \sqrt{(3)^2 + (-5)^2}$

$= \sqrt{34}$ units

Since $\sqrt{34} > 5$, the point $(3, -5)$ lies outside the circle.

6. Mid-point of the line segment joining the points $A(2a, 4)$ and $B(-2, 3b)$ is $C(1, 2a + 1)$.

Mid-point of AB $= \dfrac{(2a-2)}{2}, \dfrac{(4+3b)}{2}$ ...(1)

Mid-point of AB $= (1, 2a + 1)$ ...(2) (given)

Now, From (1) and (2), We get

$1 = \dfrac{(2a-2)}{2}$ $\Rightarrow a = 2$

and $2a + 1 = \dfrac{(4+3b)}{2}$ $\Rightarrow 10 - 4 = 3b$

$\therefore b = 2$

Hence $a = 2$ and $b = 2$

7. Section formula if a point $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ $B(x_2, y_2)$ in ratio of $m : n$.

Then,

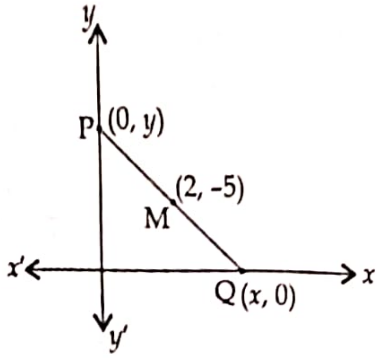
Let P divides AB in ratio of $k : 1$

$\therefore 4 = \dfrac{(6k+2)}{(k+1)}$

$\Rightarrow 4k + 4 = 6k + 2$ $\Rightarrow k = 1$

Again using section formula;

$m = \dfrac{(-3\times1 + 3\times1)}{(1+1)}$ $\therefore m = 0$



mid-point of PQ = (2, −5)
By midpoint formula,

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

$$2 = \frac{0 + x}{2} \quad \text{and} \quad -5 = \frac{y + 0}{2}$$

$$\therefore x = 4 \quad \text{and} \quad y = -10$$

The co-ordinates of P are (0,−10) and Q are (4, 0).

Short Answers

1. Let A(1, 7), B(4, 2), C(−1, −1) and D(−4, 4) be the given points.
Let's find the sides after joining these four points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(1-4)^2 + (7-2)^2}$$

$$= \sqrt{9 + 25} = \sqrt{(34)}$$

$$BC = \sqrt{(4+1)^2 + (2+1)^2}$$

$$= \sqrt{(25+9)} = \sqrt{(34)}$$

$$CD = \sqrt{(-1+4)^2 + (-1-4)^2}$$

$$= \sqrt{(9+25)} = \sqrt{(34)}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2}$$

$$= \sqrt{(25+9)} = \sqrt{(34)}$$

$$AC = \sqrt{(1+1)^2 + (7+1)^2}$$

$$= \sqrt{(4+64)} = \sqrt{(68)}$$

$$BD = \sqrt{(4+4)^2 + (2-4)^2}$$

$$= \sqrt{(64+4)} = \sqrt{(68)}$$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal.

Hence, these are the vertices of a square.

2. Let A(1, −2), B(3, 6), C(5, 10) and D(3, 2) given points.
Let's find the sides after joining these four points.

$$AB = \sqrt{(3-1)^2 + (6+2)^2}$$

$$= \sqrt{(4+64)} = \sqrt{(68)}$$

$$BC = \sqrt{(5-3)^2 + (10-6)^2}$$

$$= \sqrt{(4+16)} = \sqrt{(20)}$$

$$CD = \sqrt{(3-5)^2 + (2-10)^2}$$

$$= \sqrt{(4+64)} = \sqrt{(68)}$$

$$DA = \sqrt{(3-1)^2 + (2+2)^2}$$

$$= \sqrt{(4+16)} = \sqrt{(20)}$$

we can see that the opposite sides of the quadrilateral formed by the given four points are equal.
***i.e.*, (AB = CD) and (DA = BC).**

3. We know that by section formula,
Since, C divides A and B in the ratio 3 : 4 internally.
By section formula,

$$\frac{3x + 4(2)}{(3+4)} = -1 \quad \text{and} \quad \frac{3y + 4(5)}{(3+4)} = 2$$

$$\therefore 3x + 8 = -7 \quad \text{and} \quad 3y + 20 = 14$$

$$3x = -15 \quad \text{and} \quad 3y = -6$$

$$x = \frac{-15}{3} \quad \text{and} \quad -y = \frac{-6}{3}$$

$$x = -5 \quad \text{and} \quad y = -2$$

Hence, the co-ordinates of B are (−5, −2).

4. Using the section formula, if a point (x, y) divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$, then

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n} + \frac{my_2 + ny_1}{m+n}\right)$$

Since, P and Q are points of trisection and P is nearer to A.

$\Rightarrow AP : PB = 1 : 2$

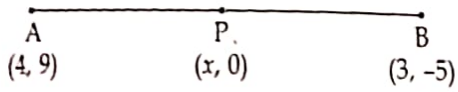
So, $P = \left(\frac{mx_2 + nx_1}{m+n} + \frac{my_2 + ny_1}{m+n}\right)$

$\Rightarrow P\left[\frac{1(5)+2(2)}{(1+2)} + \frac{1(-8)+2(1)}{(1+2)}\right]$

$\Rightarrow P(-3, 2)$ passes through $2x - y + k = 0$

$\therefore k = 8$

5. Let the coordinate of the point at which line intersects the x-axis = $(x, 0)$.



Let the Ratio be = k : 1

By using Section Formula,

$(x, 0) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$(x, 0) = \left(\frac{k(3)+1(4)}{k+1}, \frac{k(-5)+1(9)}{k+1}\right)$

$(x, 0) = \left(\frac{3k+4}{k+1}, \frac{-5k+9}{k+1}\right)$

On Equating both sides, we get

$\Rightarrow \frac{-5k+9}{k+1} = 0$

$\Rightarrow -5k = -9$

$\Rightarrow k = \frac{9}{5}$

$\Rightarrow k : 1 = \frac{9}{5} : 1 = 9 : 5$

Ratio in which x-axis divides the line segment joining the points A(4, 9) and B (3, 5) is 9 : 5.

6. **Given.** Point $P(9a - 2, -b)$ divides the line segments joining $A(3a + 1, -3)$ and $B(8a, 5)$ in the ratio 3 : 1.

Now, The coordinates of the point P which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are

$\left[\frac{(m_1x_2 + m_2x_1)}{(m_1 + m_2)}, \frac{(m_1y_2 + m_2x_1)}{(m_1 + m_2)}\right]$

Here, $m_1 : m_2 = 3 : 1$, (x_1, y_1)

$= (3a + 1, -3)$ and $(x_2, y_2) = (8a, 5)$

So, $\left[\frac{(3(8a) + 1(3a+1)}{(3+1)}, \frac{3(5) + 1(-3)}{(3+1)}\right]$

$= (9a - 2, -b)$

$\left[\frac{(24a + 3a + 1)}{4}, \frac{(15-3)}{4}\right] = (9a - 2, -b)$

$\left[\frac{(27a+1)}{4}, \frac{12}{4}\right] = (9a - 2, -b)$

Now, $\frac{(27a+1)}{4} = 9a - 2$

$27a + 1 = 4(9a - 2)$

$\Rightarrow 27a + 1 = 36a - 8$

$\Rightarrow 36a - 27a = 1 + 8$

$\Rightarrow 9a = 9$

$\Rightarrow a = \frac{9}{9}$ $\qquad \therefore$ **$a = 1$**

Also, $-b = \frac{12}{4}$

$\Rightarrow -b = 3$ $\qquad \therefore$ **$b = -3$**

Therefore, the values of a and b are 1 and -3.

7. P, Q, R, S divide the line segment AB on the ratio 1 : 4, 2 : 3, 3 : 2, 4 : 1 respectively.

From above P divides AB in 1 : 4 ratio coordinates of P using section formula,

$\frac{(mx_2 + nx_1)}{(m+n)} = \frac{(my_2 + ny_1)}{(m+n)}$

Here $m = 1$ and $n = 4$, $x_1 = 1$, $x_2 = 6$, $y_1 = 2$, $y_2 = 7$

$\therefore \left(\frac{(6+4)}{5}, \frac{(7+8)}{5}\right)$

$\Rightarrow$ **(2, 3) = coordinates of P.**

Similarly,

Coordinates of Q,

Q divide AB in 2 : 3 ratio.

$\frac{(12+3)}{5}, \frac{(14+6)}{5}$

$\Rightarrow$ **(3, 4) = coordinates of Q.**

Similarly,

R divide AB in 3 : 2 ratio.

Coordinates of R,

$\frac{(18+2)}{5}, \frac{(21+4)}{5}$

$\Rightarrow$ **(4, 5) = coordinates of R.**

Long Answers

1. Given, (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4).

Also, $a - 2b = 18$ ...(1)

We have to find the value of k and the distance AB.

The coordinates of the mid-point of the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are $\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}$.

Here,

$(x_1, y_1) = (10, -6)$ and $(x_2, y_2) = (k, 4)$

Mid-point of AB, $\left[\frac{(10+k)}{2}, \frac{(-6+4)}{2}\right] = (a, b)$

$\left[\frac{(10+k)}{2}, \frac{-2}{2}\right] = (a, b)$

Now, $\frac{(10+k)}{2} = a$

$10 + k = 2a$

$k = 2a - 10$ ...(2)

Also, $b = \frac{-2}{2}$

$b = -1$

Put $b = -1$ in (1),

$a - 2(-1) = 18$

$a + 2 = 18$

$a = 18 - 2$

$a = 16$

Put $a = 16$ in (2),

$k = 2(16) - 10$

$k = 32 - 10$ $\therefore$ $k = 22$

Therefore, the value of k is 22.

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between A(10, –6) and B(22, 4)

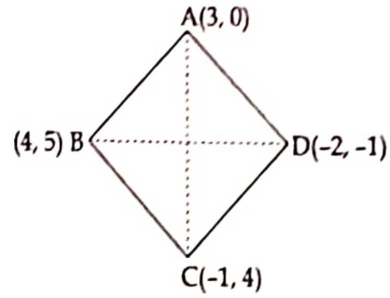
$= \sqrt{(22-10)^2 + [4-(-6)]^2}$

$= \sqrt{(12)^2 + (10)^2}$

$= \sqrt{(144+100)} = \sqrt{244} = 2\sqrt{61}$

Therefore, the distance between A & B is $2\sqrt{61}$ units.

2.



We know that the distance between the two points is given by the distance formula,

Distance formula

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Therefore, distance between A (3, 0) and C (– 1, 4) is given by

Length of diagonal AC

$= \sqrt{[3-(-1)]^2 + (0-4)^2}$

$= \sqrt{(16+16)} = 4\sqrt{2}$

The distance between B (4, 5) and D (-2, –1) is given by

Length of diagonal BD

$= \sqrt{[4-(-2)]^2 + [5-(-1)]^2}$

$= \sqrt{(36+36)}$

$= 6\sqrt{2}$

$\therefore$ **Area of the Rhombus, ABCD**

$= \frac{1}{2} \times$ *(Product of lengths of diagonals)*

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$ square units

= 24 square unit.

3. We first let the ratio be $k : 1$. In which line segment $x - 3$; $y = 0$ divides points (-2, –5) and (6, 3).

Then by using section formula coordinate of point C will be given as:

$x = \frac{[6(k) + 1(-2)]}{(k+1)}$, $y = \frac{3(k) + 1(-5)}{k+1}$

$x = \frac{6k-2}{k+1}$, $y = \frac{3k-5}{k+1}$

Therefore, coordinate of the point C

$\left(\frac{6k-2}{k+1}, \frac{3k-5}{k+1}\right)$

Also, point $C\left(\frac{6k-2}{k+1}, \frac{3k-5}{k+1}\right)$ line on the line $x - 3y = 0$. Therefore, point $C\left(\frac{6k-2}{k+1}, \frac{3k-5}{k+1}\right)$ will satisfy given line.

Substituting values in given line:

$$\frac{6k-2}{k+1} - 3\left(\frac{3k-5}{k+1}\right) = 0$$

$$\Rightarrow \frac{6k-2-9k+15}{k+1} = 0$$

$$\Rightarrow \frac{-3k+13}{k+1} \qquad \Rightarrow -3k + 13 = 0$$

$$\Rightarrow -3k = -13$$

$$\Rightarrow k = \frac{13}{3}$$

Therefore, line $x - 3y = 0$

divides line segment joining points $(-2,-5)$ and $(6, 3)$ is $13 : 3$.

To find the point of intersection or intercept we substitute the value of ratio obtained above in section formula. We have,

$$\Rightarrow C\left(\frac{6k-2}{k+1}, \frac{3k-5}{k+1}\right)$$

$$\Rightarrow C\left(\frac{6\left(\frac{13}{3}\right)-2}{\frac{13}{3}+1}, \frac{3\left(\frac{13}{3}\right)-5}{\frac{13}{3}+1}\right)$$

$$\Rightarrow C\frac{\left(\frac{72}{3}\right)}{\left(\frac{16}{3}\right)}, \frac{\left(\frac{24}{3}\right)}{\left(\frac{16}{3}\right)}$$

$$\Rightarrow C\left[\frac{72}{16}, \frac{24}{16}\right] \qquad \Rightarrow C\left(\frac{9}{2}, \frac{3}{2}\right)$$

Therefore, the coordinate of the point of intersection is $\left(\frac{9}{2}, \frac{3}{2}\right)$.

Here P is the mid-point of the line A and B.

$$P(x, y) = \frac{(-10-2)}{2}, \frac{(4+0)}{2} = (-6, 2)$$

...[Using mid-point to formula

Let the ratio be $k : 1$.

Using section formula, $-6 = \frac{k(-4)+1(-9)}{k+1}$

$\Rightarrow -6k - 6 = -4k - 9$

$\Rightarrow -6k + 4k = -9 + 6$

$\Rightarrow -2k = -3$

$\Rightarrow k = \frac{3}{2}$ or $3 : 2$

Now, y coordinate, *i.e.*, $2 = \frac{(my_2 + ny_1)}{(m+n)}$

$$\Rightarrow 2 = \frac{3(y)-2(-4)}{(3+2)}$$

$\Rightarrow 10 = 3y - 8$

$\Rightarrow 3y = 18 \qquad \therefore y = 6$

Case Based Answers

1. (*a*) We need to calculate the distance between Aditi and Akanksha house which is located at point A and B respectively.

Coordination of point

A (2, 3) and B = (2, 1)

$AB = \sqrt{(2-2)^2 + (3-1)^2}$

AB = 2 units

$\therefore$ **The distance between Aditi and Akanksha house is 2 units.**

(*b*) (*i*) Here we need to calculate the distance between the library which is located at point C and Akanksha's house which is at point A. Coordination of point

A = (2, 3) and C = (4, 1)

$AC = \sqrt{(2-4)^2 + (3-1)^2}$

$= \sqrt{8} = 2\sqrt{2}$

$AC = 2\sqrt{2}$ units

$\therefore$ **Required distance AC is $2\sqrt{2}$ units.**

Or

(*ii*) Here we need to calculate the distance between the library which is located at point C and Aditi's house which is at point B.

Coordination of point

$B = (2, 1)$ and $C = (4, 1)$

$BC = \sqrt{(2-4)^2 + (1-1)^2}$

$BC = \sqrt{(-2)^2 + (0)^2}$

$BC = 2$ units

$\therefore$ **Required distance BC is 2 units.**

(*c*) Distance between O and A

$\sqrt{(2)^2 + (3)^2} = \sqrt{(4+9)} = \sqrt{13}$ units

and distance between O and B

$= \sqrt{(2)^2 + (1)^2} = \sqrt{(4+1)} = \sqrt{5}$ units

Thus, required distance

$= \sqrt{13} - \sqrt{5}$ **units**

2. (*a*) The distance between A and C

A (4, –3) and C (8, 5)

$AC = \sqrt{(8-4)^2 + (5-(-3)^2}$

$= \sqrt{(16+64)}$

$= \sqrt{80}$

$= 4\sqrt{5}$ **units**

(*b*) (*i*) Here, we will use the section formula.

Let the coordinates of I be (x, y).

Then, by section formula

$= \frac{(1\times 8 + 2\times 7)}{(1+2)} = \left(\frac{(8+14)}{3}\right)$

$x = \left(\frac{22}{3}\right)$

then, $\left(\frac{1\times 5 + 2\times 3}{1+2}\right) = \left(\frac{5+6}{3}\right)$

$y = \left(\frac{11}{3}\right)$

Hence, The coordinates of I is $\left(\frac{22}{3}, \frac{11}{3}\right)$.

Or

(*ii*) The mid-point of A and C

$= \left(\frac{8+4}{2}, \frac{5-3}{2}\right) = (6, 1)$

(*c*) B divides the line segment joining A and C in ratio $k : 1$.

A(4, –3), C(8, 5)

$\Rightarrow 7 = \frac{(k\times 8 + 1\times 4)}{(k+1)}$

$\Rightarrow 7k + 7 = 8k + 4 \qquad \Rightarrow k = 3$

Hence 3 : 1.

3. (*a*) Clearly, the coordinates of A are (3, 3).

(*b*) (*i*) Coordinates of C are (8, 7) therefore midpoint of AC is

$\left(\frac{3+8}{2}, \frac{3+7}{2}\right) = (5.5, 5)$

Or

(*ii*) Coordinates of B and D are (4, 6) and (6, 4), respectively.

Distance between A and B

$= \sqrt{(4-3)^2 + (6-3)^2}$

$= \sqrt{(1^2 + 3^2)} = \sqrt{10}$

and Distance between A and D

$= \sqrt{(6-3)^2 + (4-3)^2}$

$= \sqrt{(3^2 + 1^2)} = \sqrt{10}$

Thus, both B and D are near to A.

(*c*) **Distance between B and C**

$= \sqrt{(8-4)^2 + (7-6)^2}$

$= \sqrt{(16+1)} = \sqrt{17}$

Distance between D and C

$= \sqrt{(8-6)^2 + (7-4)^2}$

$= \sqrt{(4+9)}$

$= \sqrt{13}$

$\therefore$ **Point A is equidistant from B and D.**

(DAY 24 SWAHA)

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11 Arithmetic Progressions

What did CBSE ask last year?

MCQs	**2 Questions (2 × 1 = 2 Marks)**
Subjective	2 Very Short Questions (2 × 2 = 4 Marks)
	1 Short Question (1 × 3 = 3 Marks)
	No Long Question
Case Based	No case Based Question

Note: *All the above typology of questions include 'Competency based Questions' labelled as* COMPETENCY.

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Introduction to the chapter

- An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. This fixed number is called the common difference of the AP.

Note: Remember that an AP can be +ve, -ve or zero

TI - n^{th} term of an A. P.

- **n^{th} term from beginning** : $\underbrace{a}_{\text{first term}} + (n-1)\underbrace{d}_{\text{common difference}}$
- **n^{th} term from last term** : $l - (n-1)\,d$

- Let $a_1, a_2, a_3, \ldots\ldots$ be an A.P. whose first term a_1 is a and the common difference is d.
Then, the second term $a_2 = a + d = a + (2-1)d$
the fourth term $a_4 = a_3 + d = (a + 2d) + d = a + 3d = a + (4-1)d$

METIC
ESSIONS

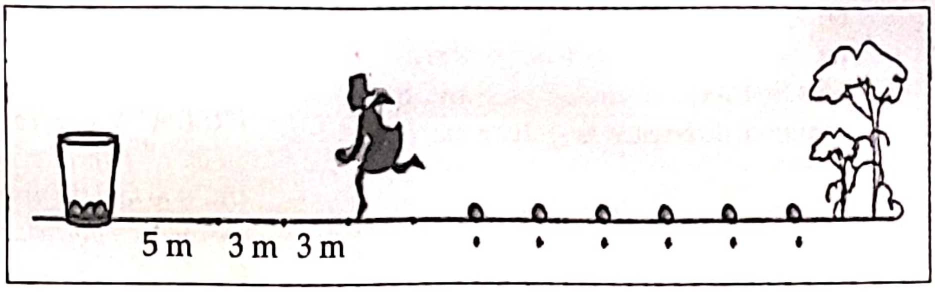
<u>Sum of first *n* Terms of A.P.</u>

- $s = \frac{n}{2}\,[2a + (n-1)d]$
- $s = \frac{n}{2}\,[a + a + (n-1)d]$
- $s = \frac{n}{2}\,(a + a_n)$

Sum from Last term of an A.P $a_c = l$

- $s = \frac{n}{2}\,(a + d)$

Note: Most important topic as tricky word-problem questions come under 5 marks and case based questions



OBJECTIVE QUESTIONS

(DAY 25)

Multiple Choice Questions

Q.1. If the sum of the n terms of an AP be $3n^2 + n$ and its common different is 6, then its first term is [CBSE 2023

(a) 2 (b) 3
(c) 1 (d) 4

Q.2. The next term of the A.P.: $\sqrt{6}, \sqrt{24}, \sqrt{54}$ is: COMPETENCY

(a) $\sqrt{60}$ (b) $\sqrt{96}$
(c) $\sqrt{72}$ (d) $\sqrt{216}$

Q.3. If $k + 24k - 6$ and $3k - 2$ are three consecutive terms of an A .P., then the value of k is: COMPETENCY

(a) 3 (b) –3
(c) 4 (d) –4

Q.4. If a_n is the n^{th} term of an arithmetic progression whose common difference is d, then which of the following statements is valid? COMPETENCY

(a) $a_{24} = a_1 + 24d$ (b) $a_{25} = a_2 + 24d$
(c) $a_{26} = a_2 + 24d$ (d) None of these

Q.5. The first term of an A.P. is p and the common difference is q, then its 10^{th} term is: [CBSE 2020

(a) $q + 9p$ (b) $p - 9d$
(c) $p + 9q$ (d) $2p + 9q$

FREE ADVICE: To find the 10^{th} term of an arithmetic progression (A.P.) when the first term is p and the common difference is q, you can use the following formula:

$T_n = p + (n-1)q$

Q.6. The value of p for which $(2p + 1)$, 10 and $(5p + 5)$ are three consecutive terms of an A.P. is: [CBSE 2020

(a) –1 (b) –2
(c) 1 (d) 2

Q.7. The common difference of the A.P. $\frac{1}{p}, \frac{(1-p)}{p}, \frac{(1-2p)}{p}$ is: COMPETENCY

(a) 1 (b) $\frac{1}{p}$
(c) –1 (d) $\frac{-1}{p}$

Q.8. In a game, a player must gather 20 flags positioned 5 meters apart in a straight line. The starting point is 10 meters away from the first flag. The player starts from the starting point, collects the 20 flags and comes back to the starting point to complete one round. What will be the total distance covered by a player upon completing one round? [CBSE 2024

(a) 105 m (b) 210 m
(c) 220 m (d) 1150 m

Q.9. In an A.P., if $d = -4$, $n = 7$, $a_n = 4$, then a is [NCERT EXEMPLAR

(a) 6 (b) 7
(c) 20 (d) 28

Q.10. In an A.P, if the common difference $d = -3$ and the eleventh term, $a_{11} = 15$ then the first term is [CBSE 2021

(a) 46 (b) 45
(c) 35 (d) 40

FREE ADVICE: You can use the formula for the n^{th} term of an arithmetic progression (A.P.) to find the first term when you have the common difference (d) and the n^{th} term (a_n).

$a_n = a + (n-1)d$

Q.11. How many two digit numbers are divisible by 3? [CBSE 2019

(a) 40 (b) 30
(c) 20 (d) 35

Q.12. Which term of the arithmetic progression (AP) 21, 18, 15, ... is 0? COMPETENCY

(a) 6th term
(b) 7th term
(c) 8th term
(d) the AP does not have 0 as any term

Q.13. Which of the following are in Arithmetic progression? COMPETENCY

(i) 2, 12, 22, 32, 42, ...

(ii) 1, 2, 4, 7, 11, 16, ...

(iii) 7, 6.5, 6, 5.5, 5, ...

(a) only (i)

(b) only (i) and (ii)

(c) only (i) and (iii)

(d) all – (i), (ii) and (iii)

Q.14. Two A.P.s have the same common difference. The Ist term of one of these is -1, and that of other is -8. The difference between their 4th term is COMPETENCY

(a) -1 (b) -8

(c) 7 (d) -9

Q.15. The missing terms in AP: ...., 13, ..., 3 are:

(a) 11 and 9

(b) 17 and 9

(c) 18 and 8

(d) 18 and 9

Q.16. If the sum of n terms of an A.P. is $2n^2 + 5n$, then its n^{th} term is COMPETENCY

(a) $4n - 3$

(b) $3n - 4$

(c) $4n + 3$

(d) $3n + 4$

Q.17. If 7 times the 7^{th} term of an A.P. is equal to 11 times its 11^{th} term, then 18^{th} term is [CBSE 2024

(a) 18 (b) 9

(c) 77 (d) 0

Q.18. Find the sum of first 10 multiples of 6. [CBSE 2019

(a) 330

(b) 320

(c) 300

(d) 325

FREE ADVICE: To find the sum of the first 10 multiples of 6, you can use the formula for the sum of an arithmetic series:

Sum $= \frac{n}{2}\,[2a + (n - 1)d]$

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statment of Reason (R). Mark the correct choice as.

(a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

(b) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*

(c) *(A) is true and (R) is false.*

(d) *(A) is false, but (R) is true.*

Q.1. Assertion: $-5, \frac{-5}{2}, 0, \frac{5}{2}$.... is in Arithmetic Progression.

Reason: The terms of an Arithmetic Progression cannot have both positive and negative rational numbers. COMPETENCY

Q.2. Assertion: The difference between any two consecutive terms in the sequence of numbers $\sqrt{6}, \sqrt{24}, \sqrt{54}, \sqrt{96}$, ... is $3\sqrt{6}$.

Reason: The sequence of numbers $\sqrt{6}, \sqrt{24}, \sqrt{54}, \sqrt{96}$, ... form an arithmetic progression. COMPETENCY

Q.3. Assertion: Let the positive numbers a, b, c be in A.P., then $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are also in A.P.

Reason: If each term of an A.P. is divided by abc, then the resulting sequence is also in A.P.

Q.4. Assertion: a, b, c are in A.P. if and only if $2b = a + c$.

Reason: The sum of first n odd natural numbers is n^2. [CBSE 2023

Q.5. Assertion: Sum of natural numbers from 1 to 100 is 5050.

Reason: Sum of n natural numbers is $\frac{n(n+1)}{2}$.

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. Write the first four terms of an Arithmetic Progression, whose first term is 3.75, and the common difference is (–1.5). COMPETENCY

Q.2. How many multiples of 4 lies between 10 and 205 ? *[CBSE 2019*

Q.3. Determine the A.P. whose third term is 16 and 7^{th} term exceed the 5^{th} term by 12. *[CBSE 2019*

Q.4. Find how many integers between 200 and 500 are divisible by 8. COMPETENCY

Q.5. The 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference. *[CBSE 2024*

Q.6. If sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$, find the n^{th} terms. *[CBSE 2019*

Q.7. The 5^{th} and 15^{th} terms of an A.P. are 13 and – 17 respectively. Find the sum of first 21 terms of the A.P. *[CBSE 2018*

Q.8. If the first term of an arithmetic progression (AP) is 5 and the common difference is (–3), then the n^{th} term of the progression is given by $T_n = 5n - 3$.
Is the above statement true or false? Justify your answer. *[CBSE 2024*

(DAY 26)

Short Answer Questions

Q.1. How many terms are there in an A.P. whose first and fifth terms are -14 and 2, respectively and the last term is 62. *[CBSE 2023*

Q.2. Rohan repays his total loan of ₹1,18,000 by paying every month starting with the first installment of ₹1,000. If he increases the installment by ₹100 every month, what amount will be paid by him in the 30^{th} installment? What amount of loan has he paid after 30^{th} installment? *[CBSE 2023*

Q.3. Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP. *[CBSE 2020*

Q.4. In a library, the arrangement of bookshelves follows a pattern where the number of books on each successive shelf increases by 10 books. The first shelf has 30 books, and the last shelf has 160 books.

(*i*) **How many shelves are there in the library?**

(*ii*) **How many total books are there in the library?**

Show your work. *[CBSE 2024*

Q.5. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an A.P. *[NCERT EXEMPLAR*

Q.6. A theatre charges ₹350 for the first ticket and ₹20 less for every subsequent ticket. The offer is valid for 12 tickets only.

(*i*) **Find the discounted price for the first four tickets.**

(*ii*) **How much would someone pay for 8 tickets?**

(*iii*) **What would be the discounted price of the 12th ticket?**

Show your work. COMPETENCY

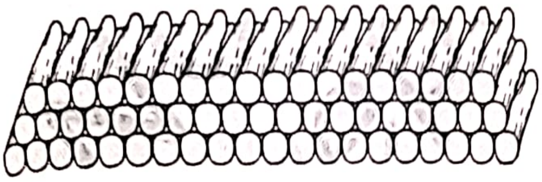
Q.7. The p^{th}, q^{th} and r^{th} terms of an AP are a, b and c respectively, show that $a(q-r) + b(r-p) + c(p-q) = 0$. COMPETENCY

Q.8. The sum of first 15 terms of an A.P. is 750 and its first term is 15. Find its 20^{th} term. COMPETENCY

(DAY 27)

Long Answer Questions

Q.1. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row? COMPETENCY



Q.2. The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term. Also, find the ratio of the sum of first 5 terms to the sum of first 21 terms. **COMPETENCY**

Q.3. The 3^{rd} and the 14^{th} terms of an arithmetic progression are (–9) and (35) respectively.

Which term of this arithmetic progression is five times the 6^{th} term? Show your work. **COMPETENCY**

Q.4. The sum of first q terms of an A.P. is $63q - 3q^2$ If its p^{th} term – 60 , find the value of p. Also, find the 11^{th} term of this A.P. **COMPETENCY**

Q.5. Find the sum of the integers between 100 and 200 that are not divisible by 9. *[NCERT EXEMPLAR*

CASE BASED QUESTIONS

Q.1. In a pathology lab, a culture test has been conducted. In the test, the number of bacteria taken into consideration in various samples is all 3-digit numbers that are divisible by 7, taken in order.

On the basis of above information, answer the following questions.

(a) How many bacteria are considered in the fifth sample?

(b) (i) How many samples should be taken into consideration?

Or

(ii) Find the total number of bacteria in the first 10 samples. **COMPETENCY**

(c) How many bacteria are there in the 7^{th} sample from the last? **COMPETENCY**

Q.2. Anuj gets pocket money from his father everyday. Out of the pocket money, he saves 2.75 on first day, 3 on second day, 3.25 on third day and so on.

On the basis of above information, answer the following questions.

(a) What is the amount saved by Anuj on 14^{th} day?

(b) (i) What is the total amount saved by Anuj in 8 days?

Or

(ii) What is the amount saved by Anuj on 30^{th} day?

(c) What is the total amount saved by him in the month of june, if he starts savings from 1^{st} june? **COMPETENCY**

Q.3. India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number

every year. It produced 16,000 sets in 6^{th} year and 22,600 sets in 9^{th} year.

(*a*) In which year, the production is ₹29,200.

(*b*) (*i*) Find the production during 8th year.

Or

(*ii*) Find the production during first 3 years. COMPETENCY

(*c*) Find the difference of the production during 7^{th} year and 4^{th} year. COMPETENCY

* * * * *

ANSWERS

Multiple Choice Answers

1. (*d*) ***Explanation.***
   Since $S_n = 3n^2 + n$ and $d = 6$
   Substituting $n = 1$
   $S_1 = 3(1)^2 + 1$
   $S_1 = 3 + 1$
   $\therefore$ $\mathbf{S_1 = 4}$

2. (*b*) ***Explanation.*** Simplify this expression:
   $\Rightarrow d = (\sqrt{4} \times \sqrt{6} - \sqrt{6})$
   $\Rightarrow d = (2\sqrt{6}) - \sqrt{6}$
   $\Rightarrow d = \sqrt{6}$
   So, Next term $= \sqrt{54} + \sqrt{6}$
   Next term $= \sqrt{6}(3+1)$
   Next term $= 96$ or $4\sqrt{6}$
   So, the next term in the A.P. $\sqrt{6}, \sqrt{24}, \sqrt{54}$ is $4\sqrt{6}$.

3. (*a*) ***Explanation.*** First term+third term is equal to twice of second term that is:
   $(k + 2) + (3k - 2) = 2(4k - 6)$
   $\Rightarrow 4k = 8k - 12$
   $\Rightarrow 4k - 8k = -12$
   $\Rightarrow -4k = -12$
   $\Rightarrow 4k = 12$
   $\Rightarrow k = \frac{12}{4} = 3$
   Hence $k = 3$

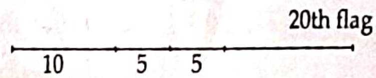
4. (*c*) Taking Option (*c*),
   $a_{26} = a_2 + 24d$
   $= a + d + 24d$ ...[$\because a_2 = a + d$
   $= a + 25d$
   Hence, option (*c*) is correct.

5. (*c*) ***Explanation.*** First term $a = p$
   Common difference, $d = q$
   10^{th} term $= a + 9d = p + 9(q) = p + 9q$

6. (*d*) Here, $a = 2p + 1$,
   $b = 10$ and $c = 5p + 5$
   We know in AP: $b - a = c - b$
   $\Rightarrow 2(10) = 2p + 1 + 5p + 5$
   $\Rightarrow 20 = 7p + 6$
   $\Rightarrow 14 = 7p$
   $\Rightarrow p = \frac{14}{7}$
   $\Rightarrow p = 2$
   Hence, the value of p is 2.

7. (*c*) The common difference of the AP is
   $\frac{1}{p}, \frac{(1-p)}{p}, \frac{(1-2p)}{p}$
   $d = \frac{1-p}{p} - \frac{1}{p} = \frac{1-p-1}{p} = \frac{-p}{p} = \mathbf{-1}$

8. (*b*) **Given:** $a = 10$, $n = 20$,
   Distance between each flag after first flag, $d = 5$



   According to question
   $a_n = a + (n - 1)\,d$
   $a_{20} = 10 + (20 - 1) \times 5$
   $= 10 + 19 \times 5$
   $= 10 + 95 = 105$ m
   $\therefore$ **Distance covered in one round**
   $= 2 \times 105$ m
   = 210 m

9. (*d*) $a_n = a + (n - 1)d$
   $4 = a + (7 - 1)(-4)$
   ...[By the given condition
   $\Rightarrow -a = -4 - 24$
   $\therefore a = 28$

10. (*b*) Given. $a_{11} = 15$

$a + 10d = 15$

$\Rightarrow a + 10(-3) = 15$

$\Rightarrow a - 30 = 15$

$\therefore a = 15 + 30 = 45$

11. (*b*) The two-digit numbers divisible by 3 are 12, 15, 18, ..., 99. Clearly, these numbers are in AP.

Here, $a = 12$ and $d = 15 - 12 = 3$

Let this AP contain n terms. Then,

$a_n = 99$

$a_n = a + (n - 1)d$

$12 + (n - 1) \times 3 = 99$

$\Rightarrow 3n + 9 = 99$

$\Rightarrow 3n = 99 - 9 = 90$

$\therefore n = 30$

Hence, there are 30 two-digit numbers divisible by 3.

12. (*c*) Given. A.P 21, 18, 15, ... is 0

$\therefore a = 21, d = 18 - 21 = -3, a_n = 0$

$\Rightarrow a_n = a + (n - 1)d$

$\Rightarrow 0 = 21 + (n - 1)(-3)$

$\Rightarrow 0 = 21 - 3n + 3$

$\Rightarrow 24 - 3n = 0$

$\Rightarrow +3n = +24$

$\therefore n = \frac{24}{3} = 8$

Hence 8^{th} term will be zero.

13. (*c*) (*i*) We have, 2, 12, 22, 32, 42, ...

$d_1 = 12 - 2 = 10; d_2 = 22 - 12 = 10$

$d_3 = 32 - 22 = 10; d_4 = 42 - 32 = 10$

As, common difference is same so it forms an AP.

(*ii*) We have, 1, 2, 4, 7, 11, 16, ...

$d_1 = 2 - 1 = 1; \quad d_2 = 4 - 2 = 2$

As Common difference is not equal, therefore, **it does not form an AP.**

(*iii*) ..., 7, 6.5, 6, 5.5, 5....

$d_1 = 6.5 - 7 = -0.5$

$d_2 = 6 - 6.5 = -0.5$

$d_3 = 5.5 - 6 = -0.5$

$d_4 = 5 - 5.5 = -0.5$

It forms an A.P.

14. (*c*) Given: $a_1 = -1$ and $a'_1 = -8$

Let d be the same common difference of two A.Ps.

So, $d_1 = d, d'_1 = d$

$a_n = a + (n - 1)d$

$a_4 - a'_4 = [a_1 + (4 - 1) d_1] - [a'_1 + (4 - 1) d'_1]$

$\therefore a_4 - a'_4 = (-1 + 3d) - [-8 + 3d]$

$= -1 + 3d + 8 - 3d = 7$

15. (*c*) $a_2 = 13$ and $a_4 = 3$

The n^{th} term of an AP:

$a_n = a + (n - 1)d$

$a_2 = a + (2 - 1)d$

$13 = a + d$ ...(*i*)

and $a_4 = a + (4 - 1)d$

$3 = a + 3d$ ...(*ii*)

Subtracting equation (*i*) from (*ii*), we get,

$-10 = 2d \quad \Rightarrow d = -5$

Now put value of d in equation (*i*), we get

$13 = a + (-5)$

$a = 18$ **(first term)**

$\therefore a_3 = 18 + (3 - 1)(-5)$

$= 18 + 2(-5) = 18 - 10 = 8$ **(third term)**

16. (*c*) Let a be the first term and d be the common difference of an A.P. and $S_n = 2n^2 + 5n$.

$S_1 = 2(1)^2 + 5 \times 1 = 2 + 5 = 7$

$S_2 = 2 \times (2)^2 + 5 \times 2 = 8 + 10 = 18$

First term $(S_1) = 7$

and second term $a_2 = S_2 - S_1 = 18 - 7 = 11$

$d = a, d_1 = 11 - 7 = 4$

Now, $a_n = a + (n-1) d$

$= 7 + (n-1) 4 = 7 + 4n - 4 = \mathbf{4n + 3}$

17. (*d*) We have $7a_7 = 11a_{11}$

$\Rightarrow 7[a + (7-1)d] = 11[a + (11-1) d]$

$\Rightarrow 7(a + 6d) = 11(a + 10d)$

$\Rightarrow 7a + 42d = 11a + 110d$

$\Rightarrow 4a = -68d \qquad \Rightarrow a = -17d$

Now, $a_{18} = a + (18-1)d = a + 17d$

$= -17d + 17d = \mathbf{0}$

18. (*a*) First 10 multiples of 6 are 6, 12, 18, ... 60

The sequence are in A.P.

Then, $a = 6$, $d = 12 - 6 = 6$, $n = 10$

Sum of 10 terms,

$S_n = \frac{n}{2} \times [2a + (n-1) \times d]$

$\therefore S_{10} = \frac{10}{2} \times [2 \times 6 + (10-1) \times 6]$

$= 5(12 + 54) = 5(66) = \mathbf{330}$

Hence, the sum of first 10 terms of an A.P. is 330.

Assertion Reason Answers

1. (c) *(A) is true and (R) is false.*
2. (c) *(A) is true and (R) is false.*

Given.

Assertion: $\sqrt{6}, \sqrt{24}, \sqrt{54}, \sqrt{96}$

$d = \sqrt{24} - \sqrt{6}$

$d = 2\sqrt{6} - \sqrt{6} = \sqrt{6}$

Hence, assertion is wrong.

Reason: $\sqrt{6}, \sqrt{24}, \sqrt{54}, \sqrt{96}$.... form an A.P.

As, common difference is equal in all cases so it forms an AP and Reason is correct.

3. (*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation. If a, b, c are in AP, then

$b - a = c - b$

$a + c = 2b \qquad ...(i)$

For $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$

$\frac{1}{bc} + \frac{1}{ab} = \frac{2}{ac} \quad \Rightarrow \quad \frac{a+c}{abc} = \frac{2}{ac}$

$\Rightarrow \frac{2b}{abc} = \frac{2}{ac} \qquad ...[\because a + c = 2b$

$\Rightarrow \frac{2}{ac} = \frac{2}{ac}$

Hence, $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are also in AP.

Alternatively

$\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$

Multiplying abc in all

$\frac{abc}{bc}, \frac{abc}{ac}, \frac{abc}{ab}$

a, b, c are in AP

Hence $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are in AP.

4. (*b*) *Both (A) and (R) are true and (R) is not the correct explanation of (A).*

Explanation. **Assertion:** a, b, c are in A.P. $b - a$

$= c - b$ ...[Since common difference is same Hence $2b = a + c$ is true.

Reason: $1 + 3 + 5 + 2n - 1$

$\therefore S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(1 + 2n - 1) = n^2$

which is also True. But it is not the correct explanation of Assertion.

5. (*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation. The sum of the first n natural number can be calculated using the formula:

$\text{Sum} = \frac{(n(n+1))}{2}$

In this case, n is 100, so;

$\text{Sum} = \frac{(100(100+1))}{2}$

$\Rightarrow \text{Sum} = \frac{10101}{2}$

$\therefore$ **Sum = 5050**

Very Short Answers

1. **Given.** First term $(a) = 3.75$
 Common difference $(d) = -1.5$
 $a_2 = a + d$
 $= 3.75 + (-1.5)$
 $= 3.75 - 1.5 = 2.25$
 $a_3 = a + 2d$
 $= 3.75 + 2(-1.5)$
 $= 3.75 - 3 = 0.75$
 $a_4 = a + 3d$
 $= 3.75 + 3(-1.5)$
 $= 3.75 - 4.5 = -0.75$
 So, first four terms of an AP
 $= 3.75, 2.25, 0.75, -0.75$.
2. The formula of n^{th} term of an A.P is
 $a_n = a + (n - 1)d$
 A.P. is 12, 16, 20 ... 204
 Since, $a = 12$ and $d = 4$
 So, $204 = 12 + (n - 1)4$
 $\Rightarrow 204 = 12 + 4n - 4$
 $\Rightarrow 204 = 8 + 4n$
 $\Rightarrow 196 = 4n \quad \therefore \quad n = 49$
 Therefore, the value of n is 49.
3. According to the question,
 $a_3 = 16 \Rightarrow a + 2d = 16$ ...(*i*)
 Using, $a_7 - a_5 = 12$
 $[a + 6d] - [a + 4d] = 12$
 $\Rightarrow 2d = 12$
 $\therefore d = 6$
 By substituting this in equation (*i*), we obtain
 $a + 2 \times 6 = 16 \Rightarrow a + 12 = 16$
 $\therefore \quad a = 4$
 Therefore, A.P. will be $4, 4 + 6,$
 $4 + 2 \times 6, 4 + 3 \times 6, ...$
 Hence, the sequence will be 4, 10, 16, 22, ...
4. Numbers between 200 and 500 divisible by 8 are 208, 216, ..., 496.
 This forms an AP 208, 216, ..., 496.
 So, first term $(a) = 208$
 Common difference $(d) = 8$
 $a + (n - 1)d = 496 \quad [\because \text{last term} = 496]$
 $208 + (n - 1)8 = 496$
 $(n - 1)8 = 288$
 $n = 37$
 Thus, there are 37 integers between 200 and 500 which are divisible by 8.
5. Given that the 17^{th} term of an AP exceeds its 10^{th} term by 7. The formula for n^{th} term of an AP is $a_n = a + (n - 1)\, d$.
 Here, a_n is the n^{th} term, a is the first term, d is the common difference and n is the number of terms.
 $\therefore a_{17} = a + (17 - 1)d$
 $\Rightarrow a_{17} = a + 16d$
 $a_{10} = a + (10 - 1)d$
 $\Rightarrow a = a + 9d$
 According to the question, $a_{17} - a_{10} = 7$ ...(Given)
 $a + 16d - (a + 9d) = 7$
 $\Rightarrow 16d - 9d = 7$
 $\Rightarrow 7d = 7$
 $\Rightarrow d = 1$
 Therefore, the common difference is 1.
6. **Given.** Sum of n terms is $S_n = 3n^2 - 4n$
 So, Sum of $n - 1$ terms is S_{n-1}
 $= 3(n - 1)^2 - 4(n - 1)$
 $= 3n^2 - 6n + 3 - 4n + 4$
 $= 3n^2 - 10n + 7$
 Sum of n terms is equal to sum of $n-1$ terms *plus* n^{th} term *i.e.,*
 $\Rightarrow a_n = S_n - S_{n-1}$
 $= 3n^2 - 4n - (3n^2 - 10n + 7)$
 $\therefore$ **n^{th} term $= 6n - 7$**
7. The first term is a.
 The common difference is d.
 The number of terms are n.
 Therefore 5^{th} term is,
 $a_5 = a + (5 - 1)d$
 $\Rightarrow 13 = a + 4d$
 $\Rightarrow a = 13 - 4d$ ...(1)
 The 15^{th} term is,
 $a_{15} = a + (15 - 1)d$
 $\Rightarrow -17 = a + 14d$ ...(2)
 By substituting value of a from (1) in equation (2), we get
 So, the common difference is -3.
 Now, $a_1 = 25$
 So, the first term is 25.
 So, the sum of the first 21 terms of the A.P is –105.

8. **Given.** First term $(a) = 5$

Common difference $(d) = -3$

n^{th} term of an A.P.

$T_n = a + (n - 1)d$

$= 5 + (n - 1)(-3)$

$= 5 - 3n + 3 = 8 - 3n$

As, n^{th} term of an A.P. is $8 - 3n$.

Hence, the above statement is false.

Short Answers

1. **Given.** $a = -14$ and $a_5 = 2$

$\therefore a + 4d = 2$

$\Rightarrow -14 + 4d = 2$

$\Rightarrow 4d = 16$

$\Rightarrow d = 4$

Now, $a_n = 62$ ...(Given)

$\Rightarrow a + (n - 1)d = 62$

$\Rightarrow -14 + (n - 1)4 = 62$

$\Rightarrow -14 + 4n - 4 = 62$

$\Rightarrow 4n - 18 = 62$

$\Rightarrow 4n = 80$

$\therefore n = \frac{80}{4} = 20$

2. Total amount of loan = ₹1,18,000

Since the amount of each installment increases by ₹100 every month.

$\therefore$ Installments paid are in AP.

Amount of first installment, a = ₹1000

Increase in amount of each installment, d = ₹100

Amount paid by Rohan in 30^{th} installment = a_{30}.

Now, $a_{30} = a + 29d = 1000 + 29 \times 100$

= ₹3900

Amount of loan still paid by Rohan after 30 installments = Total loan − amount paid in 30 installments

$= 118000 - \frac{30}{2}[2 \times 1000 + (30 - 1) \times 100]$

$= 118000 - 30[2450] = 118000 - 73500$

= **₹44500**

3. Since $a, 7, b, 23, c$ are in AP.

$\therefore 7 - a = b - 7 = 23 - b = c - 23$ ...(1)

On taking 2^{nd} and 3^{rd} expression, we get,

$b - 7 = 23 - b$

$\Rightarrow 2b = 30 \quad \Rightarrow b = 15$

On taking 1^{st} and 2^{nd} expression, we get,

$7 - a = b - 7$

$\Rightarrow 7 - a = 15 - 7 \Rightarrow a = -1$

Again, on taking 3^{rd} and 4^{th} expression, we get,

$23 - b = c - 23$

$\Rightarrow 23 - 15 = c - 23 \qquad \Rightarrow \mathbf{c = 31}$

Hence $a = -1$, $b = 15$, $c = 31$

4. (*i*) Let the total number of shelves in the library be n

No. of Books, first shelf (a) = 30 books

Common difference d, = 10

No. of Books, last shelf has (a_n) = 160 books

As we know, $a_n = a + (n - 1)d$

$\Rightarrow 160 = 30 + (n - 1)(10)$

$\Rightarrow 160 - 30 = 10n - 10$

$\Rightarrow 130 + 10 = 10n$

$\Rightarrow 10n = 140$

$\Rightarrow n = \frac{140}{10}$

$\therefore \mathbf{n = 14}$

Number of Books in first shelf $(n) = 14$.

(*ii*) Total no. of books in library

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{14}{2}[2(30) + (14 - 1)10]$

$= 7[60 + 130] = 7[190]$

$= \mathbf{1330}$

So, Total number of Books in the library = 1330.

5. It is given that:

$k^2 + 4k + 8,\ 2k^2 + 3k + 6$

and $3k^2 + 4k + 4$ are consecutive terms of an AP.

$\therefore 2k^2 + 3k + 6 - (k^2 + 4k + 8)$

$= 3k^2 + 4k + 4 - (2k^2 + 3k + 6)$

is the Common difference

By multiplying the negative sign

$2k^2 + 3k + 6 - k^2 - 4k - 8$

$= 3k^2 + 4k + 4 - 2k^2 - 3k - 6$

$\Rightarrow k^2 - k - 2 = k^2 + k - 2$

$\Rightarrow -k = k$

$\Rightarrow 2k = 0$

$\Rightarrow k = 0$

Therefore, k is 0.

6. (*i*) Charges for first ticket, $(a) = 350$

Common difference, $(d) = (-20)$

No. of tickets = 4

$a_2 = a + d$

$= 350 + (-20)$

$= 350 - 20 = 330$

$a_3 = a + 2d$

$= 350 + 2(-20)$

$= 350 - 40 = 310$

$a_4 = a + 3d$

$= 350 + 3(-20)$

$= 350 - 60 = 290$

Hence, Price for subsequent 4 tickets = 350, 330, 310, 290.

(*ii*) Price paid for 8 tickets,

$S_8 = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{8}{2}[2(350) + (8 - 1)(-20)]$

$= 4[700 - 140] = 2240$

(*iii*) Discounted Price of 12[th] ticket,

$a_{12} = a + (n - 1)d$

$= 350 + (12 - 1)(-20)$

$= 350 + 11(-20)$

$= 350 - 220 = ₹130$

7. Let A be the first term and D the common difference of A.P.

$T_p = a = A + (p - 1)D = (A - D) + pD$ ...(1)

$T_q = b = A + (q - 1)D = (A - D) + qD$ ...(2)

$T_r = c = A + (r - 1)D = (A - D) + rD$ ...(3)

Here we have got two unknowns A and D which are to be eliminated.

We multiply (1), (2) and (3) by $q - r$, $r - p$ and $p - q$ respectively and add.

$a(q - r) + b(r - p) + c(p - q)$

$= (A - D)[q - r + r - p + p - q]$

$+ D[p(q - r) + q(r - p) + r(p - q)] = 0$

8. **Given.** $a = 15$ and $S_{15} = 750$

By formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 15 + (15 - 1)d]$

$\Rightarrow 750 = \frac{15}{2}[30 + 14d]$

$\Rightarrow 750 \times 2 = 450 + 210d$

$\Rightarrow 450 + 210d = 1500$

$\Rightarrow 210d = 1500 - 450$

$\Rightarrow 210d = 1050$

$\Rightarrow d = \frac{1050}{210} = 5 \Rightarrow d = 5$

By formula, $a_n = a + (n - 1)d$

Now, $a_{20} = 15 + (20 - 1)5$

$\Rightarrow a_{20} = 15 + 95 = 110.$

Hence, the 20[th] term of the A.P. is 110.

Long Answers

1. It can be observed that the number of logs in rows are forming an A.P. Which is 20, 19, 18, 17 ....

First term, $a = 20$

Common difference, $d = 19 - 20 = -1$

Sum of the n terms, $S_n = 200$

We know that sum of n terms of AP is given by the formula

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore\ 200 = \frac{n}{2}[2 \times 20 + (n - 1)(-1)]$

$\Rightarrow 400 = n[40 - n + 1]$

$\Rightarrow 400 = n[41 - n]$

$\Rightarrow 400 = 41n - n^2$

$\Rightarrow n^2 - 41n + 400 = 0$

$\Rightarrow n^2 - 16n - 25n + 400 = 0$

$\Rightarrow n(n - 16) - 25(n - 16) = 0$

$\Rightarrow (n - 16)(n - 25) = 0$

Either $(n - 16) = 0$ or $(n - 25) = 0$

$\therefore n = 16$ or $n = 25$

The number of logs in n^{th} row will be

$a_{16} = a + (n - 1)\,d$

$a_{16} = 20 + (16 - 1) \times (-1)$

$a_{16} = 20 - 15 = 5$

Similarly, $a_{25} = 20 + (25 - 1) \times (-1)$

$\therefore a_{25} = 20 - 24 = -4$

Clearly, the number of logs in the 16^{th} row is 5. However, the number of logs in the 25^{th} row is negative 4, which is not possible.

Therefore, 200 logs can be placed in 16 rows. The number of logs in the top (16^{th}) row is 5.

2. Let a and d be the first term and common difference of given AP. it is given that:

$$\frac{a_{11}}{a_{18}} = \frac{2}{3}$$

$$\Rightarrow \frac{(a+10d)}{(a+17d)} = \frac{2}{3}$$

$\Rightarrow 3(a + 10d) = 2(a + 17d)$

$\Rightarrow 3a + 30d = 2a + 34d$

$\Rightarrow a = 4d$

Ratio of 5^{th} term to the 21^{st} term,

$$\frac{a_5}{a_{21}} = \frac{(a+4d)}{(a+20d)} \quad \text{...(i)}$$

putting $a = 4d$ in (i), we get

$$\frac{a_5}{a_{21}} = \frac{(4d+4d)}{(4d+20d)}$$

$$\Rightarrow \frac{a_5}{a_{21}} = \frac{8d}{24d}$$

$$\Rightarrow \frac{a_5}{a_{21}} = \frac{1}{3}$$

$\therefore a_5 : a_{21} = 1 : 3$

Ratio of S_5 to the S_{21} is

$$\frac{S_5}{S_{21}} = \frac{5[2a+4d]}{21[2a+20d]} \quad \text{...(ii)}$$

Putting $a = 4d$ in (ii), we get

$$\frac{S_5}{S_{21}} = \frac{5[2(4d)+4d]}{21[2(4d)+20d]}$$

$$\Rightarrow \frac{S_5}{S_{21}} = \frac{5[12d]}{21[28d]}$$

$$\Rightarrow \frac{S_5}{S_{21}} = \frac{60d}{588d}$$

$$\Rightarrow \frac{S_5}{S_{21}} = \frac{5}{49}$$

$\therefore S_5 : S_{21} = 5 : 49$

3. We have, $a_3 = -9$

$\Rightarrow a + 2d = -9$ ...(i)

$\Rightarrow a_{14} = 35$

$\Rightarrow a + 13d = 35$ ...(ii)

On subtracting equation (ii) from (i), we get

$$\begin{array}{rcrl} a & + & 2d & = -9 \\ a & + & 13d & = 35 \\ - & - & & - \\ \hline & & -11d & = -44 \end{array}$$

$$\therefore d = \frac{-44}{-11} = 4$$

Putting the value of d in equation (i)

$a + 2(4) = -9$

$a + 8 = -9$

$a = -9 - 8$

$a = -17$

ATQ, $a_n = 5a_6$

$a + (n - 1)d = 5(a + 5d)$

$-17 + (n - 1)(4) = 5[-17 + 5(4)]$

$\Rightarrow -17 + 4n - 4 = 15$

$\Rightarrow 4n = 15 + 21$

$\Rightarrow 4n = 36$

$$\therefore n = \frac{36}{4} = 9$$

4. $S_q = 63q - 3q^2$

$T_p = -60$

$S_1 = a_1 = 63 - 3 = 60$

$S_2 = a_1 + a_2 = 63(2) - 3(4) = 114$

$\therefore a_1 + a_1 + d = 114$ ...[$\because a_2 = a_1 + d$]

$d = 114 - 2(60) = -6$

Now, $T_p = a + (p - 1)d$

$\Rightarrow -60 = 60 + (p - 1)(-6)$

$$\Rightarrow \frac{-120}{-6} = (p - 1)$$

$\Rightarrow 20 = p - 1$

$\Rightarrow p = 21$

$\therefore T_{11} = a + 10d = 60 + 10(-6) = 0.$

5. The integers between 100 and 200 are 101, 102, 103, ..., 199.

Here, first term, $a = 101$

$l = 199$

$d = 1$

The n^{th} term of the series in AP is given by $a_n = a + (n-1)d$

$199 = 101 + (n-1)1$

$199 - 101 = n - 1$

$98 + 1 = n$

$n = 99$

If l is the last term of an AP, then the sum of the terms is given by

$$S = \frac{n}{2}[a + l]$$

So, $S = \frac{99}{2}[101 + 199]$

$= \frac{99}{2}[300]$

$= 99(150)$

$S = 14850$

The integers between 100 and 200 that are divisible by 9 are 108, 117, 126, ..., 198.

Therefore, the series is 108, 117, 126, ..., 198.

First term, $a = 108$

Last term, $l = 198$

Common difference, $d = 9$

The n^{th} term of the series in AP is given by $a_n = a + (n-1)d$

So, $198 = 108 + (n-1)9$

$\Rightarrow 198 - 108 = 9n - 9$

$\Rightarrow 90 + 9 = 9n$

$\Rightarrow 9n = 99$

$\Rightarrow n = 99/9$

$\therefore\ n = 11$

If l is the last term of an AP, then the sum of the terms is given by

$$S = \frac{n}{2}[a + l]$$

So, $S = \frac{11}{2}[108 + 198]$

$= \frac{11}{2}[306]$

$= 11(153)$

$S = 1683$

Sum of the integers between 100 and 200 that are not divisible by 9

= (Sum of the integers between 100 and 200) − (sum of the integers between 100 and 200 that are divisible by 9)

$= 14850 - 1683 = 13167$

Therefore, the sum of the integers between 100 and 200 that are not divisible by 9 is 13167.

Case Based Answers

1. Here the smallest 3-digit number divisible by 7 is 105. So, the number of bacteria taken into consideration is 105, 112, 119, 994

So, first term $(a) = 105$, $d = 7$ and last term $(l) = 994$

(*a*) $t_5 = a + 4d = 105 + 28 =$ **133**

(*b*) (*i*) Let n samples be taken under consideration.

Last term = 994

$\Rightarrow a + (n-1)d = 994$

$\Rightarrow 105 + (n-1)\,7 = 994$

$\therefore\ n =$ **128**

Or

(*ii*) Total number of bacteria in first 10 samples,

$$S_{10} = \frac{10}{2}[2(105) + 9(7)]$$

$=$ **1365**

(*c*) **t_7 from end** $(128 - 7 + 1)^{th}$ term from beginning $= 122^{th}$ term

$= 105 + 121(7) =$ **952**

2. Here the savings form an A.P. *i.e.,* 2.75, 3, 3.25, ...

So, $a = 2.75$, $d = 3 - 2.75 = 0.25$

(*a*) Amount saved by Anuj on 14^{th} day

$t_{14} = a + 13d$

$= 2.75 + 13(0.25)$

$=$ ₹6

(*b*) (*i*) Total amount saved by Anuj in 8 days,

$$S_8 = \frac{8}{2}[2(2.75) + 7(0.25)]$$

$=$ **₹29**

Or

(*ii*) Amount saved by Anuj on 30th day

$t_{30} = a + 29d$

$= 2.75 + 29(0.25) = \mathbf{10}$

(*c*) Number of days in June = 30

$S_{30} = \frac{30}{2}[2(2.75) + 29(0.25)]$

$= \mathbf{191.25}$

3. (*a*) $a_n = 29200,\ a = 5000,\ d = 2200$

$\therefore a_n = a + (n-1)d$

$\Rightarrow 29200 = 5000 + (n-1)2200$

$\Rightarrow 29200 - 5000 = 2200n - 2200$

$\Rightarrow 24200 + 2200 = 2200n$

$\Rightarrow 26400 = 2200n$

$\Rightarrow n = \frac{264}{22}$

$\Rightarrow n = 12$

$\therefore$ **Production was ₹29200 in 12th year**

(*b*) (*i*) $n = 8,\ a = 5000,\ d = 2200$

$\therefore a_n = a + (n-1)d$

$= 5000 + (8-1)2200$

$= 5000 + 7 \times 2200$

$= 5000 + 15400 = 20400$

$\therefore$ **The production during 8th year,**

= 20400

Or

(*ii*) $n = 3,\ a = 5000,\ d = 2200$

$\therefore S_n = \frac{n}{2} \times [2a + (n-1) \times d]$

$= \frac{3}{2} \times [2(5000) + (3-1) \times 2200]$

$S_3 = \frac{3}{2} \times (10000 + 2 \times 2200)$

$= \frac{3}{2} \times (10000 + 4400)$

$= 3 \times 7200$

$= 21600$

The production during first 3 years is 21600.

(*c*) We have, $a_4 = a + 3d$

$= 5000 + 3\ (2200)$

$= 5000 + 6600$

$= 11600$

$a_7 = a + 6d$

$= 5000 + 6 \times 2200$

$= 5000 + 13200$

$= 18200$

$\therefore a_7 - a_4 = 18200 - 11600$

$= 6600$

(DAY 27 SWAHA)

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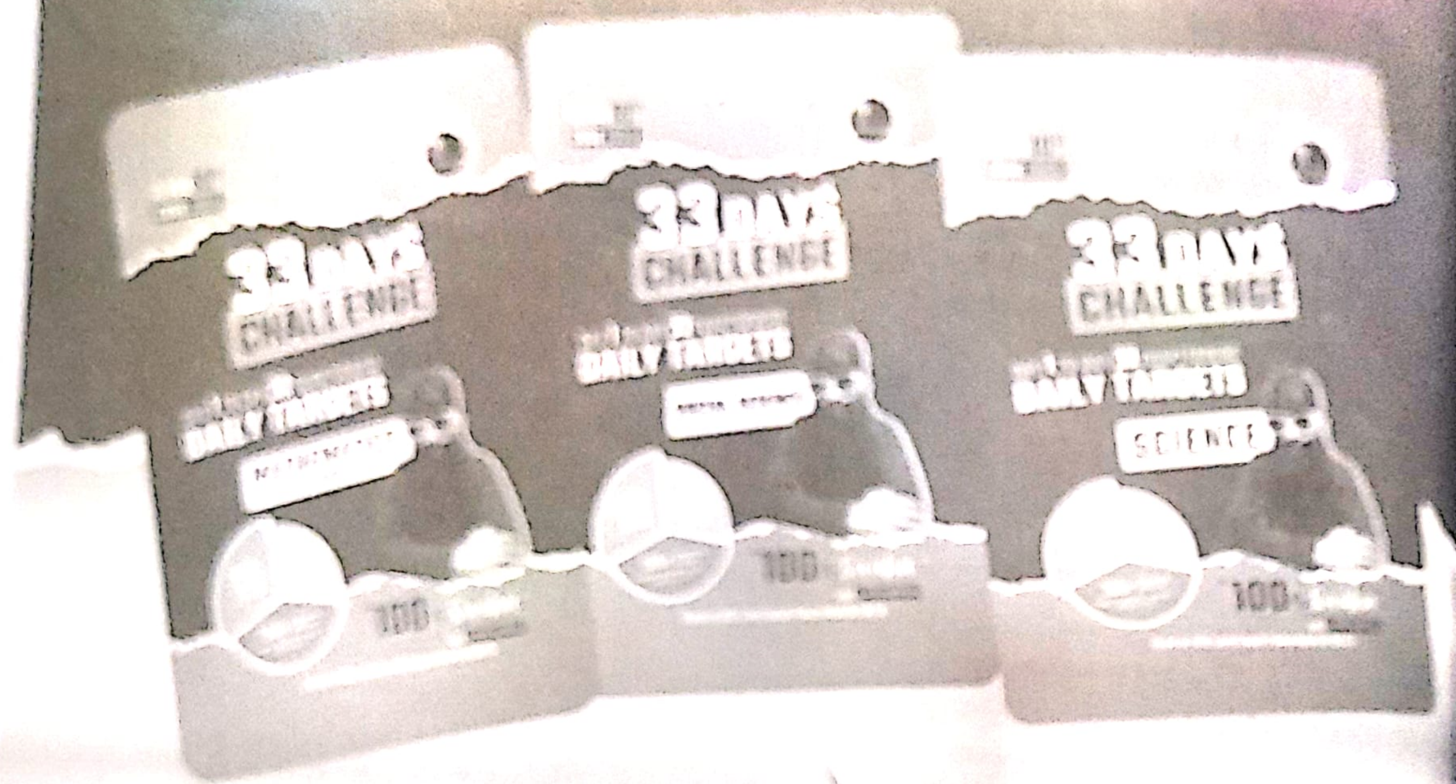

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12 Real Numbers

What did CBSE ask last year?

MCQs	1 Question (1 × 1 = 1 Marks)
Subjective	No Very Short Question
	1 Short Question (1 × 3 = 3 Marks)
	No Long Questions
Case Based	1 Case Based Question (1 + 1 + 2 = 4 Marks)

Note: All the above typology of questions include 'Competency Based Questions' labelled as

COMPETENCY

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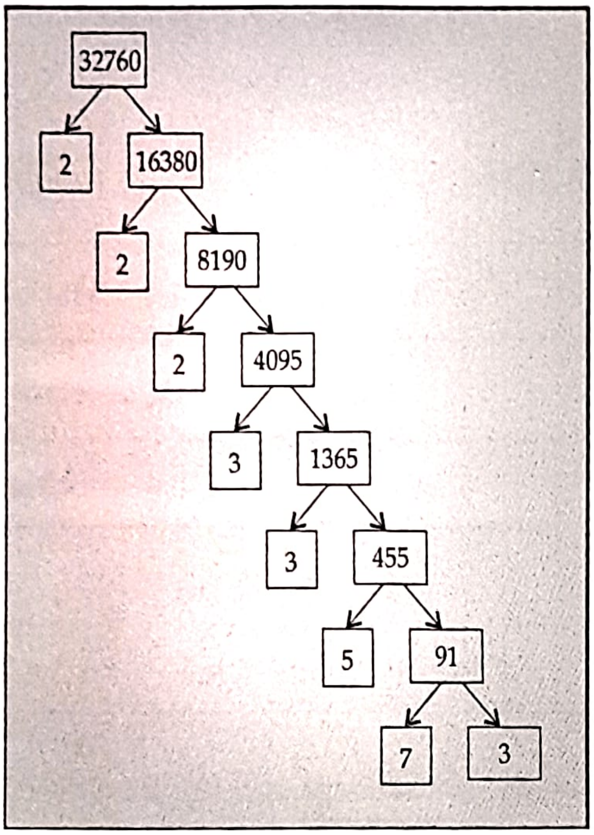
Fundamental theorem of arithmetic

❑ **Theorem** (Every composite number can be written as the product of powers of primes & the prime factorisation of the natural number is unique, except for the order of its factors)



❑ **HCF & LCM**

(For any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b i.e., the product of numbers)

Note: CBSE asks formula based questions from this topic.

$$\text{LCM}(p, q, r) = \frac{p.q.r.HCF(p,q,r)}{HCF(p,q).HCF(q,r).HCF(p,r)}$$

$$\text{HCF}(p, q, r) = \frac{p.q.r.LCM(p,q,r)}{LCM(p,q).LCM(q,r).LCM(p,r)}$$

Rational and Irrational Numbers

- **Euclid's division algorithm** (It's simple long division method we're usual to since class 5^{th})

- **Q = *Rational Numbers:*** Real numbers of the form $\frac{p}{q}$, $q \neq 0$, p, q $\in$ I are rational numbers.

> All integers can be expressed as rationals, for example, $5 = \frac{5}{1}$

- **Q′ = *Irrational Numbers:*** Real numbers which cannot be expressed in the form $\frac{p}{q}$ and whose decimal expansions are non-terminating and non-recurring.

> Roots of prime like $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc. are irrational.

- ***Prime Numbers:*** The natural numbers greater than 1 which are only divisible by 1 and the number itself are called prime numbers. Prime numbers have two factors, i.e., 1 and the number itself.
  For example, 2, 3, 5, 7 & 11 etc.

> Why can't we write the form as 2n + 1?

> 1 is not a prime number as it has only one factor.

OBJECTIVE QUESTIONS

(DAY 28)

Multiple Choice Questions

Q.1. Given that p is a non-negative integer, which of these gives positive integers that are multiple of 5? COMPETENCY

(a) $10p$ and $10p + 2$

(b) $10p$ and $10p + 3$

(c) $10p$ and $10p + 4$

(d) $10p$ and $10p + 5$

Q.2. If n is a natural number, then $2(5^n + 6^n)$ always ends with: [CBSE 2022

(a) 1 (b) 4

(c) 3 (d) 2

Q.3. If a and b are two co-prime numbers, then a^3 and b^3 are: [CBSE 2022

(a) Co-prime (b) Not co-prime

(c) Even (d) Odd

FREE ADVICE: *Agar n co-prime hai toh n^m bhi co-prime hoga or yaha pe m koi bhi natural number ho sakta hai.*

Q.4. The LCM of two numbers is 9 times their HCF. The sum of LCM and HCF is 500. Find the HCF of the two numbers. COMPETENCY

(a) 40

(b) 30

(c) 50

(d) 60

Q.5. Which of the following is an irrational number? COMPETENCY

(a) $\sqrt{5}$ (b) 3.14159265359

(c) $\frac{\sqrt{4}}{3}$ (d) $0.\overline{23517}$

Q.6. Rahul has 40 cm long red and 84 cm long blue ribbon. He cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece? COMPETENCY

(a) 4 cm as it is the LCM of 40 and 84

(b) 4 cm as it is the HCF of 40 and 84

(c) 8 cm as it is the LCM of 40 and 84

(d) 8 cm as it is the HCF of 40 and 84

FREE ADVICE: *Kisi do numbers ko equal divide karne ki baat ho toh usme highest common factor nikalna hota hai yani ki HCF.*

Q.7. If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is

(a) 4

(b) 2

(c) 1

(d) 3

Q.8. The sum of exponents of prime factors in the prime factorisation of 196 is COMPETENCY

(a) 3

(b) 4

(c) 5

(d) 2

Q.9. Which of the following is an irrational number? [CBSE 2024

(a) $5\sqrt{4}$

(b) $\frac{\sqrt{2}}{\sqrt{8}}$

(c) $6+\sqrt{5}$

(d) $\sqrt{64}-\sqrt{4}$

Q.10. The L.C.M. of x and 18 is 36 and the H.C.F. of x and 18 is 2. What is the number x?

(a) 1

(b) 2

(c) 3

(d) 4

FREE ADVICE: *LCM × HCF = First number × second number*

Q.11. The product of a non-zero rational and an irrational number is [NCERT EXEMPLAR

(a) always irrational
(b) always rational
(c) rational or irrational
(d) one

Q.12. The rational form of $0.2\overline{54}$ is in the form of $\frac{p}{q}$ then $p + q$ is COMPETENCY

(a) 14
(b) 55
(c) 69
(d) 79

Q.13. Value of $2\sqrt{3}$ is [CBSE 2020

(a) an integer (b) a rational number
(c) an irrational number
(d) a whole number

Q.14. When 2^{256} is divided by 17 the remainder would be COMPETENCY

(a) 1
(b) 16
(c) 14
(d) none of the above

Q.15. $3.\overline{27}$ is

(a) an integer
(b) a rational number
(c) a natural number
(d) an irrational number

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statment of Reason (R). Mark the correct choice as.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).
(c) (A) is true and (R) is false.
(d) (A) is false, but (R) is true.

Q.1. **Assertion:** Product of HCF and LCM of three numbers is equal to the product of those numbers. COMPETENCY
Reason: Product of HCF and LCM of two numbers is equal to the product of those numbers.

Q.2. **Assertion:** A number N when divided by 15 gives the remainder 2. Then the remainder is same when N is divided by 5.
Reason: $\sqrt{3}$ is an irrational number. COMPETENCY

Q.3. **Assertion:** The sum of an irrational number and a rational number is always irrational.
Reason: Irrational numbers cannot be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$. COMPETENCY

Q.4. **Assertion:** 2 is an example of a rational number.
Reason: The square roots of all positive integers are irrational numbers. COMPETENCY

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons. [NCERT EXEMPLAR

Q.2. Find the greatest number which when divides 1251, 9377 and 15628 leaves remainder 1, 2 and 3 respectively. [CBSE 2022

Q.3. Write the smallest number which is divisible by both 306 and 657. [CBSE 2019

Q.4. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8. [NCERT

Q.5. What is the HCF of the smallest Prime number and smallest composite number? [CBSE 2019

Q.6. HCF of 144 and 180 is expressend in the form $13m - 3$, find the value of m. [CBSE 2014

Q.7. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers. [CBSE 2018

Q.8. Explain whether the number (3 × 5 × 13 × 46 + 23) is a prime number or composite number. COMPETENCY

Q.9. Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number. [CBSE 2017

(DAY 29)

Short Answer Questions

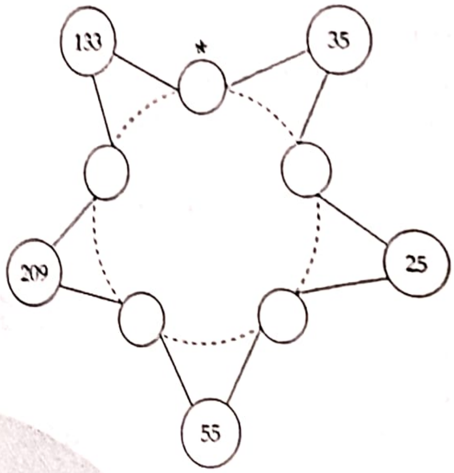
Q.1. Find the smallest pair of 4-digit numbers such that the difference between them is 303 and their HCF is 101. Show your steps. COMPETENCY

Q.2. Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers? [CBSE 2023

Q.3. An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? COMPETENCY

Q.4. Show that $(\sqrt{3}+\sqrt{5})^2$ is an irrational number. [CBSE 2015

Q.5. In the figure below, the inner circles are filled with the prime factors of the numbers given in the outer circles. Each number from 1-26 corresponds to the letter in its position in the alphabet, A-Z. For instance, 1 is A, 2 is B, and so on.



Starting clockwise from *, find the word formed by the numbers in the inner circle. Show your work. [CBSE 2024

Q.6. Prove that $\sqrt{2}$ is an irrational number. [CBSE 2019

Q.7. Show that $5-\sqrt{3}$ is irrational. [NCERT Exemplar

Q.8. If p is prime number, then prove that $\sqrt{p}$ is an irrational. COMPETENCY

Long Answer Questions

Q.1. Show that reciprocal of $3+2\sqrt{2}$ is an irrational number. [CBSE 2014

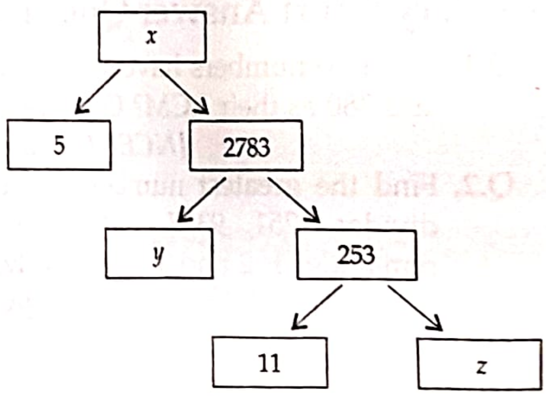
Q.2. What is the minimum possible sum of the numbers, if the LCM and the HCF of two numbers are 924 and 7 respectively? [HOTS

Q.3. Prove that $\sqrt{7}+\sqrt{11}$ is an irrational number.

CASE BASED QUESTIONS

Q.1. A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience.

Observe the following factors tree and Answer the following:



(*a*) What will be the value of x?

(*b*) (*i*) What will be the value of y?

Or, (*ii*) What will be the value of z?

(*c*) What is the prime factorisation of 13915? [CBSE 2024

Q.2. School authorities decided to break the monotony of the school academics and want to arrange a picnic for the students of class X. The bus in charge of the school has to arrange the buses for the picnic.

The students are into 3 groups according to their sections of class. The students in group A are 156, group B are 208 and group C are 260. Buses had to be hired according to the number of the students.

Observe the following factors tree and Answer the following:

(*a*) Calculate the minimum number of students that can accommodate in each bus.

(*b*) (*i*) What are the minimum number of buses required to accommodated all the students. COMPETENCY

Or

(*ii*) What is the the product of HCF and LCM of 156,208 & 260.

(*c*) If we double number of the students, then find the numbers of the buses to accommodate all students. COMPETENCY

Q.3. To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections— section A and section B of grade X. There are 32 students in section A and 36 students in section B.

Observe the following factors tree and Answer the following:

(*a*) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B?

(*b*) (*i*) If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32, 36) is ________ .

Or

(*ii*) 36 can be expressed as a product of its primes as ________ .

(*c*) If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then the LCM (p, q) is ________ .

*****

ANSWERS

Multiple Choice Answers

1. (*d*) $10p$ and $10p + 5$
2. (*d*) 5^n always ends with 5.
   6^n always ends with 6.
   Thus, $(5^n + 6^n)$ always ends with
   $5 + 6 = 11$.
   Thus,
   $2 \times (5^n + 6^n)$ always ends with
   $2 \times 11 = 22$.
   i.e., it always end with 2.
3. (*a*) Since a and b both are coprimes.
   So a^3, b^3 both are also **coprimes.**
4. (*c*) Let the HCF of two numbers be x.
   Then, LCM = $9x$
   According to the question, we have
   LCM + HCF = 500 ...[Given
   $\Rightarrow 9x + x = 500$
   $\Rightarrow 10x = 500$
   $\Rightarrow x = 50$
   Hence, the HCF of the two numbers is 50.
5. (*a*) $\sqrt{5}$, as it is non-terminating and non-recurring.
6. (*b*) The H.C.F of 40 and 84 is 4.
   Therefore the length of each piece is **4 cm.**
7. (*b*) We have, $65 = 5 \times 13$
   $117 = 3 \times 3 \times 13$
   Therefore, HCF of 65 and 117 is 13.
   So, $65m - 117 = 13$
   $\Rightarrow 65m = 130$
   $\therefore$ **$m = 2$**
8. (*b*) $196 = 2^2 \times 7^2$
   $\therefore$ **Sum of powers** $= 2 + 2 = 4$
9. (*c*) $6 + \sqrt{5}$, As Any number added to irrational number is also irrational.
10. (*d*) L.C.M × H.C.F
    = First number × Second number
    $= x \times 18 = 36 \times 2$
    $\therefore$ **Required number,** $x = 36 \times \dfrac{2}{18} = 4$
11. (*a*) The product of a non-zero rational number with an irrational number always results in an irrational number.
    Hence, the product of a non-zero rational number with an irrational number is always an **irrational** number.
12. (*c*) Express it in the form $\dfrac{p}{q}$
    Then
    $x = 0.2545454$ ...(*i*)
    Multiplying both sides by 10 we get
    $10x = 2.545454$ ...(*ii*)
    Again Multiplying both sides by 100 we get
    $1000x = 254.545454$ ...(*iii*)
    Equation (*iii*) – Equation (*ii*) gives
    $990x = 252 \quad \therefore x = \dfrac{252}{990} = \dfrac{14}{55}$
    Which is of the form $\dfrac{p}{q}$
    Then $p = 14$, $q = 55$
    $\therefore$ **Value of** $p + q = 14 + 55 = 69$
13. (*c*) Since, $\sqrt{3}$ is irrational.
    So, it is a **irrational number.**
14. (*a*) 2^{256} can be written as $(2^4)^{64}$
    $= (17 - 1)^{64}$.
    In the expansion of $(17 - 1)^{64}$, every term is divisible by 17 except $(-1)^{64}$.
    Hence remainder is 1.
    Alternatively:
    Euler's number of 17 is 16 and 256 is a multiple of 16. Hence the remainder is 1.
15. (*b*) a rational number

Assertion Reason Answers

1. (*c*) Only the product of LCM and HCF of two number is equal to the product of two numbers.
2. (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)
3. (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)
4. (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

Very Short Answers

1. No, because HCF is always a factor of LCM but here 18 is not a factor of 380.
2. First subtract the remainders from their respective numbers,

$\Rightarrow$ 1251 – 1 = 1250

$\Rightarrow$ 9377 – 2 = 9375

$\Rightarrow$ 15628 – 3 = 15625

So, According to the prime factorisation,

$\Rightarrow 1250 = 2 \times 5 \times 5 \times 5 \times 5$

$\Rightarrow 9375 = 3 \times 5 \times 5 \times 5 \times 5 \times 5$

$\Rightarrow 15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$\therefore$ **HCF(1250, 9375, 15625)**

$= 5 \times 5 \times 5 \times 5 =$ **625**

3. Smallest divisible number = LCM

LCM (306, 657) = $2 \times 3^2 \times 17 \times 73$

= **22,338.**

4. Let given that, n is an odd integer.

$a = n^2 - 1$ ...(*i*)

From equation (*i*), at $n = 1$

$a = (1)^2 - 1 = 0$

$\Rightarrow n = 1, 3, 5.....$

which is divisible by 8.

so, From equation (*i*) at $n = 3$

$a = 3^2 - 1 = 8$ **which is divisible by 8.**

5. We know that,

Smallest prime number = 2

Smallest composite number = 4

So, HC[illegible] 4) = 2

6. Number $144 = 2^4 \times 3^2$

Number $180 = 2^2 \times 3^2 \times 5$

$\Rightarrow$ H.C.F (180, 144) $= 2^2 \times 3^2 = 4 \times 9 = 36$

Given HCF = $13m - 3$

$13m - 3 = 36$

$\Rightarrow 13m = 39$

$\therefore m = 3$

7. We have

$\Rightarrow 404 = 2 \times 2 \times 101 = 2^2 \times 101$

$\Rightarrow 96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$\Rightarrow$ HCF (404, 96) = 4

$\Rightarrow$ LCM (404,96) $= 101 \times 2^5 \times 3 = 9696$

$\Rightarrow$ HCF $\times$ LCM $= 4 \times 9696 = 38784$

Also, Product of two numbers

$= 404 \times 96 = 38784$

Hence, HCF × LCM = Product of 96 and 404. (Hence Provde)

8. Given. $(3 \times 5 \times 13 \times 46) + 23$

$\Rightarrow 23[(3 \times 5 \times 13 \times 2) + 1]$

$\Rightarrow 23(390 + 1)$

$\Rightarrow 23 \times 391 = 8993$

According to fundamentals theorem of arithmetic composite number is expressed by the product of prime numbers.

$\therefore$ **(3 × 5 × 13 × 46 + 23) is a composite number.**

9. Let $5+3\sqrt{2}$ be a rational number

$\Rightarrow 5+3\sqrt{2} = \frac{p}{q}$ Where $q \neq 0$ and p and q are co-prime number

$\Rightarrow 3\sqrt{2} = \frac{p}{q} - \frac{5}{1} \quad \Rightarrow \sqrt{2} = \frac{p-5q}{3q}$

and q are integers and $g \neq 0$.

$\frac{p-5q}{3q}$ is rational number $\sqrt{2}$ is a rational number but $\sqrt{2}$ is irrational number.

This contradiction has arisen because our assumption is wrong.

So we conclude that $5+3\sqrt{2}$ is an irrational number.

Short Answers

1. **Given.** The difference between the smallest pair of 4-digit numbers is 303 and their HCF is 101.

   Assume that the smallest pair of 4-digit numbers is $(101 \times x, 101 \times y)$

or $(101x, 101y)$.

so,

$\Rightarrow 101x - 101y = 303$

$\Rightarrow 101(x - y) = 303$

$\Rightarrow x - y = \frac{303}{101}$

$\Rightarrow x - y = 3$

   Now, from the above calculations, it is evident that the required smallest pair will be of 4-digit numbers, only when $x, y > 9$

   Putting $y = 10$, it is obtained that $x = 13$.

   So, the smallest pair of 4-digit numbers
   $= (101 \times 13, 101 \times 10)$
   $= (1313, 1010)$

   The required smallest pair of 4-digit numbers, such that the difference between them is 303 and their HCF is 101, is written as (1313, 1010)

2. **Given:** Ratio of the numbers 2 : 3

   LCM of numbers = 180

   We have to find HCF

   We know that,

   Product of LCM and HCF of two numbers is equal to the product of the number

   LCM × HCF = $a \times b$

   ...[a, b are the two numbers

   Let, numbers = $2x$ and $3x$

$\Rightarrow$ LCM $= 2 \times 3 \times x = 6x$

$\Rightarrow 6x = 180$

$\Rightarrow x = \frac{180}{6} = 30$

   Numbers are $30 \times 2 = 60$ and $30 \times 3 = 90$

   Now, LCM× HCF = $a \times b$ (a, b are the numbers)

$\Rightarrow 180x = 60 \times 90$

$x = \frac{5400}{180} = 30$

   Therefore, HCF = 30.

3. Let the number of columns be x which is the largest number, which should divide both 612 and 48. It means x should be HCF of 612 and 48.

   So, We can write 612 and 48 as follows

$\Rightarrow 612 = 2 \times 2 \times 3 \times 3 \times 5 \times 17$

$\Rightarrow 48 = 2 \times 2 \times 2 \times 2 \times 3$

$\Rightarrow$ HCF(16, 48) $= 2 \times 2 \times 3 = 12$

   Thus HCF of 612 and 48 is 12 *i.e.*, 12 columns are required.

   Here we have solved using Euclid's algorithm but you can solve this problem by simple method of HCF.

4. Let us assume to the contrary that $(\sqrt{3}+\sqrt{5})^2$ is a rational number,

   then there exists a and b co-prime integers such that,

$\Rightarrow (\sqrt{3}+\sqrt{5})^2 = \frac{a}{b}$

$\Rightarrow 3 + 5 + 2\sqrt{15} = \frac{a}{b}$

$\Rightarrow 8 + 2\sqrt{15} = \frac{a}{b}$

$\Rightarrow 2\sqrt{15} = \frac{a}{b} - 8$

$\Rightarrow 2\sqrt{15} = \frac{(a-8b)}{b}$

$\Rightarrow \sqrt{15} = \frac{(a-8b)}{2b}$

$\Rightarrow \frac{(a-8b)}{2b}$ is a rational number.

   Then $\sqrt{15}$ is also a rational number

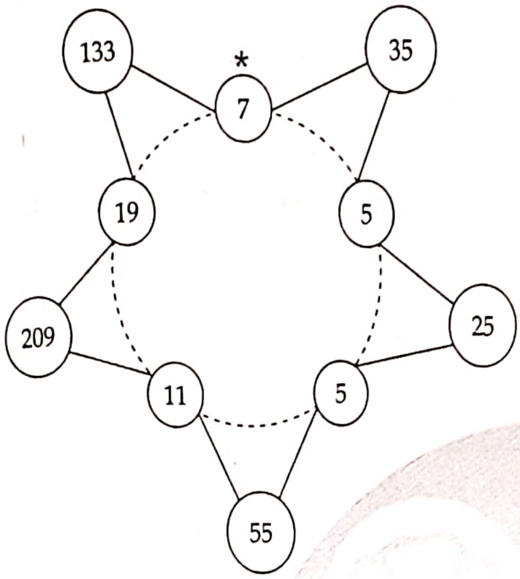
   But as we know $\sqrt{15}$ is an irrational number.

   This makes a contradiction.

   This contradiction has arisen as our assumption is wrong.

   Hence, $(\sqrt{3}+\sqrt{5})^2$ is an irrational number.

5. Redraws the factor tree diagram with the prime factors. The tree may look as follows:



Use the number to letter mapping and find the word as GEEKS.

As, word formed by number in inner circle = GEEKS

G → 7

E → 5

E → 5

K → 11

S → 19

6. From the contradiction approach also known as the Euclidian approach, let us assume that $\sqrt{2}$ is a rational number.

So, $\sqrt{2}$ can be expressed in $\left(\frac{p}{q}\right)$ form where p, q is not equal co-primes where q is not equal to 0.

$\Rightarrow \sqrt{2} = \frac{p}{q}$

Now apply square on both sides,

$\Rightarrow (\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$

The above equation can be written as,

$\Rightarrow 2 = \left(\frac{p}{q}\right)^2$

This can be rewritten as

$\Rightarrow 2q^2 = p^2$ rewrite this as $\frac{p^2}{2} = q^2$

So, from the above equations, it is clear that 2 divides p and q in the multiple of 2 so let us assume that

$\Rightarrow p = 2n \qquad \Rightarrow p^2 = 4n^2$

By comparing the above equation, we get

$\Rightarrow 2q^2 = 4n^2 \qquad \Rightarrow q^2 = 2n^2$

$\Rightarrow q^2$ is a multiple of 2.

Hence q is a multiple of 2.

Then from this one, our assumption fails, and it is a clear contradiction.

Hence $\sqrt{2}$ is an irrational number.

7. Let us assume, to the contrary, that $5-\sqrt{3}$ is rational.

That is, we can find coprime a and b $(b \neq 0)$ such that

$\Rightarrow 5-\sqrt{3} = \frac{a}{b}$

therefore, $5-\frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get

$\sqrt{3} = 5-\frac{a}{b} = \frac{(5b-a)}{b}$

Since a and b are integers, therefore $\frac{5b-a}{b}$ is also rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

$\Rightarrow$ This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

8. If possible, let $\sqrt{p}$ be a rational number.

also a and b is rational. then, $\sqrt{p} = \frac{a}{b}$

on squaring both sides, we get,

$(\sqrt{p})^2 = \frac{a^2}{b^2}$

$\Rightarrow p = \frac{a^2}{b^2}$

$\Rightarrow b^2 = \frac{a^2}{p}$ ...[p divides a^2 so, p divides a

Let $a = pr$ for some integer r

$\Rightarrow b^2 = \frac{(pr)^2}{p} \qquad \Rightarrow b^2 = \frac{p^2r^2}{p}$

$\Rightarrow b^2 = pr^2$

$\Rightarrow r^2 = \frac{b^2}{p}$ ...[p divides b^2 so, p divides b

Thus p is a common factor of a and b.

But this is a contradiction, since a and b have no common factor.

This contradiction arises by assuming $\sqrt{p}$ a rational number.

Hence, $\sqrt{p}$ is irrational.

Long Answers

1. First of all, rationalise the denominator of the reciprocal of $3 + 2\sqrt{2}$.

$\Rightarrow \dfrac{1}{(3+2\sqrt{2})} \times \dfrac{(3-2\sqrt{2})}{(3-2\sqrt{2})}$

$\Rightarrow \dfrac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2} = 3 - 2\sqrt{2}$

After rationalising its denominator, we get $(3 - 2\sqrt{2})$ as a result.

Now, let us assume that $(3 - 2\sqrt{2})$ is an irrational number. So, taking a rational number *i.e.*, 3 and subtracting from it.

We have;

$[3-2\sqrt{2}-3] = -2\sqrt{2}$

As a result, we get $(-2\sqrt{2})$ which is an irrational number.

Hence, the reciprocal of $(3 + 2\sqrt{2})$ is an irrational number.

2. Given that,

$\Rightarrow$ HCF = 7 and LCM = 924

Let the two number be $7p$ and $7q$, where 7 is the HCF of $7p$ & $7q$,

so, p & q are co-prime

$\Rightarrow LCM_{(p,q)} \times HCF_{(p,q)} = p \times q$

$\Rightarrow 924 \times 7 = 7p \times 7q$

$\Rightarrow 7pq = 924$

$\Rightarrow pq = 132$

$\Rightarrow pq = 1 \times 132$ or 132×1

$\therefore p + q = 132 + 1 = 133$

$\Rightarrow pq = 3 \times 4$ or 4×33

$\Rightarrow p + q = 33 + 4 = 37$

$\Rightarrow 11 \times 12$ or 12×11

$\Rightarrow p + q = 11 + 12 = 23$

$\therefore$ **The minimum possible sum,**

$p + q = 7x + 7y$

(HCF P + HCF Q)

$\Rightarrow 7 \times 11 + 7 \times 12 =$ **161**

3. Let us assume that $\sqrt{7}+\sqrt{11}$ is rational number.

Let us $\sqrt{7}+\sqrt{11} = \dfrac{a}{b}$

so, co - primes $b \neq 0$.

$\Rightarrow \sqrt{7} = \left(\dfrac{a}{b}\right) - \sqrt{11}$

Squaring on both side we get,

$\Rightarrow 7 = \left(\dfrac{a^2}{b^2}\right) - 2\times\dfrac{a}{b}\sqrt{11} + (\sqrt{11})^2$

$\Rightarrow \left(\dfrac{2a}{b}\right)\sqrt{11} = \dfrac{a^2}{b^2} + 11 - 7$

$\Rightarrow \left(\dfrac{2a}{b}\right)\sqrt{11} = \dfrac{a^2}{b^2} + 4$

$\Rightarrow \sqrt{11} = \left[\dfrac{(a^2+4b^2)}{b^2}\right]\dfrac{b}{2a}$

$\Rightarrow \sqrt{11} = \dfrac{(a^2+4b^2)}{2ab}$

since, a and b are integer $\dfrac{(a^2+4b^2)}{2ba}$ is rational number,

So, $\sqrt{11}$ is rational number.

This contradicts the fact that $\sqrt{11}$ is irrational number.

Hence $\sqrt{7}+\sqrt{11}$ is irrational number.

Case Based Answers

1. (*a*) Here, $x = 5 \times 2783 =$ **13915**

(*b*) (*i*) Here, $2783 = 253 \times y$

$\Rightarrow \dfrac{2783}{253} = y$

$\therefore$ **$y = 11$**

Or

(*ii*) Here, $253 = 11 \times z$

$\Rightarrow \dfrac{253}{11} = z$

$\therefore$ **$z = 23$**

(*c*) By Factor Tree, $13915 = 5 \times 2783$

$= 5 \times 253 \times 11$

$= 5 \times 11 \times 23 \times 11$

$= \mathbf{5 \times 11^2 \times 23}$

2. (*a*) Given. Numbers are: 156, 208 & 260

So, we have to calculate

HCF of 156, 208 and 260

$156 = 2^2 \times 3 \times 13$

$208 = 2^4 \times 13$

$260 = 2^2 \times 5 \times 13$

Then HCF $= 2^2 \times 13 = $ **52**

(*b*) (*i*) Since, 52 students can go in a bus

Then, for group A, buses required

$= \frac{156}{52} = 3$

for group B, buses required

$= \frac{208}{52} = 4$

for group C, buses required

$= \frac{260}{52} = 5$

$\therefore$ **Total Buses = 3 + 4 + 5 = 12**

Or

(*ii*) Prime factors of

$156 = 2^2 \times 3 \times 13$

$208 = 2^4 \times 13$

$260 = 2^2 \times 5 \times 13$

and HCF $= 2^2 \times 13 = 52$

LCM $= 2^4 \times 3 \times 5 \times 13 = 3120$

$\therefore$ **HCF × LCM = 52 × 3120 = 162240**

(*c*) Let consider there are three groups of students,

So, Group A = 156 × 2 = 312

Group B = 208 × 2 = 416

Group C = 260 × 2 = 520

Since, 52 students can go in a bus,the required number of the buses are

Group A $= \frac{312}{52} = 6$

Group B $= \frac{416}{52} = 8$

Group C $= \frac{520}{52} = 10$

$\therefore$ **Total buses = 6 + 8 + 10 = 24**

3. (*a*) Given.

Number of students in section A = 32.

Number of students in section B = 36.

Prime factorization of 32 and 36:

$32 = 2^5$

$36 = 2^2 \times 3^3$

$\therefore$ LCM(32, 36) $= 2^5 \times 3^2 = 288$

Therefore, the minimum number of books required for students among Section A and B is 288 books.

(*b*) (*i*) We have to find HCF(32, 36)

Factors of 32 are 1, 2, 4, 8, 16 and 32 Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

So, HCF(32, 36) = 4

Or

(*ii*) If we write 36 as a product of all of its prime factors, we can find the prime factorization of 36.

We can write 36 as a product of prime factors $= 2^2 \times 3^2$.

(*c*) Given.

$p = ab^2 = a \times b \times b$

$q = a^3b = a \times a \times a \times b$

L.C.M of p, q is $a^3 b^2$.

(DAY 29 SWAHA)

*****

13 Pair of linear equations in two variables

What did CBSE ask last year?

MCQs	1 Question (1 × 1 = 1 Mark)
Subjective	No Very Short Questions
	No Short Questions
	1 Long Question (1 × 5 = 5 Marks)
Case Based	1 Question (1 + 1 + 2 = 4 Marks)

Note: *All the above typology of questions include 'Competency based Questions' labelled as* COMPETENCY.

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Introduction to the chapter

- A pair of values of variables 'x' and 'y' which satisfy both the equations in the given system of equations is said to be a solution of the simultaneous pair of linear equations.
- A pair of linear equations in two variables can be represented and solved, by—(i) Graphical method,
  (ii) Algebraic method.
  - **(i) Graphical method.** The graph of a pair of linear equations in two variables is presented by two lines.
  - **(ii) Algebraic methods.** Following are the methods for finding the solutions (s) of a pair of linear equations:
    (a) Substitution method, (b) Elimination method.
- There are several situations which can be mathematically represented by two equations that are not linear to start with. But we allow them so that they are reduced to a pair of linear equations.
- ***Consistent system.*** A system of linear equations is said to be consistent if it has at least one solution.
- ***Inconsistent system.*** A system of linear equations is said to be inconsistent if it has no solution.

PAIR OF
EQUATIONS
VARIA

LINEAR
IN TWO
BLES

Algebraic Methods of Solving a Pair of Linear Equations

- ❑ Substitution method
- ❑ Elimination method

In case of Boat questions

If the speed of boat is x m/s and streams y m/s

- ◆ Upstream $(x - y)ms^{-1}$
- ◆ Downstream $(x + y)ms^{-1}$

- ❑ While taking a number

One's digit = y
Ten's digit = x
$[10x + y]$ = Number format

Note: Short & long questions can be asked to get solutions through any of the given method (thus, practice both the methods), situational questions may come under case based questions.

Conditions for Consistency

Let the two equations be:

$a_1x + b_1y + c_1 = 0; a_2x + b_2y + c_2 = 0$

Then,

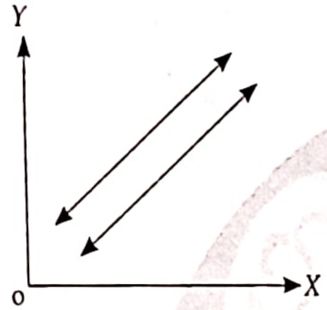
Relationship between coefficient or the pair of equations	Graph	Number of Solutions	Consistency of System
1. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution	Consistent
2. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	Inconsistent
3. $\frac{a_1}{a}$ [illegible] c_1	Co-incident lines	Infinite solutions	Consistent

OBJECTIVE QUESTIONS

(DAY 30)

Multiple Choice Questions

Q.1. Given below is a graph showing two lines that never intersect. These are represented by two linear equations.



Which of these can be said about the number of solution(s) of the above pair of linear equations? **COMPETENCY**

(*a*) They have infinitely many solutions.
(*b*) They have a unique solution.
(*c*) They do not have a solution.
(*d*) Nothing can be said about the number of solutions unless the algebraic form of these equations are known.

Q.2. The condition for the system of linear equations $ax + by = c$ and $lx + my = n$ have a unique solution is **COMPETENCY**

(*a*) $am \neq bl$ (*b*) $al \neq bm$
(*c*) $al \neq bm$ (*d*) $am = bl$

Q.3. The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are: **COMPETENCY**

(*a*) intersecting
(*b*) parallel
(*c*) coincident
(*d*) either intersecting or parallel

FREE ADVICE: You can see that the equation is inconsistent; it doesn't have a unique solution. This means that the two lines are coincident (the same line). They overlap and have infinitely many points in common.

Q.4. Two lines are given to be parallel. The equation of one of the lines is $3x - 2y = 5$. The equation of the second line can be: [CBSE 2022

(*a*) $9x + 8y = 7$
(*b*) $-12x - 8y = 7$
(*c*) $-12x + 8y = 7$
(*d*) $12x + 8y = 7$

Q.5. In a ΔABC, $\angle A = x$, $\angle B = (3x - 2)$, $\angle C = y$. Also, $\angle C - \angle B = 9$. Then value of $\angle C$ is. [CBSE 2022

(*a*) 82° (*b*) 135°
(*c*) 155° (*d*) 145°

Q.6. The value of k, for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions is : [CBSE 2020

(*a*) ±1 (*b*) 1
(*c*) –1 (*d*) 2

FREE ADVICE: For a pair of linear equations to have infinitely many solutions, they must follow:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Q.7. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$ has no solution, is: [CBSE 2020

(*a*) –2 (*b*) –3
(*c*) 3 (*d*) 2

FREE ADVICE: For the system of equations to have no solution, it means that

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Q.8. The pair of equations $y = 0$ and $y = -5$ has: **COMPETENCY**

(*a*) one solution
(*b*) two solutions
(*c*) infinitely many solutions
(*d*) no solution

Q.9. If the lines $3x + 2ky - 2 = 0$ and $2x + 5y + 1 = 0$ are parallel, then what is the value of k? **COMPETENCY**

(a) $\frac{4}{15}$ (b) $\frac{15}{4}$

(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

Q.10. The pair of equations $3x - 5y = 7$ and $-6x + 10y = 7$ have COMPETENCY

(a) a unique solution

(b) infinitely many solutions

(c) no solution

(d) two solutions

Q.11. If a pair of linear equations given by $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$ has infinitely many solutions, then which of the following is definitely true? COMPETENCY

(a) $\frac{l_1}{l_2} = \frac{n_2}{n_1}$ (b) $l_1 l_2 \neq m_1 m_2$

(c) $\frac{l_1}{l_2} \neq \frac{m_1}{m_2}$ (d) $l_1 m_2 = l_2 m_1$

Q.12. The pair of equations $5x - 15y = 8$ and $3x - 9y = \frac{24}{5}$ has [NCERT EXEMPLAR

(a) one solution

(b) two solutions

(c) infinitely many solutions

(d) no solution

Q.13. Harsh correctly solved a pair of linear equations in two variables and found their only point of intersection as (3, –2). One of the lines was $x - y = 5$. Which of the following could have been the other line? COMPETENCY

I : $3x - 3y = 15$

II : $2x - 3y = 12$

III : $2x - 3y = 14$

(a) only I

(b) only II

(c) only I and II

(d) only II and III

Q.14. The ratio of a two-digit number and the sum of its digits is 7 : 1. How many such two-digit numbers are possible? COMPETENCY

(a) 11 (b) 24

(c) 39 (d) 4

Q.15. If the system of equations

$2x + 3y = 7$

$(a + b)x + (2a - b) y = 21$

has infinitely many solutions, then

(a) $a = 1, \quad b = 5$

(b) $a = 5, \quad b = 1$

(c) $a = -1, \quad b = 5$

(d) $a = 5, \quad b = -1$

Q.16. The value of k for which the system of equations:

$x + 2y = 5$

$3x + ky + 15 = 0$ has no solution is COMPETENCY

(a) 6 (b) -6

(c) 32 (d) None of these

Q.17. For what value of k, the following system of equations $kx + 2y = 3$, $3x + 6y = 10$ has a unique solution.

(a) $k = 1$ (b) $k > 1$

(c) $k < 1$ (d) $k \neq 1$

FREE ADVICE: To have a unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Q.18. Given below is a pair of linear equations in two variables.

$4x + 2y = 18$

$3x - 6y = 6$

Which of the following pairs of equations have the same number of solution(s) as the given pair? [CBSE 2024

(a) $3a + 3b = 18; a + b = 6$

(b) $a - b = 4; b - a = 4$

(c) $6a - 2b = 10; 3a + b = 5$

(d) $7a + 9b = 27; 28a + 36b = 76$

Q.19. The sum of the digits of a two-digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is [NCERT EXEMPLAR

(a) 25 (b) 72

(c) 63 (d) 36

Q.20. The value of x and y satisfying the two equations $32x + 33y = 31$, $33x + 32y = 34$ respectively are: [CBSE 2022

(a) 2, -1 (b) -1, 4

(c) 1, -2 (d) -1, -4

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

Q.1. Assertion: The graph of the linear equations $3x + 2y = 12$ and $5x - 2y = 4$ gives a pair of intersecting lines.

Reason: The graph of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ gives a pair of intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. COMPETENCY

Q.2. Assertion: The value of $q = \pm 2$ if $x = 3, y = 1$ is the solution of the line $2x + y - q^2 - 3 = 0$.

Reason: The solution of the line will satisfy the equation of the line.

Q.3. Assertion: If the pair of lines are coincident, then we say that pair of lines is consistent and it has a unique solution.

Reason: If the pair of lines are parallel, then the pair has no solution and is called inconsistent pair of equations. COMPETENCY

Q.4. Assertion: The pair of linear equations $x - 2y + 3 = 0$ and $3x + 4y - 11 = 0$ has a unique solution.

Reason: The pair of linear equations $x - 2y + 3 = 0$ & $3x + 4y - 11 = 0$ represents a pair of coincident lines. COMPETENCY

Q.5. Assertion: If one equation of a pair of dependent linear equations is $-3x + 5y - 2 = 0$. The second equation will be $-6x + 10y - 4 = 0$.

Reason: The condition for dependent linear equations is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. COMPETENCY

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. Find c, if the system of equations $cx + 3y + (3 - c) = 0$, $12x + cy - c = 0$ has infinitely many Solutions. [CBSE 2019

Q.2. For what value of k, does the system of equations

$2x + 3y = 7$

$(k - 1)x + (k + 2)y = 3k$

have an infinite number of solutions? [CBSE 2019

Q.3. A pair of linear equations is shown below. COMPETENCY

$(k - 1)x + y = k + 1$

$(k^2 - 1)x + (k + 1)y = 1 - k^2$

If $k > 1$, then how many solutions does this pair of equations have?

Q.4. For what value of p, will the following pair of linear equations has infinitely many solutions:

$(p - 3)x + 3y = p$; $px + py = 12$. COMPETENCY

Q.5. Solve the following pair of linear equations:

$3x - 5y = 4$

$2y + 7 = 9x$ [CBSE 2019

(DAY 31)

Short Answer Questions

Q.1. Solve for x and y:

$\frac{ax}{b} - \frac{by}{a} = a + b$, $ax - by = 2ab$ [CBSE 2023

Q.2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5; 2x - 3y = 7$

(ii) $2x - 3y = 8; 4x - 6y = 9$ **COMPETENCY**

Q.3. The equations of the lines l_1, l_2, and l_3 are given by $5x + 3y = 2p$, $35x + 21y = pq$, and $100x + 4qy = 240$, respectively, where p and q are real numbers.

(*i*) For what values of p and q does the line l_3 coincide with l_1? Show your steps.

(*ii*) For the values of p and q found in question (*i*), are the lines l_1 and l_2 parallel?

Justify your answer. **COMPETENCY**

Q.4. Determine the value of m and n so that the following pair of linear equations has infinite number of solutions:

$(2m - 1)x + 3y = 5; 3x + (n - 1)y = 2.$

[CBSE 2013

Q.5. Find those integral values of m for which the x-coordinate of the point of intersection of lines $y = mx + 1$ and $3x + 4y = 9$ is an integer. **COMPETENCY**

Long Answer Questions

Q.1. Draw the graphs of the lines $x = -2$ and $y = 3$. Write the vertices of the figure, formed by these lines, the x-axis and the y-axis. Also, find the area of the figure.

[NCERT

Q.2. If $2x + y = 23$ and $4x - y = 19$ then find the value of $5y - 2x$ and $\left(\frac{y}{x}\right) - 2$

[CBSE 2020

Q.3. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father. **COMPETENCY**

Q.4. Anisha lives 15 km away from her school. She walks to the bus stop and takes a bus to school everyday.

If she goes to the nearest bus stop, she needs to walk for 3 km and cover the rest by bus. This takes her 1.5 hours. If she walks to a bus stop further away, she needs to walk for 5 km and cover the rest by bus. This takes her 2 hours.

Frame equations and solve them to find the average speed Anisha walks at, as well as the average speed of the bus. Show your steps. *[CBSE 2024*

Q.5. The ten's digit of a number is twice its unit's digit. The number obtained by interchanging the digits is 36 less than the original number. Find the original number. *[CBSE 2024*

CASE BASED QUESTIONS

Q.1. A book store shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizes etc. He takes a fixed charge for the first two days and an additional charge for subsequent day. Aditi paid ₹22 for a book and kept for 6 days; while Akanksha paid ₹16 for keeping the book for 4 days.

Assume that the fixed charge be x and additional charge (per day) be y.

On the basis of above information, answer the following questions.

[CBSE 2022

(*a*) What is the algebraic representation of the situation of amount paid by Akanksha?

(*b*) (*i*) What are the fixed charges for a book?

Or

(*ii*) What is the total amount paid by both, if both of them have kept the book for 2 more days?

(*c*) What is the algebraic representation of the situation of amount paid by Aditi? **COMPETENCY**

Q.2. A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is ₹9000 and from batch II is 26,000. Assume that each poor child pays x per month and each rich child pays y per month.

On the basis of above information, answer the following questions

[CBSE 2023

(*a*) Represent the information given above in terms of x and y.

(*b*) (*i*) Find the monthly fee paid by a poor child. **COMPETENCY**

Or

(*ii*) Find the difference in the monthly fee paid by a poor child and a rich child.

(*c*) If there are 10 poor and 20 rich children in batch III, what is the total monthly collection of fees from batch III? **COMPETENCY**

Q.3. Two schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey x per student and Cricket y per student. School 'P' decided to award a total of ₹9,500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award ₹7,370 for the two games to 4 and 3 students respectively.

On the basis of above information, answer the following questions.

[CBSE 2024

(*a*) Represent the given information algebraically (in terms of x and y).

(*b*) (*i*) What is the prize amount for hockey?

Or

(*ii*) Prize amount on which game is more and by how much?

(*c*) What will be the total prize amount if there are 2 students each from two games?

*****

ANSWERS

Multiple Choice Answers

1. (c) They do not have a solution.
As lines are parallel to each other so, these lines have no solution.

2. (a) $ax + by = c$ and $lx + my = n$
Comparing $ax + by - c = 0$
with $a_1x + b_1y + c_1 = 0$
$a_1 = a, b_1 = b, c_1 = -c$
Comparing $lx + my - n = 0$
with $a_2x + b_2y + c_2 = 0$
$a_2 = l, b_2 = m, c_2 = -n$
$\Rightarrow$ For a unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{a}{l} \neq \frac{b}{m}$
$\Rightarrow am \neq bl$

3. (c) **Given.** $2x = 5y + 6$ or $2x - 5y - 6 = 0$ ...(1)
and $15y = 6x - 18$ or $6x - 15y - 18 = 0$ ....(2)
Since, $\frac{2}{6} = \frac{-5}{-15} = \frac{-6}{-18} = \frac{1}{3}$
Therefore, the lines are coincident.

4. (c) As given lines are parallel.
$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ should be satisfied.

5. (a) **Given.** In a ΔABC,
$\angle A + \angle B + \angle C = 180^\circ$ ...(By angle-sum property)
and $\angle C - \angle B = 9$
$\Rightarrow y - (3x - 2) = 9$
$\Rightarrow y = 7 + 3x$
Here, $x + (3x - 2) + y = 180^\circ$
$\Rightarrow 4x - 2 + 7 + 3x = 180^\circ$
$\Rightarrow 7x = 175^\circ \quad \Rightarrow x = 25^\circ$
$\therefore y = 7 + 3 \times 25 = 82^\circ$
$\therefore$ **$\angle A = x = 25^\circ$**
and **$\angle B = 3x - 2 = 73^\circ$**
and **$\angle C = y = 82^\circ$**

6. (b) Pair of linear equations
$a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$ has infinitely many solutions if
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
Given. $kx + y = k^2$ and $x + ky = 1$ has infinitely many solutions.
$\Rightarrow \frac{k}{1} = \frac{1}{k} = \frac{k^2}{1}$ ...(i)
Comparing $\frac{k}{1} = \frac{1}{k}$, we get
$k^2 = 1$
$\Rightarrow k = \pm 1$
But $k \neq -1$ because it does not satisfy (i).
$\therefore k = 1$

7. (d) We have, $x + y = 4$ ...(1)
and $2x + ky = 3$ ...(2)
For no solution : $\frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$
$\therefore k = 2$

8. (d) Since, given variable y has different values. So, the pair of equations has no solution.

9. (b) The condition for parallel lines is:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Hence, $\frac{3}{2} = \frac{2k}{5}$
$\therefore k = \frac{15}{4}$

10. (c) Here, $\frac{3}{-6} = \frac{-5}{10} = \frac{7}{7}$
We can compare above with $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, which gives no solution.

11. (d) We have, $l_1x + m_1y + n_1 = 0$
and $l_2x + m_2y + n_2 = 0$
For infinite many solutions
$\frac{l_1}{l_1} = \frac{m_1}{m_2} \neq \frac{n_1}{n_2}$

Taking first two, we have

$$\frac{l_1}{l_2} = \frac{m_1}{m_2}$$

$\therefore l_1 m_2 = l_2 m_1$

12. (c) Here, $a_1 = 5, b_1 = -15, c_1 = -8$

and $a_2 = 3, b_2 = -9, c_2 = \frac{-24}{5}$

So, $\frac{a_1}{a_2} = \frac{5}{3}; \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3}$

and $\frac{c_1}{c_2} = \frac{-8}{\left(\frac{-24}{5}\right)} = \frac{-40}{-24} = \frac{5}{3}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{5}{3}$

Therefore, the given pair of equations have infinitely many solutions.

13. (b) Given that one of the line is $x - y = 5$

The only point of intersection of the two lines is 3, -2

When we substitute $x = 3, y = -2$ in

$3x - 3y = 15$, we get

$9 + 6 = 15$

So, L.H.S = R.H.S

Now consider $2x - 3y = 12$ and when we substitute $x = 3, y = -2$, we get

$6 + 6 = 12$

So, L.H.S = R.H.S

Here, only one point of intersection for the two lines.

So, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

If we consider the lines $x - y = 5$ and

$3x - 3y = 15$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{5}{15} = \frac{1}{3}$

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ If has infinitely many solutions.

If we consider the lines $x - y = 5$ and

$2x - 3y = 12$

$\frac{a_1}{a_2} = \frac{1}{2} \neq \frac{b_1}{b_2} = \frac{1}{3}$

So, the unique solution is given by the line $2x - 3y = 12$.

14. (c) Let x and y be the unit' digit and ten's digit respectively.

According to question,

$= \frac{10y + x}{x + y} = \frac{7}{1}$

$\Rightarrow 10y + x = 7x + 7y$

$\Rightarrow 3y = 6x$

$\Rightarrow y = 2x$ ...(i)

There are 4 digits *i.e.*, 21, 42, 63 and 84, that satisfies (*i*).

15. (b) For these equations to represent the same line, the ratios of the coefficients must be the same:

$$\frac{2}{(a + b)} = \frac{3}{(2a - b)}$$

Now, let's solve for a and b:

$\left(\frac{2}{3}\right) = \frac{(a + b)}{(2a - b)}$

$2(2a - b) = 3(a + b)$

$\Rightarrow 4a - 3a = 3b + 2b$

$\Rightarrow a = 5b \quad \Rightarrow \frac{a}{b} = \frac{5}{1}$

So, a and b must have this proportional relationship.

16. (a) For no solution, we have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow k = 6$

Therefore, the given system of equations will have no solutions, if $k = 6$.

17. (d) The given equations are:

$kx + 2y = 3$

$3x + 6y = 10$

For a unique solution,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

where[$a_1 = k, a_2 = 3, b_1 = 2, b_2 = 6$.

$\Rightarrow \frac{k}{3} \neq \frac{2}{6}$

$\Rightarrow k \neq 1$

$\therefore$ For all values of k except 1, the given linear equations will have a unique solution.

18. (c) We have, $4x + 2y = 18$

and $3x - 6y = 6$

$\frac{a_1}{a_2} = \frac{4}{3}$; $\frac{b_1}{b_2} = \frac{2}{-6} = \frac{-1}{3}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, Unique solution

Now taking equations of option (c), we have,

$6a - 2b = 10$ ...(i)

$3a + b = 5$ ...(ii)

$\therefore \frac{a_1}{a_2} = \frac{6}{3} = \frac{2}{1}$ and $\frac{b_1}{b_2} = \frac{-2}{1} = \frac{-2}{1}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, option (c) is correct.

19. (d) Let the digits in the tens place be x and digit in the ones place be y.

Sum of the digit is 9.

Therefore, we have

$x + y = 9$ ...(1)

The two digit number is of the form $10x + y$.

$\therefore$ We have $10x + y + 27 = 10y + x$

$\Rightarrow 9x - 9y = -27$

$\Rightarrow x - y = -3$ ...(2)

On subtracting equation (2) from (1) and eliminating y, we get

$2x = 6 \quad \Rightarrow x = 3$

On substituting value of x in equation (1), we get

$x + y = 9$

$\Rightarrow 3 + y = 9$

$\Rightarrow y = 6$

$\therefore$ **Required Number $= 10 \times 3 + 6$**

$= 30 + 6 = 36$

20. (a) Given. $32x + 33y = 31$

or $32x = 31 - 33y$ ...(1)

$33x + 32y = 34$ ...(2)

Now divide (1) by 32 throughout we have,

$x = \frac{31}{32} - \frac{33y}{32}$ ...(3)

Now substitute this value of x in equation (2) we have,

$\Rightarrow 33\left(\frac{31 - 33y}{32}\right) + 32y = 34$

Now multiply by 32 throughout we have,

$\Rightarrow 33(31 - 33y) + (32)^2 y = 34(32)$

Now simplify this we have,

$\Rightarrow 1023 - 1089y + 1024y = 1088$

$\Rightarrow 1023 - 65y = 1088$

$\Rightarrow 65y = 1023 - 1088 = -65$

Now divide by 65 throughout we have,

$\Rightarrow y = \frac{-65}{65} = -1$

Now substitute this value in equation (3), we have

$\Rightarrow x = \frac{31 - 33(-1)}{32} = 2$

So the values of x and y satisfying the two equations are $(x, y) = (2, -1)$.

Assertion Reason Answers

1. (a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

EXPLANATION: *Pair of linear equations:*

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Consistent

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ *...[unique solution and lines intersects each other.*

and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ *...[infinite solutions and line coincide each other.*

Inconsistent

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ *...[No solution , lines are parallel to each other.*

2. (a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

EXPLANATION: *The equation of the given line is $2x + y - q^2 - 3 = 0$.*

Substituting $x = 3$ and $y = 1$ in the equation, we get

$2(3) + (1) - q^2 - 3 = 0$

$\Rightarrow 7 - q^2 - 3 = 0$

$\Rightarrow q^2 = 4$

$\therefore q = \sqrt{4}$ or ± 2

3. (d) (A) is false, but (R) is true.

4. (c) (A) is true and (R) is false.

We have, $x - 2y + 3 = 0$

$3x + 4y - 11 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}; \frac{b_1}{b_2} = \frac{-2}{4} = \frac{-1}{2}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

It has unique solution and a pair of linear equations represents intersecting lines.

5. (c) (A) is true and (R) is false.

EXPLANATION: *To find the second equation of the pair of dependent linear equations, we can multiply the given equation by a constant and get the second equation.*

Let's try multiplying the given equation by :

$-3x + 5y - 2 = 0$ ...[Given equation

$2(-3x + 5y - 2) = 0$

$-6x + 10y - 4 = 0$

Hence, the second equation of the pair of dependent linear equations is $-6x + 10y - 4 = 0$.

Very Short Answers

1. For the equation, $cx + 3y = c - 3$ and $12x + cy = c$

$\frac{c}{12} = \frac{3}{c} = \frac{c-3}{c}$

$\Rightarrow \frac{c}{12} = \frac{3}{c}$ and $\frac{c}{12} = \frac{c-3}{c}$

$\Rightarrow c^2 = 36$ and $c^2 = 12c - 36$

$\Rightarrow c = \pm 6$ and $c^2 - 12c + 36 = 0$

$\Rightarrow c = \pm 6$ and $(c - 6)^2 = 0$

$\Rightarrow c = \pm 6$ and $c = 6$

$\Rightarrow c = 6$

Alternatively,

If system of equations are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ & they have infinitely many solutions then it must satisfy the following condition:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

From given system of equations,

$a_1 = c, \quad b_1 = 3, \quad c_1 = 3 - c$

and $a_2 = 12, \quad b_2 = c, \quad c_2 = -c$

Putting these values in condition of infinitely many solutions we get,

$\frac{c}{12} = \frac{3}{c} = \frac{(3-c)}{(-c)}$

By solving the first two,

$\frac{c}{12} = \frac{3}{c}$

$\Rightarrow cc = 12 \times 3 \qquad \Rightarrow c^2 = 36$

$\Rightarrow c = \pm\sqrt{(36)} \qquad \therefore c = \pm 6$

2. The given system may be written as

$2x + 3y - 7 = 0$

$(k - 1)x + (k + 2)y - 3k = 0$

The given system of equations is of the form:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, \quad b_1 = 3, \quad c_1 = -7$

$a_2 = (k - 1), \ b_2 = k + 2, \ c_2 = -3k$

For infinite number of solutions, we have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$

$\frac{2}{k-1} = \frac{3}{k+2}$ and $\frac{3}{k+2} = \frac{-7}{-3k}$

$\Rightarrow 2k + 4 = 3k - 3$ and $9k = 7k + 14$

$\Rightarrow k = 7$ and $k = 7$

Therefore, the given system of equations will have infinitely many solutions, if $k = 7$.

3. **Given.** $(k - 1)x + y = k + 1$

and $(k^2 - 1)x + (k + 1)y = 1 - k^2$

$$\frac{a_1}{a_2} = \frac{k-1}{k^2-1}$$

$$= \frac{(k-1)}{(k+1)(k-1)} = \frac{1}{k+1}$$

$$\frac{b_1}{b_2} = \frac{1}{k+1} \text{ and } \frac{c_1}{c_2} = \frac{k+1}{1-k^2}$$

$$= \frac{k+1}{(k+1)(1-k)} = \frac{-1}{(1-k)} = \frac{-1}{(k-1)}$$

as, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ hence for $k > 1$ there is no solution.

4. The equations $(p-3)x + 3y - p = 0$ and $px + py - 12 = 0$.

For infinite many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here $a_1 = p-3$, $b_1 = 3$, $c_1 = -p$,
$a_2 = p$, $b_2 = p$, $c_2 = -12$

$$\frac{p-3}{p} = \frac{3}{p} = \frac{-p}{-12}$$

Solving $\frac{3}{p} = \frac{-p}{-12}$

$\Rightarrow p^2 = 36 \qquad \Rightarrow p = \pm 6$

Now, solving $\frac{p-3}{p} = \frac{-p}{-12}$

$\Rightarrow -12p + 36 = -p^2$

$\Rightarrow p^2 - 12p + 36 = 0$

$\Rightarrow (p-6)^2 = 0 \quad \therefore p = 6$

Hence, the value of $p = 6$.

5. Given linear equations:

$3x - 5y = 4$ ...(i)

$2y + 7 = 9x$ or $9x - 2y = 7$ ...(ii)

Multiply (i) by 3,

$9x - 15y = 12$ ...(iii)

Subtract equation (iii) from (ii) we get

$13y = -5$

$\therefore y = -\frac{5}{13}$

Put the value of y in equation (i)

$$3x - 5\left(\frac{-5}{13}\right) = 4$$

$\therefore x = \frac{9}{13}$

Short Answers

1. $\frac{ax}{b} - \frac{by}{a} = a + b$ ...(i)

$ax - by = 2ab$ ...(ii)

Mutiply (i) by (b), we get

$$\therefore \left(\frac{ax}{b} - \frac{by}{a}\right) b = b(a+b)$$

$$\Rightarrow ax - \frac{b^2y}{a} = ab + b^2 \quad \text{...(iii)}$$

Subtracting (iii) from (ii), get

$$\Rightarrow ax - by - ax + \frac{b^2y}{a} = 2ab - ab - b^2$$

$$\Rightarrow -by + \frac{b^2y}{a} = ab - b^2$$

$$\Rightarrow by\left(\frac{b}{a} - 1\right) = b(a-b)$$

$$\Rightarrow by\left(\frac{b-a}{a}\right) = b(a-b)$$

$\therefore y = -a$

Putting the value y in (ii), we get

$ax - b(-a) = 2ab$

$\Rightarrow ax + ab = 2ab$

$\Rightarrow ax = ab \qquad \therefore x = b$

2. For any pair of linear equation

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Consistent means pair of linear equations will have an unique solution or infinitely many solutions.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (Intersecting lines or Unique Solution)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (Coincident lines or Infinitely many Solutions)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (Parallel lines or No Solution)

(i) $3x + 2y = 5$; $2x - 3y = 7$

$$\frac{a_1}{a_2} = \frac{3}{2};\ \frac{b_1}{b_2} = \frac{2}{(-3)};\ \frac{c_1}{c_2} = \frac{(-5)}{(-7)} = \frac{5}{7}$$

From the above, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, lines are intersecting and have a unique solution.

Hence, the pair of equations is consistent.

(ii) $2x - 3y = 8; 4x - 6y = 9$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{(-6)} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-8}{(-9)} = \frac{8}{9}$$

From the above,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, these lines are parallel and have no solution.
Hence, the pair of equations is inconsistent.

3. (i) Using the condition for coincident lines for l_1 and l_3.

$$\Rightarrow \frac{a_1}{a_3} = \frac{l_1}{l_3} = \frac{c_1}{c_3}$$

On Taking, I and II we have,

$$\frac{a_1}{a_3} = \frac{c_1}{c_3} \quad \Rightarrow \frac{5}{100} = \frac{2p}{240}$$

$$\Rightarrow 100 \times 2p = 5 \times 240$$

$$\Rightarrow 2p = \frac{5 \times 240}{100}$$

$$\Rightarrow 2p = 12$$

$$p = \frac{12}{2} = 6$$

Now, Taking I and II, $\frac{a_1}{a_3} = \frac{b_1}{b_3}$

$$\Rightarrow \frac{5}{100} = \frac{3}{4q} \quad \Rightarrow 20q = 300$$

$$\therefore q = \frac{300}{20} = 15$$

(ii) Using the condition if $l_1 || l_2$

$$\Rightarrow \frac{a_1}{a_2} = \frac{5}{35} = \frac{1}{7}; \quad \frac{b_1}{b_2} = \frac{3}{21} = \frac{1}{7}$$

$$\frac{c_1}{c_2} = \frac{2(6)}{15(6)} = \frac{2}{15};$$

As, $\frac{1}{7} = \frac{1}{7} \neq \frac{2}{15}$ hence it concludes that l_1 parallel to l_2.

4. This system of equation is of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where $a_1 = 2m - 1$, $b_1 = 3$, $c_1 = -5$ and $a_2 = 3$, $b_2 = n - 1$ and $c_2 = -2$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The given system of equations will have infinite number of solutions, if

$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{-5}{-2}$$

$$\Rightarrow \frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$$

$$\Rightarrow \frac{2m-1}{3} = \frac{5}{2} \text{ and } \frac{3}{n-1} = \frac{5}{2}$$

$$\Rightarrow 4m - 2 = 15 \text{ and } 6 = 5n - 5$$

$$\Rightarrow 4m = 17 \text{ and } 5n = 11$$

$$\Rightarrow m = \frac{17}{4} \text{ and } n = \frac{11}{5}$$

Hence, the given system of equations will have infinite number of solutions, if $m = \frac{17}{4}$ and $n = \frac{11}{5}$.

5. Given equations of lines are:

$$3x + 4y = 9 \quad ...(1)$$

$$\text{and } y = mx + 1 \quad ...(2)$$

Substituting (2) in (1)

$$3x + 4(mx + 1) = 9$$
$$3x + 4mx + 4 = 9$$
$$\Rightarrow x(3 + 4m) = 5$$
$$\Rightarrow x = \left(\frac{5}{3 + 4m}\right)$$

For x to be an integer, $3 + 4m$ must evenly divide 5 *i.e.*, $3 + 4m$ must be a factor of 5.

So,

$3 + 4m = 5$	$3 + 4m = -5$	$3 + 4m = 1$	$3 + 4m = -1$
$m = \frac{(5-3)}{4}$	$m = \frac{(-5-3)}{4}$	$m = \frac{(1-3)}{4}$	$m = \frac{(-1-3)}{4}$
$m = \frac{1}{2}$	$m = -2$	$m = -\frac{1}{2}$	$m = -1$

Thus, the integral values of m are -1 and -2.

Therefore, the number of integral values of m are 2.

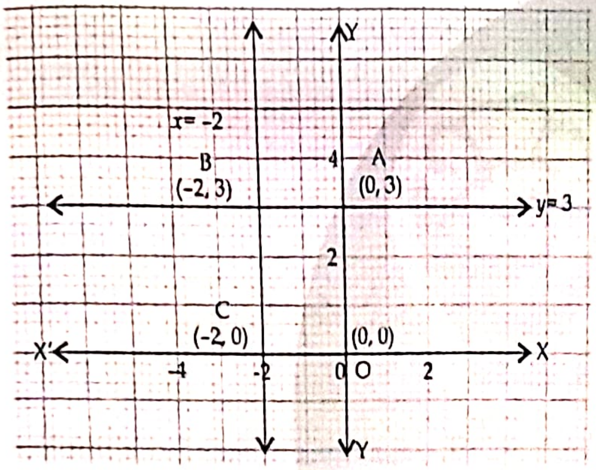
Long Answers

1. The graph of $x = -2$ is a line parallel to the y-axis at a distance of 2 units to the left of it.

   Hence, the line l is the graph of $x = -2$.

   The graph of $y = 3$ is a line parallel to the x-axis at a distance of 3 units above it.

   Hence, the line m is the graph of $y = 3$.



   A is a point on the y-axis at a distance of 3 units above the x-axis.

   Thus, the coordinates of A are (0, 3).

   C is a point on the x-axis at a distance of 2 units to the left of the y-axis.

   Thus, the coordinates of C are (-2, 0).

   B is the solution of the pair of equations $x = -2$ and $y = 3$.

   Thus, the coordinates of B are (-2, 3).

   Hence, The vertices of the rectangle OABC are O (0, 0), A (0, 3), B (-2, 3), C (-2, 0).

   The length and breadth of this rectangle are 2 units and 3 units.

   Area of a rectangle = *length* × *breadth.*

   The area of rectangle OABC = 2 × 3 = 6 **sq. units.**

   Therefore, the area of rectangle OABC = 2 × 3 = 6 sq. units.

2. We have the linear equations as,

   $2x + y = 23$ ...(*i*)

   $4x - y = 19$ ...(*ii*)

   Adding the equation (*i*) and (*ii*),

   $6x = 42$

   $\therefore\ x = 7$

   Substituting x value in (*i*), we get,

   $2(7) + y = 23$

   $14 + y = 23$

   $y = 23 - 14$

   $\therefore\ y = 9$

   Substituting the values of x and y in $5y - 2x$ and $\left(\frac{y}{x}\right) - 2$, we get,

   $5y - 2x = 5 \times 9 - 2 \times 7$

   $= 45 - 14 = \mathbf{31}$

   and $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{-5}{7}$

   Therefore, the values of $(5y - 2x)$ and $\frac{y}{x} - 2$ are 31 and $\frac{-5}{7}$ respectively.

3. Let, the sum of age of two children be 'x' years the age of father be 'y' years.

   Then $y = 3x$ ...(*i*)

   After 5 years,

   the sum of children's age = $(x + 5 + 5)$

   $= (x + 10)$ yrs .

   Father's age = $(y + 5)$ yrs.

   Then, $y + 5 = 2(x + 10)$ ...(*ii*)

   $\Rightarrow y + 5 = 2x + 20$

   $\Rightarrow 3x + 5 = 2x + 20$ **...[From (*i*)**

   $\Rightarrow x = 15$

   From eq. (*i*), $y = 45$

   Hence, the present age of father is 45 years.

4. Let the average speed while walking $= x$ km/h

   and the average speed of Bus = y km/h

   $\frac{3}{x} + \frac{12}{y} = 1.5$

   According to Question

   $\frac{3}{x} + \frac{12}{y} = \frac{3}{2}$ ...(*i*) **...[Time = $\frac{\text{Distance}}{\text{Speed}}$**

   and $\frac{5}{x} + \frac{10}{y} = 2$ ...(*ii*)

   Let $\frac{1}{x} = m$ and $\frac{1}{y} = n$

$\Rightarrow 3m + 12n = \frac{3}{2}$ | $5m + 10n = 2$

$\Rightarrow 2(3m + 12n) = 3$ ...(iii) | ...(iv)

Now, solving (iii) and (iv) by elimination we get,

$\Rightarrow 5\,(6m + 24n = 3)$ ...(v)

$\Rightarrow 30m + 120n = 15$

$\Rightarrow 6\,(5m + 10n = 2)$

$\Rightarrow 30m + 60n = 12$ ...(vi)

$\Rightarrow 30m + 120n = 15$

$\Rightarrow 30m + 60n = 12$ ...(vii)

$\quad - \quad - \quad -$

$60n = 3$

$\therefore n = \frac{3}{60} = \frac{1}{20}$

Putting value of n in equation (iii)

$\Rightarrow 6m + 24\left(\frac{1}{20}\right) = 3$

$\Rightarrow 6m + \frac{6}{5} = 3$

$\Rightarrow \frac{30m + 6}{5} = 3$

$\Rightarrow 30m + 6 = 15$

$\Rightarrow 30m = 15 - 6$

$\Rightarrow 30m = 9 \qquad \therefore m = \frac{9}{30} = \frac{3}{10}$

Now,

$\frac{1}{x} = m \qquad \frac{1}{y} = n$

$\frac{1}{x} = \frac{3}{10} \qquad \frac{1}{y} = \frac{1}{20}$

$x = \frac{10}{3}$ km/h $\qquad y = 20$ km/h

So, Average speed while walking = 10 km/h

Average speed of Bus = 20 km/h

5. Let the two digit number be $10y + x$

So, the unit's digit be x and ten's digit $= y$

Then, $y = 2x$

Number obtained by interchanging its digits be $10x + y$. ...(i)

Then, $(10y + x) - (10x + y) = 36$

$\Rightarrow 10y + x - 10x - y = 36$

$9y - 9x = 36\,;\ y - x = 4$ ...(ii)

Solving eq. (i) and eq. (ii)

$2x - x = 4 \qquad \therefore x = 4$

Putting the value of x in eq. (i)

$y = 8$

Hence, the original number is

$10y + x = 84$

Case Based Answers

1. (a) Akanksha kept book for 4 days.

Amount paid by Akanksha for first 2 days $= x$

Amount paid by Akanksha for next 2 days $= 2y$

Required equation, $x + 2y = 16$

(b) (i) From above questions we get

$x + 2y = 16$ ...(i)

$x + 4y = 22$ ...(ii)

Subtracting (ii) from (i),

$x + 2y - x - 4y = 16 - 22$

$-2y = -6 \quad \therefore y = 3$

Putting $y = 3$ in eq (i),

$x + 2 \times 3 = 16 \quad \Rightarrow x + 6 = 16$

$\therefore x = 10$

Thus, fixed charges for a book = ₹10

Or

(ii) Total amount paid by Akanksha for two more days.

$(16 + 2 \times 3) =$ ₹22

Total amount paid by Aditi for two more days.

$(22 + 2 \times 3) =$ ₹28

$\therefore$ **Total amount paid by both of them,**

$22 + 28 =$ ₹50

(c) Aditi kept book for 6 days.

Amount paid by Aditi for first 2 days $= x$

Amount paid by Aditi for next 4 days $= 4y$

A.T.Q, $x + 4y = 22$

2. (a) Since, each poor child pays x and each rich child pays y.

In batch I, 20 poor and 5 rich children pays 9000 can be represented as $20x + 5y = 9000$ and In batch II, 5 poor and 25 rich children pays 26,000 can be represented as $5x + 25y = 26{,}000$.

(b) (i) As we have, $20x + 5y = 9000$ ...(1)

and $5x + 25y = 26000$

or $x + 5y = 5{,}200$ ...(2)

On subtracting (2) from (1), we get

$\Rightarrow 19x = 3800$

$\therefore x = ₹200$

Monthly fee paid by a poor child $= ₹200$

Or

(ii) Put the value of x in equation (ii), we get

$200 + 5y = 5200$

$5y = 5200 - 200$

$y = 1000$

$y - x = 1000 - 200 = ₹800$

Hence, difference in the monthly fee paid by a poor child and a rich child is ₹800.

(c) Total monthly fee $= 10x + 20y$

$= 10(200) + 20(1000)$

$= 2{,}000 + 20{,}000 = ₹22{,}000$

3. (a) Algebraic equations are:

$5x + 4y = ₹9500$ ...(i)

and $4x + 3y = ₹7370$ ...(ii)

(b) (i) Multiply by 3 in equation (i) and by 4 in equation (ii)

$15x + 12y = 28{,}500$ ...(iii)

$16x + 12y = 29{,}480$ ...(iv)

On subtracting equation (iii) from equation (iv), we get

$x = 980$

$\therefore$ **Prize amount for hockey $= ₹980$**

Or, (ii) Now, put this value in equation (i), we get

$5 \times 980 + 4y = 9500$

$\Rightarrow 4y = 9500 - 4900 = 4600$

$\Rightarrow y = 1150$

Prize amount for cricket $= 1150$

Difference $= 1150 - 980 = 170$

$\therefore$ **Prize amount for cricket is ₹170 more than hockey.**

(c) Total prize amount for 2 students each from two games

$= 2x + 2y$

$= 2(x + y) = 2(980 + 1150)$

$= 2 \times 2130 = ₹4260$

(DAY 31 SWAHA)

* * * * *


10th
33 DAYS CHALLENGE
DAILY TARGETS
MATHEMATICS
"100% Swaha"
10th
33 DAYS CHALLENGE
DAILY TARGETS
SOCIAL SCIENCE
"100% Swaha"
10th
33 DAYS CHALLENGE
DAILY TARGETS
SCIENCE
"100% Swaha"

14 Quadratic Equations

What did CBSE ask last year?

MCQs	1 Question ($1 \times 1 = 1$ Mark)
Subjective	No Very Short Questions
	1 Short Question ($1 \times 3 = 3$ Marks)
	1 Long Question ($1 \times 5 = 5$ Marks)
Case Based	No Case Based Question Asked

Note: All the above typology of questions include 'Competency based Questions' labelled as **COMPETENCY**.

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Introduction to the chapter

- A quadratic polynomial of the form $ax^2 + bx + c$, where $a \neq 0$ and a, b, c are real numbers is called a quadratic equation.
  When $ax^2 + bx + c = 0$.

  Here a and b are the coefficients of x^2 and x respectively and 'c' is a constant term.
- Any value is a solution of a quadratic equation if and only if it satisfies the quadratic equation.

Note: Very short & short questions come frequently from this topic.

QUADRATIC

EQUATIONS

Nature of Roots

- Quadratic formula. The roots, i.e., α and β of a quadratic equation $ax^2 + bx + c = 0$ are given by

  $$\frac{-b \pm \sqrt{D}}{2a} \text{ or } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ provided } b^2 - 4ac = 0$$

  Here the value $b^2 - 4ac$ is known as the discriminant and is generally denoted by D. 'D' helps us to determine the nature of roots for a given quadratic equation. Thus, $D = b^2 - 4ac$.

  The rules are:

  - If $D = 0 \Rightarrow$ The roots are Real and Equal.
  - If $D > 0 \Rightarrow$ The two roots are Real and unequal.
  - If $D < 0 \Rightarrow$ No Real roots exist.

- If α and β are the roots of the quadratic equation, then quadratic equation is

  $$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

  Or x^2 - (sum of roots) x + product of roots = 0

  ...where $\left[\begin{array}{l} \text{Sum of roots } (\alpha + \beta) = \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \dfrac{-b}{a} \\[2em] \text{Product of roots } (\alpha\beta) = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{c}{a} \end{array}\right.$

Note: *Situational problems can be asked as short questions & case based questions as well.*

OBJECTIVE QUESTIONS

(DAY 32)

Multiple Choice Questions

Q.1. If the quadratic equation $ax^2 + bx + c = 0$ has two real and equal roots, then prime c is equal to [CBSE 2023

(a) $\frac{b}{2a}$ (b) $\frac{b}{7a}$

(c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

Q.2. If 'p' is a root of the quadratic equation $x^2 - (p + q)x + k = 0$ then the value of 'k' is COMPETENCY

(a) $-pq$ (b) pq

(c) $p + q$ (d) $pq - p^2$

FREE ADVICE: We can use the fact that the sum of the roots of a quadratic equation is equal to the negative coefficient of the linear term.

Q.3. What is the quadratic equation in x whose roots are 2 and –5. [CBSE 2021

(a) $x^2 + 3x - 10$ (b) $x^2 + 3x + 10$

(c) $x^2 + 4x - 10$ (d) $x^2 + 3x - 20$

Q.4. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - x - 4$. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$. [CBSE 2024

(a) $\frac{15}{4}$ (b) $\frac{16}{4}$

(c) $\frac{14}{4}$ (d) 8

Q.5. If one zeros of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of k. [CBSE 2021

(a) 10 (b) 12

(c) –10 (d) 14

Q.6. For what values of k are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ are real and equal? COMPETENCY

(a) ± 10 (b) ± 12

(c) ± 9 (d) ± 8

FREE ADVICE: For the roots of a quadratic equation to be real and equal, the discriminant (Δ) must be equal to zero. The discriminant is given by the formula: $\Delta = b^2 - 4ac$.

Q.7. Which of the following is a quadratic equation? COMPETENCY

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$

(b) $-2x^2 = (5 - x)\left[2x - \left(\frac{2}{5}\right)\right]$

(c) $(k + 1)x^2 + \left(\frac{3}{2}\right)x = 7$, where $k = -1$

(d) $x^3 - x^2 = (x - 1)^3$

Q.8. Which of the following equations has the sum of its roots as 3? COMPETENCY

(a) $2x^2 - 3x + 6 = 0$

(b) $-x^2 + 3x - 3 = 0$

(c) $\sqrt{2x^2} - \frac{3}{\sqrt{2x}} + 1 = 0$

(d) $3x^2 - 3x + 3 = 0$

Q.9. The sum of a number z and its reciprocal is 4. Which of these correctly represents the above statement? COMPETENCY

(a) $z^2 + 1 = 4$ (b) $z^2 + z = 4$

(c) $z^2 + 1 = 4z$ (d) $z^2 + 1 = -4z$

Q.10. Look at the quadratic equation below:

$-y^2 + 8y - 18 = 0$

Which of these can be said about the nature of roots of the above quadratic equation? COMPETENCY

(a) Real and unequal roots

(b) Real and equal roots

(c) No real roots

(d) Cannot say

Q.11. The quadratic equation $x^2 + 8x + h = 0$ has equal roots. COMPETENCY

Which of these is the value of h?

(a) 8 (b) 0

(c) 4 (d) 16

Q.12. If the quadratic equation $px^2 - 2\sqrt{5}\,px + 15 = 0$ has two equal roots, then find the value of p. COMPETENCY

(a) 2 (b) 4

(c) 3 (d) 5

FREE ADVICE: If a quadratic equation has two equal roots, it means that the discriminant (Δ) of the equation is equal to zero.

Q.13. The value of λ for which $(x^2 + 4x + \lambda)$ is a perfect square, is: *[CBSE 2020*

(a) 16 (b) 9

(c) 1 (d) 4

Q.14. Which of these equations will definitely have no real roots, for any value of a and b other than zero? COMPETENCY

(i) $2x^2 - bx - b^2 = 0$

(ii) $a^2 x - ax + 2 = 0$

(iii) $x^2 + ax + b = 0$

(a) only (i) (b) only (ii)

(c) only (iii) (d) both (i) and (ii)

Q.15. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real? COMPETENCY

(a) $k < 4$ (b) $k > 4$

(c) $k \leq 4$ (d) $k \geq 4$

FREE ADVICE: For the roots of a quadratic equation to be real, the discriminant (Δ) must be greater than or equal to zero.

Q.16. Find the nature of the roots of the quadratic equation $2x^2 - 4x + 3 = 0$. *[CBSE 2019*

(a) Two real roots

(b) One real roots

(c) No real roots

(d) Infinite roots

Q.17. Write the discriminant of the quadratic equation $(x + 5)^2 = 2(5x - 3)$. *[CBSE 2019*

(a) 124 (b) 136

(c) –124 (d) 130

Q.18. When does a quadratic equation have only one real root?

(a) When the discriminant is zero.

(b) When the discriminant is positive.

(c) When the discriminant is negative.

(d) When the leading coefficient is zero.

Q.19. Which of these is a QUADRATIC equation having one of its roots as zero? COMPETENCY

(i) $x^3 + x^2 = 0$

(ii) $x^2 - 2x = 0$

(iii) $x^2 - 9 = 0$

Option:

(a) only (i) (b) only (ii)

(c) only (i) and (ii) (d) only (ii) and (iii)

Assertion Reason Questions

In the following question, a statement of Assertion (A) is followed by statement of Reason (R). Mark the correct choice as.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

Q.1. Assertion: If $5 + \sqrt{7}$ is a root of a quadratic equation with rational co-efficients, then its other root is $5 - \sqrt{7}$.

Reason (R): Square roots of a quadratic equation with rational co-efficients occur in conjugate pairs.

Q.2. Assertion: The equation $x^2 + 5x + 1 = (x - 4)^2$ is a quadratic equation.

Reason: Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$ is called a quadratic equation.

Q.3. Assertion: If $2x^2 + 3x + 4 = 0$ and $ax^2 + bx + c = 0$ have a common root, then $a : b : c = 2 : 3 : 4$ (a, b and c are real numbers). COMPETENCY

Reason: For a quadratic equation in x with real coefficients, complex roots occur in conjugate pairs.

Q.4. Assertion: $3x^2 - 6x + 3 = 0$ has repeated roots.

Reason: The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.

Q.5. Assertion: The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary.

Reason: If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

SUBJECTIVE QUESTIONS

Very Short Answer Questions

Q.1. Find the sum and product of the roots of the quadratic equation $2x^2 + 9x + 4 = 0$. [CBSE 2023

Q.2 One of the roots of the quadratic equation $ax^2 + 4x + a = 0$ is (-2).
Find the value of a. Show your work. COMPETENCY

Q.3. Given below is an equation, where p is a real number.
$px^2 + 4x + 4 = 0$
For what value of p will this equation not be a quadratic equation? Justify your answer. COMPETENCY

Q.4. Solve: $\sqrt{2x+9} + x = 13$. COMPETENCY

Q.5. Solve for x:

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$ [CBSE 2016

Q.6. For what value of k, are the roots of the quadratic $(k+4)x^2 + (k+1)x + 1 = 0$ equal? COMPETENCY

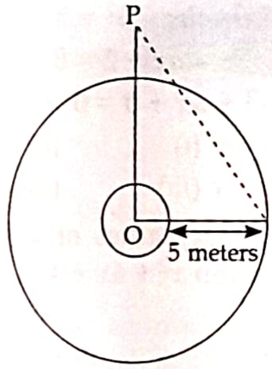
Q.7. Check whether the given equations is quadratic or not: $(x + 2)^3 = x^3 - 4$ [NCERT

Q.8. Find the nature of the roots of the quadratic equation $4x^2 - 5x + 3 = 0$. [CBSE 2023

Q.3. One of the roots of the quadratic equation $x^2 + 12x - k = 0$, is thrice the other. Find the value of k? [CBSE 2023

Q.4. One root of the quadratic equation $2x^2 - 8x - k$ is $\frac{5}{2}$. Find the other root and the value of k. COMPETENCY

Q.5. A circular garden has a concentric circular fountain area with centre O. The distance between the circumference of the fountain area and that of the garden is 5 meters as shown in the figure below.



The height of the fountain OP is 5 times the radius of the fountain area.
Find the radius of the fountain area if the shortest distance between the top of the fountain and the circumference of the garden is 17 meters. Show your work. [CBSE 2024

Q.6. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained. COMPETENCY

Q.7. Find the value of 'p' for which the quadratic equation $px(x - 2) + 6 = 0$ has two equal real roots. COMPETENCY

Short Answer Questions

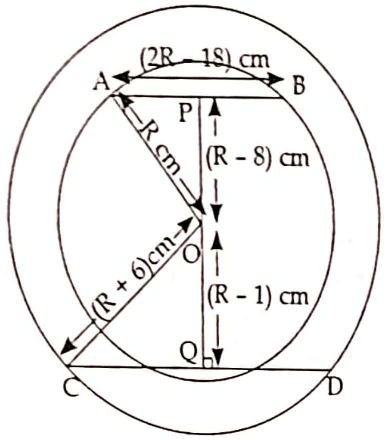
Q.1. If α and β are roots of the quadratic equation $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are α^2 and β^2. [CBSE 2023

Q.2. The sum of two numbers is 15. If the sum of their reciprocal is $\frac{3}{10}$. Find smallest number. [CBSE 2023

(DAY 33)

Long Answer Questions

Q.1. In the figure below, two concentric circles have centre O. Their radii are R and (R + 6)cm respectively.



(**Note:** The figure is not to scale.)

Find the lengths of the chords AB and CD. Show your work. COMPETENCY

Q.2. Solve for: $\frac{(x+1)}{(x-1)} - \frac{(x-1)}{(x+1)} = \frac{5}{6}$, $x \neq 1, -1$.

[CBSE 2018

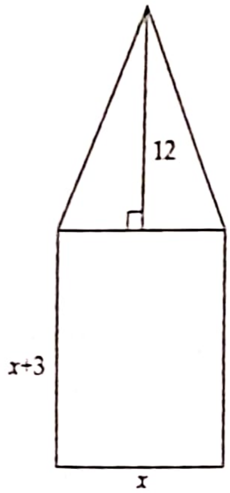
Q.3. If α, β, are zeroes of polynomial $p(x) = 5x^2 + 5x + 1$ then find the value of

(*i*) $\alpha^2 + \beta^2$

(*ii*) $\alpha^{-1} + \beta^{-1}$

COMPETENCY

Q.4. A rectangular park is to be designed whose breadth is 3m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.



[CBSE 2024

Q.5. At present Aditi's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Aditi and Nisha.

[NCERT EXEMPLAR

CASE BASED QUESTIONS

Q.1. Raj and Ajay are very close friends. Both their families decides to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km.

Based on the above situation, answer the following questions:

(*a*) What will be the distance covered by Ajay's car in two hours?

(*b*) (*i*) Write quadratic equations which describes the speed of Raj's car?

Or

(*ii*) What is the speed of Raj's car?

(*c*) How much time did Ajay take to travel 400 km? COMPETENCY

Q.2. The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.

Based on the situation, answer the following questions:

(*a*) Let speed of the stream be x km/hr. then what will be the speed of the motorboat in upstream?

(*b*) (*i*) What is the relation between speed, distance and time?

Or

(*ii*) What is the speed of current?

(*c*) How much time did the boat take going downstream? **COMPETENCY**

Q.3. Due to some technical problems, an aeroplane started late by one hour from its starting point. The pilot decided to increase the speed of the aeroplane by 100 km/hr from its usual speed, to cover a journey of 1200 km in time.

Based on the above situation. Now, answer the following questions:

(*a*) What is the nature of the roots of the quadratic equation formed in this situation?

(*b*) What is the values of k for which the difference between the roots of the equation $x^2 + kx + 3 = 0$ is 2.

(*c*) What is the values of m and n if $x = 2$ and $x = 3$ are solutions of the equation $3x^2 - mx + 2n = 0$.

COMPETENCY

*****

ANSWERS

Multiple Choice Answers

1. (*d*) For the equation to have equal roots, the discriminant must be equal to zero.

$D = b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac \qquad \Rightarrow c = \frac{b^2}{4a}$

∴ Option D is correct.

2. (*b*) We have, $x^2 - (p+q)x + k = 0$

On comparing with $ax^2 + bx + c = 0$, we have

$a = 1, b = -(-p+q), c = k$

As we know,

$\alpha + \beta = \frac{-b}{a} = \frac{-[-(p+q)]}{1}$

$\Rightarrow \alpha + \beta = p + q \qquad ...(i)$

But $\alpha = p \qquad ...(ii)$ [Given

On putting value of α in (*i*), we have

$p + \beta = p + q$

$\beta = q \qquad ...(iii)$

$\alpha\beta = \frac{c}{a}$

$p \cdot q = \frac{k}{1}$ ...[From (*ii*) & (*iii*)

3. (*a*) Sum of the roots $= 2 + (-5) = -3$

Product of the roots $= 2 \times (-5) = -10$

Hence the required Quadratic equation is $x^2 -$ (sum of the zeros)x + product of the zeroes $= 0$

$= x^2 - (-3)x + (-10) = 0$

$= x^2 + 3x - 10$

4. (*a*) Given, $f(x) = x^2 - x - 4$

Sum of zeros $= \alpha + \beta = 1$

Product of zeros $= \alpha\beta = -4$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

$= \left\{\frac{(\alpha+\beta)}{\alpha\beta}\right\} - \alpha\beta = \left(\frac{1}{-4}\right) - (-4) = \frac{15}{4}$

5. (*c*) If one zero of the quadratic equation is 2 that means

$(2)^2 + 3 \times 2 + k = 0$

$\Rightarrow 4 + 6 + k = 0$

$\therefore k = -10$

6. (*c*) As we know, for real equal roots

$D = b^2 - 4ac = 0$

here, $b = 2k, a = 3, c = 27$

putting these values

$D = (2k)^2 - (4 \times 3 \times 27)$

$0 = 4k^2 - 324$

$\Rightarrow 4k^2 = 324$

$\Rightarrow k = \sqrt{81}$

$\therefore k = \pm 9$

7. (*d*) Taking option (*d*),

$x^3 - x^2 = (x-1)^3$

$x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$2x^2 - 3x + 1 = 0$

which represents a quadratic equation.

8. (*b*) Taking option (*b*), $-x^2 + 3x - 3 = 0$

Sum of the roots $= -\left(\frac{-b}{a}\right) = -\left(\frac{3}{-1}\right) = 3$

9. (*c*) According to question, $\frac{z}{1} + \frac{1}{z} = 4$

$\Rightarrow \frac{z^2+1}{z} = 4$

$\therefore z^2 + 1 = 4z$

10. (*c*) We have, $-y^2 + 8y - 18 = 0$

$D = b^2 - 4ac$

$\Rightarrow D = (8)^2 - 4(-1)(-18)$

$\Rightarrow D = 64 - 72$

$\Rightarrow D = -8 \qquad \Rightarrow D < 0$

Therefore, no real roots.

11. (*d*) $x^2 + 8x + h = 0$

For equal roots, $D = 0$

$b^2 - 4ac = 0$

$(8)^2 - 4(1)(h) = 0$

$\Rightarrow 64 - 4h = 0 \qquad \Rightarrow -4h = -64$

$\therefore h = \frac{64}{4} = 16$

12. (*c*) If in equation $ax^2 + bx + c = 0$ the two roots are equal.

Then $b^2 - 4ac = 0$

In equation $px^2 - 2\sqrt{5}px + 15 = 0$

$a = p, b = -2\sqrt{5}$ and $c = 15$

Then $b^2 - 4ac = 0$

$\Rightarrow \left(-2\sqrt{5}p\right)^2 - 4 \times p \times 15 = 0$

$\Rightarrow 20p^2 - 60p = 0$

$\Rightarrow 20p(p-3) = 0$

So when $p - 3 = 0$

$\Rightarrow p = 3$

$p \neq 0$ as it makes coefficient of $x^2 = 0$.

Hence, $p = 3$

13. (*d*) **Given:** $x^2 + 4x + \lambda$

On comparing with $ax^2 + bx + c$

Here, $a = 1$, $b = 4$, $c = \lambda$

If $x^2 + 4x + \lambda = 0$ then it is a perfect square and has equal roots.

D(discriminant) = $b^2 - 4ac$

$D = 4^2 - 4 \times 1 \times \lambda$

$D = 16 - 4\lambda$

$D = 0$

$0 = 16 - 4\lambda$

$\Rightarrow 4\lambda = 16$

$\therefore \lambda = \frac{16}{4} = 4$

14. (*b*) Taking option (*b*), $a^2x - ax + 2 = 0$

$D = b^2 - 4ac$

$\Rightarrow D = (-a)^2 - 4(a^2)(2)$

$\Rightarrow D = a^2 - 8a^2$

$\Rightarrow D = -7a^2$

$D < 0$ hence, no real roots.

15. (*c*) For a Quadratic Equation

$ax^2 + bx + c = 0$ to have real roots,

$b^2 - 4ac \geq 0$

$\Rightarrow b^2 \geq 4ac$

$\Rightarrow 16 \geq 4k$

$\therefore$ $k \leq 4$

16. (*c*) Let's find the value of $b^2 - 4ac$ by substituting values of *a*, *b* and *c* in

$2x^2 - 4x + 3 = 0$

$(-4)^2 - 4(2)(3)$

$\Rightarrow 16 - 24 = -8$

The value of $b^2 - 4ac < 0$.

Hence the roots are **not real.**

17. (*c*) $(x+5)^2 = 2(5x-3)$

$\Rightarrow x^2 + 10x + 25 = 10x - 6$

...$[(a+b)^2 = a^2 + 2ab + b^2$

$x^2 + 31 = 0$

Here $a = 1$, $b = 0$ and $c = 31$

$D = 0 - (4 \times 1 \times 31)$

$\therefore$ **D = –124**

18. (*a*) A quadratic equation has only one real root when the discriminant is zero.

19. (*b*) Taking Option (*b*),

As $x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, x = 2$

Assertion Reason Answers

1. (*a*) *Explanation.*

x^2 – (sum of roots)x + (*product of roots*) = 0

In this case, the sum of the roots is

$5+\sqrt{7}+5-\sqrt{7} = 10$.

The product of the roots is $(5+\sqrt{7})(5-\sqrt{7})$, which equals $25 - 7 = 18$.

So, the equation becomes:

$x^2 - 10x + 18 = 0$

Now, if you solve this equation, you will find that the roots are indeed $5+\sqrt{7}$ and $5-\sqrt{7}$.

2. (*d*) *Explanation*: We have,

$x^2 + 5x + 1 = (x-4)^2$

$\Rightarrow x^2 + 5x + 1 = x^2 - 8x + 16$

$13x - 15 = 0$

It is not of the form $ax^2 + bx + c = 0$

3. (*a*) *Explanation*: $2x^2 + 3x + 4 = 0$.

$$x = \frac{-3 \pm \sqrt{9-32}}{2 \times 2}$$

$\therefore$ Roots of the quadratic are complex.

$\therefore$ Both root are common in the equations $2x^2 + 3x + 4 = 0$ and $ax^2 + bx + c = 0$.

Because complex roots occur in pairs.

$\therefore \frac{a}{2} = \frac{b}{3} = \frac{c}{4}$

$\therefore a : b : c = 2 : 3 : 4$

4. (*c*) *Explanation.* Assertion $3x^2 - 6x + 3 = 0$

$D = b^2 - 4ac = (-6)^2 - 4(3)(3)$

$= 36 - 36 = 0$

Roots are repeated as D = 0.

Reason is false, as roots are equal for D = 0

5. (*a*) *Explanation.* $x^2 + 2x + 2 = 0$

$D = b^2 - 4ac \quad \Rightarrow D = (2)^2 - 4 \times 1 \times 2$

$= 4 - 8 = -4 < 0$ **Discriminant,**

As we know roots are imaginary when $D < 0$. So, both A and R are correct and R explains A.

Very Short Answers

1. **Given.** $2x^2 + 9x + 4 = 0$

$\therefore$ **Sum of roots** $= \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$= \dfrac{-9}{2}$

and **Product of roots** $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$= \dfrac{4}{2} = \mathbf{2}$

2. We have, $ax^2 + 4x + a = 0$ ...(*i*)

Here one root $(x) = -2$

Putting value of x in (*i*), we get

$a(-2)^2 + 4(-2) + a = 0$

$a(4) - 8 + a = 0$

$\Rightarrow 5a - 8 = 0 \quad \Rightarrow 5a = 8$

$\Rightarrow a = \dfrac{8}{5}$

3. We have, $px^2 + 4x + 4 = 0$

For $p = 0$ equation will not be a quadratic equation as if $p = 0$ then it will make term with degree 2 as zero, the equation will turn into linear equation.

4. **Given.**

$\sqrt{2x+9} + x = 13$ ...(*i*)

$\sqrt{2x+9} = 13 - x$

On squaring, we get

$2x + 9 = x^2 + 169 - 26x$

$\Rightarrow x^2 + 160 - 28x = 0$

$\Rightarrow x^2 - 20x - 8x + 160 = 0$

$\Rightarrow (x - 20)(x - 8) = 0$

$\therefore x = 20$ or $x = 8$

But $x = 20$, does no satisfy (*i*)

$\therefore$ **$x = 8$ is the solution of (*i*)**

5. **Given.** $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Multiplying the equation by $\sqrt{3}$, we get

$3x^2 - 2 \times \sqrt{3} \times \sqrt{2} \times x - 6 = 0$

$(\sqrt{3}x)^2 - 2 \times \sqrt{3} \times \sqrt{2} \times x + (\sqrt{2})^2 - (\sqrt{2})^2 - 6 = 0$

$(\sqrt{3}x - \sqrt{2})^2 - 2 - 6 = 0$

$(\sqrt{3}x - \sqrt{2})^2 = 8$

$(\sqrt{3}x - \sqrt{2}) = \pm 8$

$(\sqrt{3}x - \sqrt{2}) = 8$ or $(\sqrt{3}x - \sqrt{2}) = -8$

$\therefore x = \dfrac{(8 + \sqrt{2})}{\sqrt{3}}$ or $x = \dfrac{(-8 + \sqrt{2})}{\sqrt{3}}$

6. As roots are equal

$\Rightarrow D = b^2 - 4ac = 0$

$\Rightarrow (k + 1)^2 - 4(k + 4) \times 1 = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k + 1 = 16$

$\Rightarrow (k - 1)^2 = 4^2$

$\therefore k - 1 = \pm 4$

$\Rightarrow$ **$k = 1 + 4, 1 - 4$ or $k = 5, -3$**

7. Here, LHS $= (x + 2)^3 = x^3 + 6x^2 + 12x + 8$

...$[(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Now A.T.Q, $x^3 + 6x^2 + 12x + 8 = x^3 - 4$

$6x^2 + 12x + 12 = 0$

$\Rightarrow x^2 + 2x + 2 = 0$

The equation is of the form $ax^2 + bx + c = 0$

Hence, the given equation is a **quadratic equation**.

8. **Given.** $4x^2 - 5x + 3 = 0$

Comparing with $ax^2 + bx + c = 0$,

we get, $a = 4$, $b = -5$, $c = 3$.

Therefore,

$D = b^2 - 4ac$

$\Rightarrow D = 25 - 4(4)(3)$

$= 25 - 48 = -23; \ 23 < 0$

Hence, the given equation has **no real** roots.

Short Answers

1. **Given.** $x^2 - 7x + 10 = 0$

$\Rightarrow x^2 - 2x - 5x + 10 = 0$

$x(x - 2) - 5(x - 2) = 0$

$\Rightarrow (x - 2)(x - 5) = 0 \quad \Rightarrow x = 2, 5$

$\alpha = 2, \beta = 5$

Sum of the roots, $s = \alpha^2 + \beta^2 = 2^2 + 5^2 = 29$

Product of the roots $= \alpha^2\beta^2 = (10)^2 = 100$

Required polynomial, $x^2 + 29x + 100$

2. Let first number = x
then the other number = $15 - x$
According to the question

$\frac{1}{x}+\frac{1}{15}-x=\frac{3}{10}$

$10(15 - x + x) = 3(x(15 - x))$
$150 = 45x - 3x^2$
$3x^2 - 45x + 150 = 0$
$x^2 - 15x + 50 = 0$
$x^2 - 10x - 5x + 50 = 0$
$(x - 10)(x - 5) = 0$
$\therefore$ **$x = 10, x = 5$**

3. Let α be the one root of the equation.
So, other root = 3α

Sum of roots = $\frac{-b}{a}$, where, $b = 12, a = 1$

$\Rightarrow \alpha + 3\alpha = -12$

$\Rightarrow 4\alpha = -12$

$\Rightarrow \alpha = -3$

Product of roots = $\frac{c}{a}$ ..(*i*)

$\Rightarrow \alpha \times 3\alpha = -k$ ...*where* [$c = -k, a = 1$

$\Rightarrow 3 \times (3)^2 = -k$

$\therefore$ **$k = -27$** ...**[From (*i*)**

4. We have, $2x^2 - 8x - k$ and $\alpha = \frac{5}{2}$

$\therefore \alpha + \beta = \frac{-b}{a}$

$\Rightarrow \frac{5}{2} + \beta = -\frac{(-8)}{2}$

$\Rightarrow \beta = \frac{8}{2}-\frac{5}{2}$ $\therefore \beta = \frac{3}{2}$

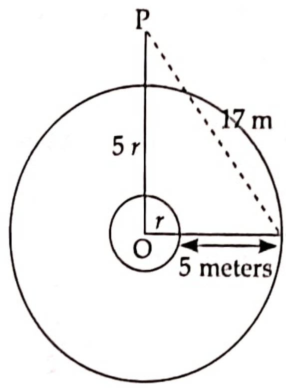
Since $\frac{5}{2}$ is a root of $2x^2 - 8x - k$, therefore putting $x = \frac{5}{2}$ in the equation will be = 0
Substituting value of x.

$$2\left(\frac{5}{2}\right)^2-8\left(\frac{5}{2}\right) - k = 0$$

$\therefore k = \frac{-15}{2}$

Similarly, if we put $x = \frac{3}{2}$, then k will be $\frac{-15}{2}$.

5. Let the radius of fountain area = r metres



Height of fountain (OP) = $5r$ metres
Shortest distance between top and circumference of garden (PQ) = 17 metres
Radius of circular garden (OQ)= $(r + 5)$ m
Now, In Right angled triangle OPQ,

$(H)^2 = (B)^2 + (P)^2$

$(PQ)^2 = (OQ)^2 + (PO)^2$

$\Rightarrow (17)^2 = (r + 5)^2 + (5r)^2$

$\Rightarrow 289 = r^2 + 25 + 10r + 25r^2$

$\Rightarrow 26r^2 + 10r + 25 - 289 = 0$

$\Rightarrow 26r^2 + 10r - 264 = 0$

$\Rightarrow 2(13r^2 + 5r - 132) = 0$

$\Rightarrow 13r^2 + 5r - 132 = 0$

$\Rightarrow 13r^2 + 44r - 39r - 132 = 0$

$\Rightarrow (13r^2 + 44r) + (-39r - 132) = 0$

$\Rightarrow r(13r + 44) - 3(13r + 44) = 0$

$\Rightarrow 13r + 44 = 0$ or $r - 3 = 0$

$\Rightarrow 13r = -44$ $\therefore r = 3$

$\therefore r = -\frac{44}{13}$

...**[Rejected, (As r cannot be negative)**

Hence, the Radius of fountain area is 3 metres.

6. $x^2 + px + 16 = 0$ have equal roots if
$D = p^2 - 4(16)(1) = 0$
$p^2 = 64$
$\Rightarrow p = \pm 8$
$\therefore x^2 \pm 8x + 16 = 0$
$\Rightarrow (x \pm 4)^2 = 0$ $\Rightarrow x \pm 4 = 0$
$\therefore$ **Roots are $x = -4$ and $x = 4$**

7. Given quadratic equation, $px(x - 2) + 6 = 0$
$\Rightarrow px^2 - 2px + 6 = 0$ ...(*i*)
By comparing equation (*i*) with
$ax^2 + bx + c = 0$, we get

$a = p$, $b = -2p$ and $c = 6$.
Since, given that the given quadratic equation has equal roots.

$\therefore \quad D = b^2 - 4ac = 0$

...[$\because$ for equal roots D = 0

$\Rightarrow \quad (-2p)^2 - 4 \times p \times 6 = 0$

$\Rightarrow \quad 4p^2 - 24p = 0 \quad \Rightarrow 4p(p - 6) = 0$

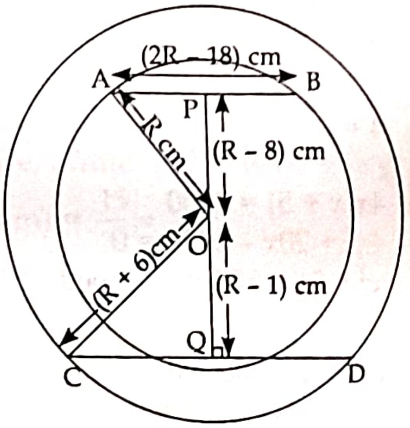
$\Rightarrow \quad p = 0$ or $p - 6 = 0$

$\Rightarrow \quad p = 0$ or $p = 6$ but $p \neq 0$

($\because$ if $p = 0$ then given equation is not quadratic equation).

Long Answers

1. As the perpendicular drawn from centre of circle to chord bisects the chord.



$AP = \frac{1}{2} \times AB$

$\Rightarrow \quad AP = \frac{1}{2}(2R - 18)$

$AP = \frac{1}{2} \times 2(R - 9)$

AP = (R – 9) cm

Now, In ΔAOP,

$(H)^2 = (B)^2 + (P)^2$

...[Using pythagoras theorem

$(OA)^2 = (OP)^2 + (AP)^2$

$(R)^2 = (R - 8)^2 + (R - 9)^2$

$R^2 = R^2 + 64 - 2(8)(R) + R^2 + 81 - 2(R)(9)$

$\Rightarrow \quad R^2 = R^2 + 64 - 16R + R^2 + 81 - 18R$

$\Rightarrow \quad R^2 - 34R + 145 = 0$

$\Rightarrow \quad R^2 - 29R - 5R + 145 = 0$

$\Rightarrow \quad R(R - 29) - 5(R - 29) = 0$

$\Rightarrow \quad (R - 29)(R - 5) = 0$

$\Rightarrow \quad R - 29 = 0$ or $R - 5 = 0$

$\Rightarrow \quad R = 29$ or $R = 5$

If R = 29,	If R = 5,
AB = (2R – 18)	AB = (2R – 18)
= (2(29) – 18)	= 2(5) – 18
= 58 – 18	= 10 – 18 = –8
AB = 40 cm	**(Rejected as it gives negative value)**

Hence, R = 29 cm

and OQ = (R – 1) = (29 – 1) = **28 cm**

OC = (R + 6) = (29 + 6) = 35 cm

Now, In ΔCOQ

$CQ = \sqrt{(35)^2 - (28)^2}$

...[Using pythagoras theorem

$\Rightarrow \quad CQ = \sqrt{1225 - 784}$

$\Rightarrow \quad CQ = \sqrt{441} \qquad \therefore CQ = 21$ cm

Now, CD = 21 × 2 = 42 cm

...[CQ × 2 = CD, Perpendicular from centre to chord bisects the chord.

$\therefore$ **CD = 42 cm**

2. we have, $\frac{[(x+1)^2 - (x-1)^2]}{(x-1)(x+1)} = \frac{5}{6}$

$\Rightarrow 6(4x) = 5(x^2 - 1)$

$\Rightarrow 5x^2 - 24x - 5 = 0$

$\Rightarrow 5x^2 - 25x + x - 5 = 0$

$\Rightarrow 5x(x - 5) + 1(x - 5) = 0$

$\Rightarrow (5x + 1)(x - 5) = 0$

$\therefore$ **$x = -\frac{1}{5}$ and $x = 5$**

3. The given polynomial,

$p(x) = 5x^2 + 5x + 1$

Since α and β are the zeroes of $p(x)$,

$\alpha + \beta = \frac{-5}{5}$

$\Rightarrow \alpha + \beta = -1$ ...(1)

and $\alpha\beta = \frac{1}{5}$ ...(2)

(*i*) Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-1)^2 - 2\left(\frac{1}{5}\right)$

$= 1 - \frac{2}{5} = \frac{(5-2)}{5} = \frac{3}{5}$

(*ii*) Now, $\alpha^{-1} + \beta^{-1}$

$$= \frac{1}{\alpha}+\frac{1}{\beta} = \frac{(\beta+\alpha)}{(\alpha\beta)}$$

$$= \frac{(-1)}{\left(\frac{1}{5}\right)} = -5$$

4. Let the length of the rectangular park be x m then, breadth be $(x - 3)$ m

$\therefore$ Area of rectangular park = $x(x - 3)$ m^2

Area of isosceles triangular park

$$= \frac{1}{2} \times (x - 3) \times 12\text{m}^2 = 6(x - 3)\ \text{m}^2$$

According to the given condition,

$x(x - 3) - 6(x - 3) = 4$

$x^2 - 3x - 6x + 18 = 4$

$x^2 - 9x + 14 = 0$

$x^2 - 7x - 2x + 14 = 0$

$x(x - 7) - 2(x - 7) = 0$

$(x - 2)(x - 7) = 0$

$x = 2$ or 7 $\qquad \therefore x = 7$ m

$x \neq 2$(As breadth can't be negative)

and $x - 3 = (7 - 3)$m = 4 m

Hence, length and breadth of the rectangular park is 7 m and 4 m respectively.

5. Let the age of Nisha be x.

Then, Aditi's age = $x^2 + 2$

As per given condition,

$(x^2 + 2) + [(x^2 + 2) - x] = 10x - 1$

$2x^2 + 4 - x - 10x + 1 = 0$

$2x^2 - 11x + 5 = 0$

On factoring,

$2x^2 - 10x - x + 5 = 0$

$2x(x - 5) - 1(x - 5) = 0$

$(2x - 1)(x - 5) = 0$

Now, $2x - 1 = 0$

$2x = 1$

$\Rightarrow x = \frac{1}{2}$

Also, $x - 5 = 0$

$x = 5$

Age of Nisha = $\frac{1}{2}$ years is not possible.

So, the age of Nisha = 5 years.

Age of Aditi = $(5)^2 + 2$

= 25 + 2 = 27 years.

Therefore, the present ages of Nisha and Aditi are 5 years and 27 years.

Case Based Answers

1. (*a*) As Ajay travels 5 km/h faster than Raj, then speed of Ajay's car = $(5 + x)$km/h

We can say, in 1 hour Ajay travels $(5 + x)$ km.

Then in 2 hours, Ajay will travel = $2(5 + x)$ km

Thus the distance covered by Ajay's car in 2 hours is **$2(x + 5)$ km**.

(*b*) (*i*) As we know, Time = $\frac{\text{Distance}}{\text{Speed}}$

$$\Rightarrow 4 = \frac{400}{x} - \frac{400}{(x+5)}$$

$$\Rightarrow 4 = \frac{(400[x + 5 - x])}{(x(x + 5))}$$

$\Rightarrow 4x(x + 5) = 2000$

$\Rightarrow 4x^2 + 20x - 2000 = 0$

$\Rightarrow$ $\mathbf{x^2 + 5x - 500 = 0}$ (speed of Raj's car)

Or

(*ii*) $x^2 + 5x - 500 = 0$

$\Rightarrow (x + 25)(x - 20) = 0$

$\Rightarrow x = -25$ or $x = 20$

As speed can not be negative, so speed of Raj's car, $\mathbf{x = 20}$

(*c*) Speed of Ajay's car = $(5 + x)$ km/hour

from last answer we know,

$x = 20$ km/hour

Thus, speed of Ajay's car = (20 + 5)

= 25 km/hour

We can say, Ajay travels 25 km in 1 hour

Then Ajay travels in 1 km = $\frac{1}{25}$ hour

$\therefore$ Ajay travels in 400 km = $\frac{400}{25}$ hours

= 16 hours

Ajay took 16 hours to travel 400 km.

2. (*a*) **According to the question:**

Speed of motor boat in upstream

= $(20 - x)$ km/hr

(b) (i) Time = $\frac{\text{Distance}}{\text{Speed}}$

Or

(ii) According to Question,

$$\frac{15}{20-x}-\frac{15}{20+x}=1$$

$\Rightarrow \frac{20+x-(20-x)}{(20)^2-(x)^2}=\frac{1}{15}$

$\Rightarrow 2x \times 15 = 400 - x^2$

$\Rightarrow x^2 + 30x - 400 = 0$

$\Rightarrow x^2 + 40x - 10x - 400 = 0$

$\Rightarrow x(x + 40) - 10(x + 40) = 0$

$\Rightarrow$ **$x = 10$ and $x = -40$**

...[Since speed cannot be -ve

$\therefore$ **Speed of Current, $x = 10$**

(c) Distance = 15 km

Speed = 20 + 10 = 30 km/h

Hence, Time (boat took going down-stream) = $\frac{15}{30}=\frac{1}{2}$ hr.

3. (a) According to the question,

$$(x + 100)\left(\frac{1200}{x}-1\right) = 1200$$

$\Rightarrow (x + 100)(1200 - x) = 1200x$

$\Rightarrow 1200x - x^2 + 120000 - 100x = 1200x$

$\Rightarrow x^2 + 100x - 120000 = 0$

The quadratic equation formed in this situation is $x^2 + 100x - 120000 = 0$

Discriminant, D = $(100)^2 - 4 \times 1 \times (-120000)$

= 10000 + 480000

= 490000 > 0

As D > 0, roots are real and distinct.

(b) The roots of the given quadratic equation $x^2 + kx + 3 = 0$

$$x = \frac{(-b\pm\sqrt{(b^2-4ac)}}{2a}=\frac{-k\pm\sqrt{k^2-12}}{2}$$

The difference between the roots = 2

$\Rightarrow \frac{(-k+\sqrt{k^2-12})}{2}+\frac{(k+\sqrt{(k^2-12)}}{2}=2$

$\Rightarrow \frac{2\sqrt{(k^2-12)}}{2}=2$

Therefore, $\sqrt{(k^2-12)}=2$

Squaring both sides, we get $k^2 - 12 = 4$

$\Rightarrow k^2 = 16$ $\quad\Rightarrow$ **$k = \pm 4$**

(c) It is given that $x = 2$ and $x = 3$ are solutions of the equation

$3x^2 - mx + 2n = 0.$

$\Rightarrow 3(2)^2 - 2m + 2n = 0$ ...(i)

and $3(3)^2 - 3m + 2n = 0$ ...(ii)

On solving equation (i), we get

$12 - 2m + 2n = 0$

$\Rightarrow 2m - 2n = 12$

$\Rightarrow m - n = 6$ ...(iii)

Now, on solving equation (ii), we get

$27 - 3m + 2n = 0$

$\Rightarrow 3m - 2n = 27$ ...(iv)

Solving the equations (iii) and (iv),

$$\begin{array}{r} 3m - 3n = 18 \\ \pm 3m \mp 2n = \pm 27 \\ \hline -n = -9 \end{array}$$

$\therefore n = 9$

and putting value of n in (iii), we get

$m - 9 = 6$ $\quad\therefore$ **$m = 15$**

(DAY 33 SWAHA)

* * * * *

DAY 33

"Congratulate yourself on completing your 33 days journey. Share your experience with others via video review on 'Amazon' , 'FlipKart' , and 'Instagram'—

@padhle.akshay."

— Akshay Bhaiya

1 Sample Question Paper

Time allowed : 3 hours *Maximum marks : 80*

General Instructions:

1. *This question paper contains 38 questions.*
2. *This Question Paper is divided into 5 **Sections A, B, C, D** and E.*
3. *In **Section A**, Questions no. **1-18** are multiple choice questions (MCQs) and Q. 19 and Q. 20 are Assertion-Reason based questions of 1 mark each.*
4. *In **Section B**, Questions no. **21-25** are very short answer (VSA) type questions, carrying **02** marks each.*
5. *In **Section C**, Questions no. **26-31** are short answer (SA) type questions, carrying **03** marks each.*
6. *In **Section D**, Questions no. **32-35** are long answer (LA) type questions, carrying **05** marks each.*
7. *In **Section E**, Questions no. **36-38** are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.*
8. *All Questions are compulsory. However, an internal choice in **2 Questions of Section B**, **2 Questions of Section C** and **2 Questions of Section D** has been provided. An internal choice has been provided in all the **2 marks questions of Section E**.*
9. *Draw neat and clean figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.*
10. *Use of calculators is not allowed.*

SECTION-A

Section A consists of 20 questions of 1 mark each.

Q.1. What is the greatest possible speed at which a driver can drive a car 190 km and 342 km in an exact number of hours? 1

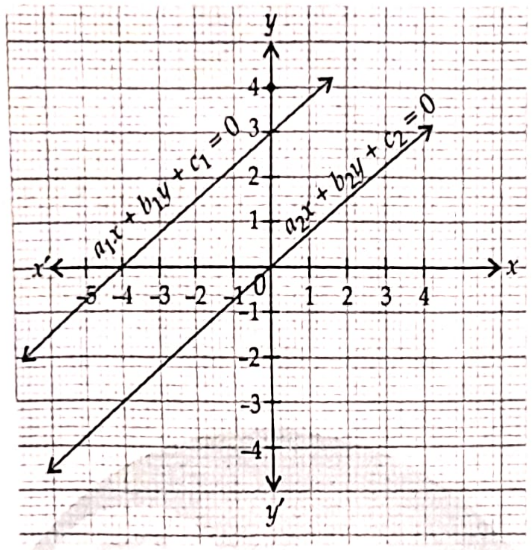
(*a*) 38 km/hr
(*b*) 68 km/hr
(*c*) 32 km/hr
(*d*) 28 km/hr

Q.2. Find the value of k for which the following system of equations has no solutions, find the value of k. 1

$2x + ky = 1;$ $3x - 5y = 7$

(*a*) $k \neq -10/3$
(*b*) $k = -5/7$
(*c*) $k = -10/3$
(*d*) $k = 10/3$

Q.3. The given pair of linear equations is non-intersecting. Which of the following statements is true? **1**



(a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (c) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (d) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Q.4. Two APs have the same common difference. The first term of one of these is –1 and that of the other is –8. Then the difference between their 4^{th} terms is **1**

(a) 1 (b) –7 (c) 7 (d) 9

Q.5. If the mean of the following distribution is 2.6, then the value of y is: **1**

Variable (x)	1	2	3	4	5
Frequency	4	5	y	1	2

(a) 3 (b) 8 (c) 13 (d) 24

Q.6. The roots of the equation $x^2 - 3x - m(m + 3) = 0$, where m is a constant is: **1**

(a) m (b) $-m$ (c) $m + 3$ (d) $m + 3, -m$

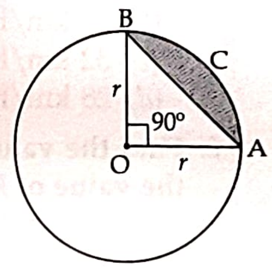
Q.7. If the quadratic equation $x^2 - 8x + k = 0$ has real roots, then **1**

(a) $k < 16$ (b) $k \leq 16$ (c) $k > 16$ (d) $k \geq 16$

Q.8. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$ is the first negative term? **1**

(a) 26 (b) 27 (c) 28 (d) 29

Q.9. In the given figure, the area of the segment ACB is **1**



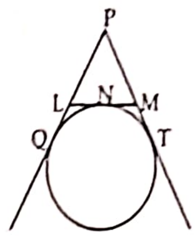
(a) $\frac{r^2}{4}(\pi - 2)$ (b) $\frac{r^2}{4}(\pi + 3)$

(c) $\frac{r^2}{4}(\pi - 2)$ (d) $\frac{r^2}{4}(\pi + 3)$

Q.10. In which quadrant the point P that divides the line segment joining the points A(2, –5) and B(5, 2) in the ratio 2 : 3 lies? **1**

(a) I^{st} (b) II^{nd} (c) III^{rd} (d) IV^{th}

Q.11. If PQ = 28 cm, then find the perimeter of ΔPLM. 1

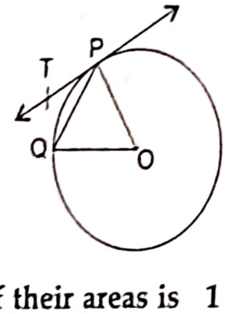


(a) 28 cm (b) 42 cm

(c) 56 cm (d) 70 cm

Q.12. In the given figure, O is the centre of a circle, PQ is a chord and PT is the tangent at P.

If $\angle POQ = 70°$, then calculate $\angle TPQ$. 1



(a) 65° (b) 55°

(c) 45° (d) 35°

Q.13. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is 1

(a) 22 : 7 (b) 14 : 11 (c) 7 : 22 (d) 11 : 14

Q.14. The value of $\cos 1°.\cos 2°.\cos 3°.\cos 4°$ .... $\cos 90°$ is 1

(a) 1 (b) 0 (c) -1 (d) 2

Q.15. If $5 \tan \beta = 4$, then $\dfrac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} =$ 1

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) 6

Q.16. Two concentric circles are of radii 7 cm and r cm respectively, where $r > 7$. A chord of the larger circle of length 48 cm touches the smaller circle. Find the value of r. 1

(a) 20 cm (b) 21 cm (c) 24 cm (d) 25 cm

Q.17 The diameter and height of a cylinder and a cone are equal. Write the ratio of volume of cylinder to the volume of the cone. 1

(a) 3:1 (b) 1:3 (c) 2:1 (d) 1:2

Q.18. A number x is selected at random from the numbers 1, 2, 3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16. 1

(a) 0.1 (b) 0.2 (c) 0.4 (d) 0.5

DIRECTION: *In the question numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option–*

(a) *Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).*

(b) *Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).*

(c) *Assertion (A) is true but Reason (R) is false.*

(d) *Assertion (A) is false but Reason (R) is true.*

Q.19. **Assertion (A):** The probability of getting a bad egg in a lot of 400 is 0.035. The number of good egg in the lot is 386. 1

Reason (R): If the probability of an event is p, the probability of its complementary event will be $1 - p$.

Q.20. **Assertion (A):** The point (0, 4) lies on y-axis. 1

Reason (R): The x-coordinate on the point on y-axis is zero.

SECTION-B

Section B consists of 5 questions of 2 marks each.

Q.21. Find the smallest pair of 4-digit numbers such that the difference between them is 303 and their HCF is 101. Show your steps. **2**

Q.22. A 3.5 cm chord subtends an angle of 60° at the centre of a circle. **2**

What is the arc length of the minor sector? Draw a rough figure and show your steps.

Note: Take π as $\frac{22}{7}$.

Or

The length of the minute hand of a clock is 6 cm. Find the area swept by it when it moves from 7:05 p.m. to 7:40 p.m.

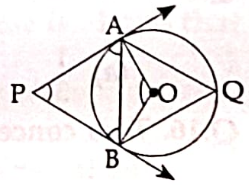
Q.23. Find the zeroes of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeroes and the co-efficients of the polynomial. **2**

Q.24. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B < 90°$; $A > B$, find A and B. **2**

Or

If $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$, and $x \sin \theta = y \cos \theta$, then prove that $x^2 + y^2 = 1$.

Q.25. In the given figure, O is the centre of circle. Find $\angle AQB$, given that PA and PB are tangents to the circle and $\angle APB = 75°$. **2**



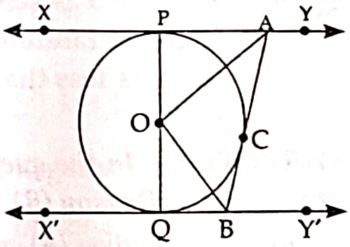
SECTION-C

Section C consists of 6 questions of 3 marks each.

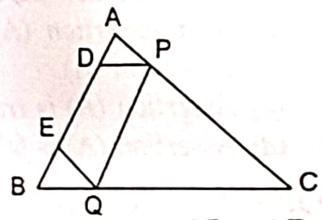
Q.26. Prove that a parallelogram circumscribing a circle is a rhombus. **3**

Or

In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B, what is the measure of $\angle AOB$?



Q.27. There are two points D and E on side AB of ΔABC such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$. **3**



Or

$\Delta ABC \sim \Delta PQR$. AD is the median to BC and PM is the median to QR. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Q.28. Prove that $7 - 6\sqrt{5}$ is an irrational number given that $\sqrt{5}$ is an irrational number. **3**

Q.29. A flight left 30 minutes later than the scheduled time and in order to reach its destination 1500 km away in time it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed. **3**

Q.30. If (–5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots; then find the values of p and k. 3

Q.31. Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is 3

(*i*) 8?

(*ii*) 13?

(*iii*) less than or equal to 12?

SECTION-D

Section D consists of 4 questions of 5 marks each.

Q.32. Water is flowing at the rate of 15 km/hour through a pipe of diameter 14 cm into a cuboid pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm? 5

Q.33. Find the mean of the following frequency distribution: 5

Class interval	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54	55 – 59
Frequency	14	22	16	6	5	3	4

Or

If the mode of the following distribution is 57.5, find the value of x. 5

C.I.	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
f	6	10	16	x	10	5	2

Q.34. From the top of a vertical tower, the angles of depression of two cars, in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. [Use $\sqrt{3} = 1.73$] 5

Or

The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 cm vertically above the first, its angle of elevation is 30°. Find the height of the tower.

Q.35. A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8 : 5, then find the ratio between the radius of their bases to their heights. 5

SECTION-E

Case study based questions are compulsory.

Q.36. A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, painfings etc. Shown here is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons.

A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern.

For Solutions

2 Sample Question Paper

Time allowed : 3 hours *Maximum marks : 80*

General Instructions:

1. *This question paper contains 38 questions.*
2. *This Question Paper is divided into 5* **Sections A, B, C, D** *and E.*
3. *In* **Section A**, *Questions no.* **1-18** *are multiple choice questions (MCQs) and* **Q. 19** *and* **Q. 20** *are Assertion-Reason based questions of* **1** *mark each.*
4. *In* **Section B**, *Questions no.* **21-25** *are very short answer (VSA) type questions, carrying* **02** *marks each.*
5. *In* **Section C**, *Questions no.* **26-31** *are short answer (SA) type questions, carrying* **03 marks each.**
6. *In* **Section D**, *Questions no.* **32-35** *are long answer (LA) type questions, carrying* **05 marks each.**
7. *In* **Section E**, *Questions no.* **36-38** *are case study based questions carrying* **4** *marks each with sub parts of the values of 1, 1 and 2 marks each respectively.*
8. *All Questions are compulsory. However, an internal choice in* **2 Questions of Section B, 2 Questions of Section C** *and* **2 Questions of Section D** *has been provided. An internal choice has been provided in all the* **2 marks questions of Section E.**
9. *Draw neat and clean figures wherever required. Take* $\pi = \frac{22}{7}$ *wherever required if not stated.*
10. *Use of calculators is not allowed.*

SECTION-A

Section A consists of 20 questions of 1 mark each.

Q.1. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is **1**

(*a*) ab (*b*) a^2b^2 (*c*) a^3b^2 (*d*) a^3b^3

Q.2. If the difference of Mode and Median of a data is 24, then the difference of median and mean is **1**

(*a*) 8 (*b*) 12 (*c*) 24 (*d*) 36

Q.3. Let p be a prime number. The quadratic equation having its roots as factors of p is **1**

(*a*) $x^2 - px + p = 0$ (*b*) $x^2 - (p + 1)x + p = 0$

(*c*) $x^2 + (p + 1)x + p = 0$ (*d*) $x^2 - px + p + 1 = 0$

Q.4. The heights of plants in Dipti's garden are recorded in the table given below. The median plant height is 55 cm. **1**

Heights of plants (in cm)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of plants	$2x$	4	$4x$	8	4

Which of the following is the value of x?

(a) 1

(b) 2

(c) 8

(d) The value of x cannot be found without knowing the total number of plants.

Q.5. Find the value of m so that the quadratic equation $mx(x - 7) + 49 = 0$ has two equal roots. 1

(a) 0, 4 (b) 0 (c) 4 (d) 2

Q.6. Find how many two-digit numbers are divisible by 6? 1

(a) 12 (b) 13 (c) 14 (d) 15

Q.7. Find the common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots\ldots$. 1

(a) 1 (b) –1 (c) 2 (d) –2

Q.8. If the quadratic equation $x^2 + 4x + k =$ has real and equal roots, then 1

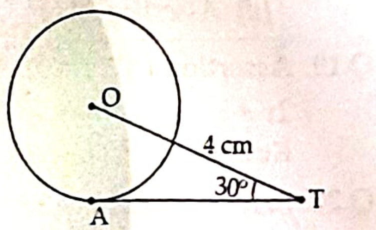
(a) $k < 4$ (b) $k > 4$ (c) $k = 4$ (d) $k \geq 4$

Q.9. The length of tangent drawn to a circle of radius 9 cm from a point 41 cm from the centre is: 1

(a) 40 cm (b) 9 cm

(c) 41 cm (d) 50 cm

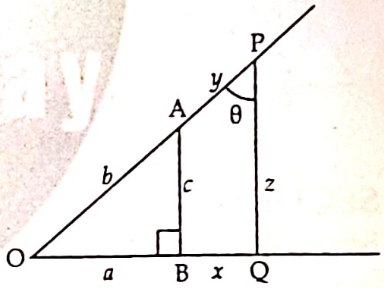
Q.10. In the given figure, AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°. Then AT is equal to 1



(a) 4 cm (b) 2 cm

(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

Q.11. In the figure shown here, lines AB and PQ are parallel to each other. All measurements are in centimetres. 1
Which of the following gives the value of cos θ?



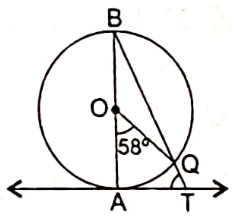
(a) $\frac{b}{c}$ (b) $\frac{c}{b}$

(c) $\frac{c}{b+y}$ (d) $\frac{a+x}{b+y}$

Q.12. If $3x = \text{cosec}\ \theta$ and $\frac{3}{x} = \cot\theta$, find the value of $3\left(x^2 - \frac{1}{x^2}\right)$. 1

(a) 3 (b) $\frac{-1}{3}$ (c) $\frac{1}{3}$ (d) –3

Q.13. In the given Fig., AB is the diameter of a circle with centre O and AT is a tangent. If ∠AOQ = 58°, find ∠ATQ. 1



(a) 59° (b) 51°

(c) 61° (d) 69°

Q.14. The volumes of two spheres are in the ratio 64 : 27. Find their radii if the sum of their radii is 21 cm. 1

(a) 9, 12 (b) 12, 9 (c) 15, 6 (d) 18, 3

Q.15. If cos (A + B) = 0 and sin (A – B) = $\frac{1}{2}$, then the value of A and B where A and B are acute angles are: 1

(a) 60°, 30° (b) 30°, 60° (c) 45°, 45° (d) 90°, 0°

Q.16. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park is **1**

(*a*) 10 m (*b*) 15 m (*c*) 20 m (*d*) 24 m

Q.17. A cylinder, a cone and a hemisphere are of equal base and have the same height. What is the ratio of their volumes? **1**

(*a*) 3 : 1 : 2 (*b*) 1 : 2 : 3 (*c*) 3 : 2 : 1 (*d*) 2 : 3 : 1

Q.18. A box contains cards numbered 3, 5, 7, 9, ..., 35, 37. A card is drawn at random from the box. Find the probability that the number on the drawn card is a prime number. **1**

(*a*) $\frac{11}{35}$ (*b*) $\frac{11}{18}$ (*c*) $\frac{5}{18}$ (*d*) $\frac{1}{3}$

DIRECTION: *In the questions number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option–*

(*a*) *Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).*

(*b*) *Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).*

(*c*) *Assertion (A) is true but Reason (R) is false.*

(*d*) *Assertion (A) is false but Reason (R) is true.*

Q.19. Assertion (A): The value of $q = \pm 2$, if $x = 3$, $y = 1$ is the solution of the line $2x + y - q^2 - 3 = 0$. **1**

Reason (R): The solution of the line will satisfy the equation of the line.

Q.20. Statement A (Assertion) : If in a ΔABC, a line DE ||BC, intersects AB in D and AC in E, then $\frac{AB}{AD} = \frac{AC}{AE}$. **1**

Statement R (Reason) : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

SECTION-B

Section B consists of 5 questions of 2 marks each.

Q.21. A forester wants to plant 66 apple trees, 88 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also, he wants to make distinct roots of the trees (only one type of tree in one row). Find the minimum number of rows required. **2**

Q.22. If a point A(0, 2) is equidistant from the points B(3, p) and C(p, 5), then find the value of p. **2**

Or, Find the value of y for which the distance between the points A(3, –1) and B(11, y) is 10 units.

Q.23. If α and β are zeroes of $p(x) = kx^2 + 4x + 4$, such that $\alpha^2 + \beta^2 = 24$, find k. **2**

Q.24. If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$. **2**

Or, Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

Q.25. A semicircle MON is inscribed in another semicircle. Radius OL of the larger semicircle is 6 cm.
Find the area of the shaded segment in terms of π. Draw a rough figure and show your steps. **2**

SECTION-C

Section C consists of 6 questions of 3 marks each.

Q.26. Prove that $\sqrt{5}$ is an irrational number. **3**

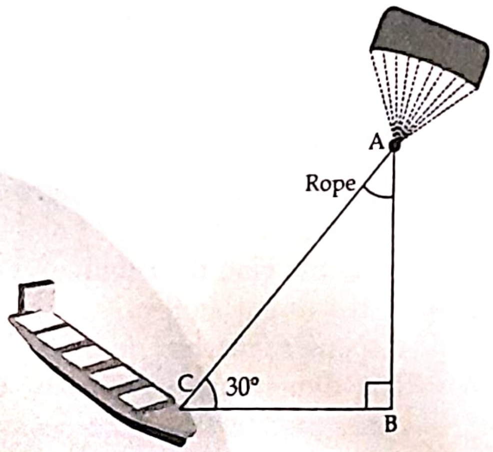
Q.27. Solve the following for x: $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$. **3**

Or, For what values of k, the roots of the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ are equal?

Q.28. 'Skysails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively. **3**

Based on the given figure related to sky sailing, answer the question:

What should be the length of the rope of the kite sail in order to pull the ship at the angle 30° and be at a vertical height of 200 m?

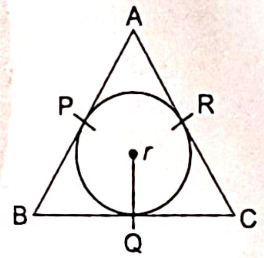


Q.29. In the given figure, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively. **3**

Prove that:

(*i*) AB + CQ = AC + BQ

(*ii*) Area (ΔABC) = $\frac{1}{2}$(Perimeter of ΔABC) × r



Q.30. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD produced to E. Prove that EL = 2 BL. **3**

Q.31. Cards numbered from 11 to 60 are kept in a box. If a card is drawn at random from the box, find the probability that the number on the drawn card is: **3**

(*i*) an odd number
(*ii*) a perfect square number
(*iii*) divisible by 5
(*iv*) a prime number less than 20

Or

Five cards—the ten, jack, queen, king and ace of diamonds, are well shuffled with their faces downwards. One card is then picked up at random.

(*a*) What is the probability that the drawn card is the queen?

(*b*) If the queen is drawn and put aside, and a second card is drawn, find the probability that the second card is

(*i*) an ace (*ii*) a queen.

SECTION-D

Section D consists of 4 questions of 5 marks each.

Q.32. Find the value of p, if the mean of the following distribution is 18: **5**

x_i	13	15	17	19	$20 + p$	23
f	8	2	3	4	$5p$	6

Or

The following table shows the marks obtained by 100 students of Class X in a school during a particular academic session. find the mode of this distribution.

Marks	*No. of Students*
Less than 10	7
Less than 20	21
Less than 30	34
Less than 40	46
Less than 50	66
Less than 60	77
Less than 70	92
Less than 80	100

Q.33. (*a*) From the top of a hill, the angles of depression of two consecutive kilometer stones due east are found to be 30° and 45°. Find the height of the hill. **3**

(*b*) A kite is flying at a height of 45 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string. **2**

Q.34. A restaurant stores ice-cream in a box with a dispenser attached for filling ice-cream cones. The dimensions of the box and the ice-cream cones used by the restaurant are shown in Figure 1. To make each serving of dessert, the cone is first filled with ice-cream and then topped with a hemispherical scoop of ice-cream taken from the same box, as shown in Figure 2. **5**

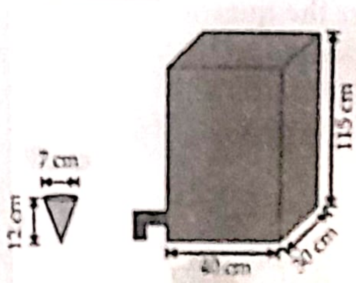


Figure 1

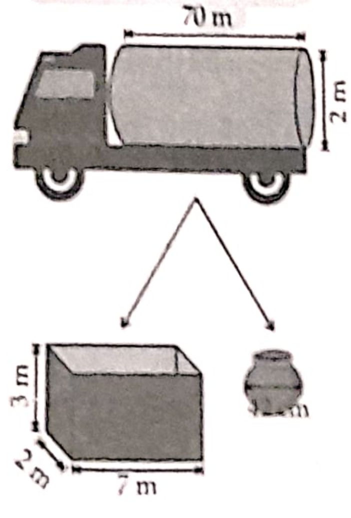

Figure 2

Note: The figures are not to scale.

Approximately how many desserts can be served out of a completely filled box of ice-cream? Show your steps.

Note: (Take π as $\frac{22}{7}$).

Or, A right-circular cylindrical water tanker supplies water to colonies on the outskirts of a city and to nearby villages. Each colony has a cuboidal water tank. In villages, people come with matkas (spherical clay pots) to fill water for their household.



Note: The figures are not to scale.

(i) How many colonies in total would one full tanker be able to supply?

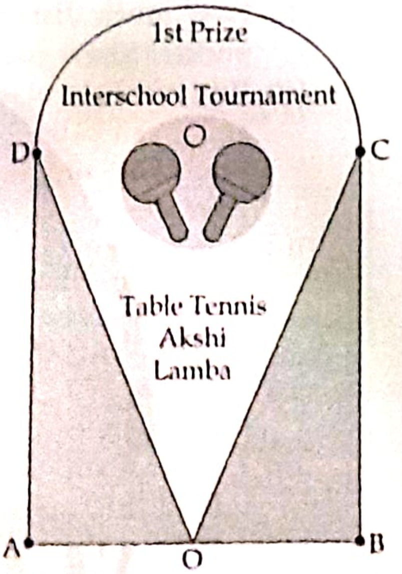
(ii) If a tanker supplies water to 3 colonies and then goes to a village where 400 people fill their matkas, roughly how much water is supplied by the tanker in all? Give your answer in m^3. Show your work.

Note: Assume all the tanks/matkas are completely filled without any loss of water; Take π as $\frac{22}{7}$; Use $1000000\ cm^3 = 1\ m^3$.

35. Students of a class are made to stand in rows. If 4 students are extra in a row, there would be 2 rows less. If 4 students are less in a row, there would be 4 more rows. Find the number of students in the class. 5

SECTION-E

Case study based questions are compulsory.



Q.36. Shown here is the trophy shield Akshi received on winning an international Table tennis tournament. The trophy is made of a glass sector DOC supported by identical wooden right triangles ΔDAO and ΔCOB. Also, $AO = 7$ cm and $AO : DA = 1 : \sqrt{3}$ (Use $\sqrt{3} = 1.73$)

Based on the given information, answer the following questions:

(i) Find $\angle DOC$. 1

(ii) Find the area of the wooden triangles. 1

(iii) Find the area of the shape formed by the glass portion. 2

Or

If Akshi wants to decorate the boundary of the glass portion with glitter tape, then find the length of the tape she needs.

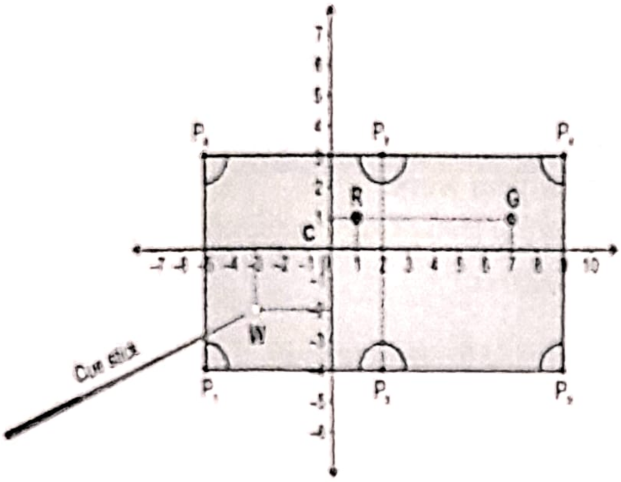
Q.37. Answer the following questions based on the information given below.

Raycasting is a technique used in the creation of computer games. The basic idea of raycasting is as follows: the map is a 2D square grid. Using rays generated from an object, this 2D map can be transformed into a 3D perspective. One of the methods involves sending out a ray from the player's location. To determine how far he/she is from a wall or an obstacle, the distance between the player's coordinates and the coordinate of the wall is calculated. If the player is near the obstacle, it looks larger and vice-versa.

Shown below is a game, Wolf 3D, which was created using raycasting.

Riju wants to create an online snooker game using raycasting. The game in the creation stage on a coordinate map is shown below.

The snooker table has six pockets (P_1, P_2, P_3, P_4, P_5 and P_6) and he has shown three balls - white (W), red (R) and green (G) on the table. The objective of the game is to use the white ball to hit the coloured balls into the pockets using a cue stick.



(i) How much distance will a ray travel if sent from the green ball to the nearest pocket? Show your work. 1

(ii) Riju wants to place a yellow ball at the midpoint of the line connecting the white and green balls.
Find the coordinates of the point at which he should place the yellow ball. Show your steps. 1

(iii) Riju is running a trial on his game. He struck the white ball in a way that it rebound off the rail (line connecting P_4 and P_6) and went into the pocket P_2. 2

- After the rebound, the ball crossed the x-axis at point $X(\frac{2}{7}, 0)$ on the way to the pocket.
- The ratio of the distance between the rail and point X and the distance between point X and the pocket was 3 : 4.
Find the coordinates of the point at which the ball struck the rail. Show your steps.

Or, (iii) Riju wants to hit a blue ball placed at (-1, -3) into pocket P_5 along a straight path. Would the red ball lie on the straight path between the blue ball and P_5? Justify your answer. 2

Q.38. On 8th November, 2016 the GOI announced the demonetisation of ₹500 and ₹1000 notes of Mahatma Gandhi Series.
According to the order of our Prime Minister Modiji, people went to the Banks and deposited the same and could get the new currency notes of denomination ₹100, ₹500 and ₹2,000 only.
A person went to the bank and deposited total ₹50,000. A cashier got total 70 notes in all.

(i) Form the pair of linear equations of the given problem. 1

(ii) How many notes of new denomination of ₹2,000 and ₹500 in exchange of deposit of ₹50,000 will the person receive? 1

(iii) How many ₹500 notes did the person deposit, as per the equation in (i) while depositing an amount of ₹50,000? 2

Or

(iii) With reference to (ii), calculate the number of ₹1,000 notes the cashier received.

* * * * *

For Solutions

3 Sample Question Paper

Time allowed : 3 hours | *Maximum marks : 80*

General Instructions:

1. *This question paper contains 38 questions.*
2. *This Question Paper is divided into 5 **Sections A, B, C, D** and E.*
3. *In **Section A**, Questions no. **1-18** are multiple choice questions (MCQs) and **Q. 19** and **Q. 20** are Assertion-Reason based questions of **1** mark each.*
4. *In **Section B**, Questions no. **21-25** are very short answer (VSA) type questions, carrying **02** marks each.*
5. *In **Section C**, Questions no. **26-31** are short answer (SA) type questions, carrying **03** marks each.*
6. *In **Section D**, Questions no. **32-35** are long answer (LA) type questions, carrying **05** marks each.*
7. *In **Section E**, Questions no. **36-38** are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.*
8. *All Questions are compulsory. However, an internal choice in **2 Questions of Section B**, **2 Questions of Section C and 2 Questions of Section D** has been provided. An internal choice has been provided in all the **2 marks questions of Section E**.*
9. *Draw neat and clean figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.*
10. *Use of calculators is not allowed.*

SECTION-A

Section A consists of 20 questions of 1 mark each.

Q.1. If the sum of zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k. 1

(*a*) 3 (*b*) 6 (*c*) 9 (*d*) 12

Q.2. What is the greatest possible speed at which a man can walk 52 km and 91 km in an exact number of hours? 1

(*a*) 17 km/hours (*b*) 7 km/hours
(*c*) 13 km/hours (*d*) 26 km/hours

Q.3. If the system of equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then k = ... 1

(*a*) -1 (*b*) 0 (*c*) 1 (*d*) 2

Q.4. If $ax + by = a^2 - b^2$ and $bx + ay = 0$, find the value of $(x + y)$.

(*a*) b (*b*) $a + b$ (*c*) a (*d*) $a - b$

Q.5. How many solutions does the pair of equations $y = 0$ and $y = -5$ has? 1

(*a*) infinite (*b*) two (*c*) one (*d*) No solution

Q.6. For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P.? **1**

(*a*) 8 (*b*) 18 (*c*) 10 (*d*) 5

Q.7. The n^{th} term of an A.P. is $7 - 4n$. Find its common difference. **1**

(*a*) -4 (*b*) -3 (*c*) -2 (*d*) 4

Q.8. Write the nature of roots of the quadratic equation $9x^2 - 6x - 2 = 0$. **1**

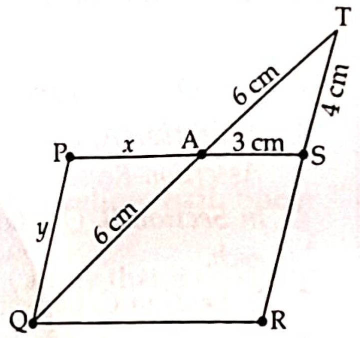
(*a*) No real roots (*b*) 2 equal real roots
(*c*) 2 distinct real roots (*d*) More than 2 real roots

Q.9. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b. **1**

(*a*) 3, -6 (*b*) -3, 6 (*c*) -6, 3 (*d*) 6, -3

Q.10. In given figure, PQRS is a parallelogram, if AT = AQ = 6 cm, AS = 3 cm and TS = 4 cm, then **1**

(*a*) $x = 4, y = 5$
(*b*) $x = 2, y = 3$
(*c*) $x = 1, y = 2$
(*d*) $x = 3, y = 4$

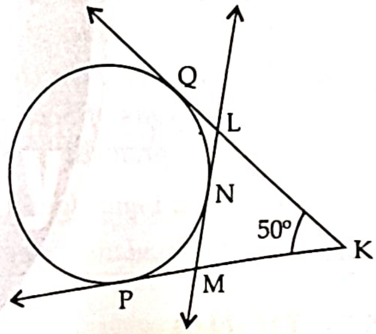


Q.11. Shown here is a circle with 3 tangents KQ, KP and LM. QL = 2 cm and KL = 6 cm. **1**

PM = $\frac{1}{2}$ KL. Note: The figure is not to scale.

What is the measure of ∠NMK?

(*a*) 50°
(*b*) 65°
(*c*) 80°
(*d*) cannot be uniquely determined with the given information.



Q.12. If $\Delta ABC \sim \Delta PQR$, perimeter of ΔABC = 32 cm, perimeter of ΔPQR = 48 cm and PR = 6 cm, then find the length of AC. **1**

(*a*) 4 cm (*b*) 5 cm (*c*) 6 cm (*d*) 2 cm

Q.13. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then calculate the height of the wall. **1**

(*a*) 5 m (*b*) 7.5 m
(*c*) 10 m (*d*) 12.5 m

Q.14. $\sqrt{3}\cos^2 A + \sqrt{3}\sin^2 A$ is equal to: **1**

(*a*) 1 (*b*) $\frac{1}{\sqrt{3}}$

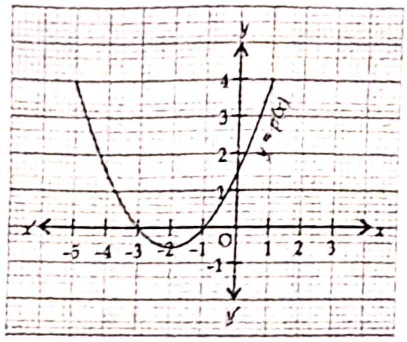
(*c*) $\sqrt{3}$ (*d*) 0

Q.15. Find all the zeroes of $f(x) = x^3 - 2x$. **1**

(*a*) $\sqrt{2}$ (*b*) $-\sqrt{2}$ (*c*) $\pm\sqrt{2}$ (*d*) $\pm\sqrt{2}, 0$

Q.16. In the given figure, the graph of some polynomial $p(x)$ is given. The zeroes of the polynomial is/are: 1

(*a*) 1.5
(*b*) –3
(*c*) –1
(*d*) –3, –1



Q.17. The total surface area of a solid hemisphere of radius 7 cm is: 1

(*a*) 447 π cm^2
(*b*) 239 π cm^2
(*c*) 174 π cm^2
(*d*) 147 π cm^2

Q.18. A bowl contains 3 red and 2 blue marbles. Roohi wants to pick a red marble. Which of the following changes could she make so that the probability of picking a red marble is greater than it was before? 1

(*i*) Adding a red marble (*ii*) Removing a blue marble
(*iii*) Adding 1 red and 1 blue marble

(*a*) only (*i*) (*b*) only (*i*) and (*ii*) (*c*) only (*i*) and (*iii*) (*d*) All of the above

DIRECTION: *In the questions numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option–*

(*a*) *Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).*
(*b*) *Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).*
(*c*) *Assertion (A) is true but Reason (R) is false.*
(*d*) *Assertion (A) is false but Reason (R) is true.*

Q.19. **Assertion (A):** Common difference of the AP –7, –3, 1, 5, 9, ... is 4. 1

Reason (R): Common difference of the AP $a, a + d, a + 2d,$ .... is given by $d = a_2 - a_1$ term.

Q.20. **Assertion (A):** If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason (R): Circumference = $2\pi \times$ radius 1

SECTION-B

Section B consists of 5 questions of 2 marks each.

Q.21. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together? 2

Q.22. Find the coordinates of a point P, which lies on the line segment joining the points A(–2, –2) and B(2, –4) such that AP = $\frac{3}{7}$ AB. 2

Or, Find the ratio in which the y-axis divides the line segment joining the points (–4, –6) and (10, 12). Also find the coordinates of the point of division.

Q.23. Check whether the three lines represented by the equations given below intersect at a common point. 2

$$2x + y - 1 = 0; \quad 4x + 3y + 5 = 0; \quad 5x + 4y + 8 = 0$$

Show your work.

Q.24. Prove that: $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A}$. **2**

Or

If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Q.25. Two coins are tossed simultaneously. What is the probability of getting **2**

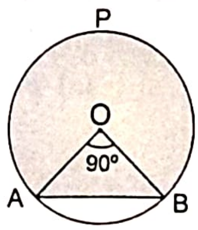
(*i*) Atleast one head?
(*ii*) Atmost one tail?
(*iii*) A head and a tail?

SECTION-C

Section C consists of 6 questions of 3 marks each.

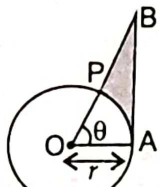
Q.26. National Art convention got registrations from students from all parts of the country, of which 60 are interested in music, 84 are interested in dance and 108 students are interested in handicrafts. For optimum cultural exchange, organisers wish to keep them in minimum number of groups such that each group consists of students interested in the same artform and the number of students in each group is the same. Find the number of students in each group. Find the number of groups in each artform. How many rooms are required if each group is allotted a room? **3**

Q.27. Find the area of the major segment APB, in the given figure of a circle of radius 35 cm and $\angle AOB = 90°$. [Use $\pi = \frac{22}{7}$] **3**

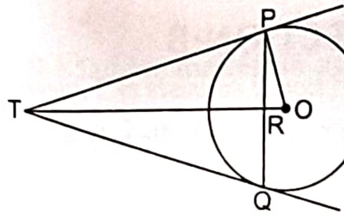


Or

Given figure is shown a sector OAP of a circle with centre O, containing $\angle\theta$. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right)$. **3**



Q.28. In the given figure, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP. **3**



Q.29. The sum of first q terms of an A.P. is $63q - 3q^2$ If its pth term – 60 , find the value of p. Also, find the 11th term of this A.P. **3**

Or

Find the sum of the integers between 100 and 200 that are not divisible by 9.

Q.30. Prove that: $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2\,\text{cosec}\,A$. **3**

Q.31. (*a*) The probability of guessing the correct answer to a certain question is $\frac{x}{12}$. If the probability of guessing the wrong answer is $\frac{3}{4}$, find x. **1**

(*b*) If a student copies the answer, then its probability is $\frac{2}{6}$. If he doesn't copy the answer, then the probability is $\frac{2y}{3}$. Find the value of y. **2**

SECTION-D

Section D consists of 4 questions of 5 marks each.

Q.32. Find the mean of following frequency distribution: **5**

Class interval	25 – 29	30 – 35	35 – 39	40 – 44	45 – 49	50 – 54	55 – 59
frequency	14	22	16	6	5	3	4

Or

The median of the following data is 50. Find the values of 'p' and 'q', if the sum of all frequencies is 90. Also find the mode. **5**

Marks obtained	20–30	30–40	40–50	50–60	60–70	70–80	80–90
Number of students	p	15	25	20	q	8	10

Q.33. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this, will the car reach the tower? Give your answer to nearest minutes. (Use $\sqrt{3}$ = 1.73) **5**

Or

The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of 3000 $\sqrt{3}$ m, find the speed of the aeroplane.

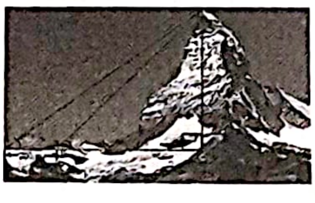
Q.34. A military tent of height 8.25 m is in the form of a right circular cylinder of base diameter 30 m and height 5.5 m surmounted by a right circular cone of same base radius. Find the length of canvas used in making the tent, if the breadth of the canvas is 1.5 m. **5**

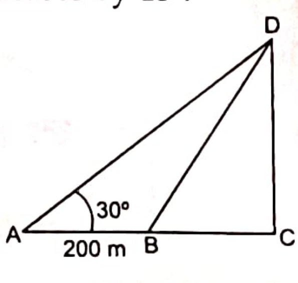
Q.35. A man sold a chair and a table together for ₹760 thereby making a profit of 25% on chair and 10% on table. By selling them together for ₹767.50 he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each. **5**

SECTION-E

Case study based questions are compulsory.

Q.36. This figure shows the mountain standing in a vertical form. The angle of elevation of the top of the mountain from a certain point is 30°. If the observer moves 200 m towards the mountain, the angle of elevation of the top increases by 15°.





(*i*) What is the angle when the observer moves towards the mountain? 1

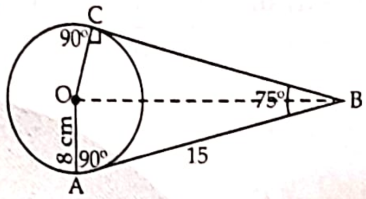
(*ii*) What is the height of the mountain? 1

(*iii*) Calculate the distance between point B and C. 2

Or

What is the total distance between point A and C? 2

Q.37. "Narendra Modi in Gujarat: Delivering a speech to mark the centenary of the Sabarmati Ashram and 150th birth anniversary of Shrimad Rajkhandraji, a guru to Mahatma Gandhi. PM Modi said unleashing violence against others went against the ideals of the father of the Nation. In a rare occasion, Modiji was seen spinning a charkha at the Sabarmati Ashram.



(*i*) What is the relation between BC and AB? 1

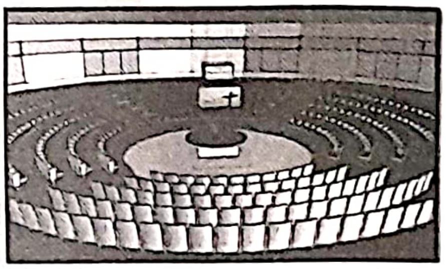
Or

(*i*) Calculate the number of tangents that can be drawn at a point of the circle.

(*ii*) What is the length of OB. 2

(*iii*) Find $\angle AOC$. 1

Q.38. A school auditorium has to be constructed with a capacity of 2000 people. The chairs in the auditorium are arranged in a concave shape facing towards the stage in such a way that each succeeding row has 5 seats more than the previous one.



Based on the given information, answer the following questions:

(*i*) If the first row has 15 seats, then how many seats will be there in 12th row? 1

(*ii*) If there are 15 rows in the auditorium, then how many seats will be there in the middle row? 1

(*iii*) If total 1875 guests were there in the auditorium for a particular event, then how many rows will be needed to make all of them sit? 2

Or

(*iii*) If total 1250 guests were there in the auditorium for a particular event, then how many rows will be left blank out of total 30 rows? 2

* * * * *

4 Sample Question Paper

For Solutions

Time allowed : 3 hours *Maximum marks : 80*

General Instructions:

1. *This question paper contains 38 questions.*
2. *This Question Paper is divided into* **5 Sections A, B, C, D** *and E.*
3. *In* **Section A,** *Questions no.* **1-18** *are multiple choice questions (MCQs) and* **Q. 19** *and* **Q. 20** *are Assertion-Reason based questions of* **1** *mark each.*
4. *In* **Section B,** *Questions no.* **21-25** *are very short answer (VSA) type questions, carrying* **02** *marks each.*
5. *In* **Section C,** *Questions no.* **26-31** *are short answer (SA) type questions, carrying* **03** *marks each.*
6. *In* **Section D,** *Questions no.* **32-35** *are long answer (LA) type questions, carrying* **05** *marks each.*
7. *In* **Section E,** *Questions no.* **36-38** *are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.*
8. *All Questions are compulsory. However, an internal choice in* **2 Questions of Section B, 2 Questions of Section C** *and* **2 Questions of Section D** *has been provided. An internal choice has been provided in all the* **2 marks questions of Section E.**
9. *Draw neat and clean figures wherever required. Take* $\pi = \frac{22}{7}$ *wherever required if not stated.*
10. *Use of calculators is not allowed.*

SECTION-A

Section A consists of 20 questions of 1 mark each.

Q.1. Find the value of a so that the point (3, a) lies on the line represented by $2x - 3y = 5$. **1**

(*a*) $\frac{1}{3}$ (*b*) 1 (*c*) 3 (*d*) $-\frac{1}{3}$

Q.2. If the sum of first n odd natural numbers is equal to k times the sum of first n even natural numbers, then k is equal to **1**

(*a*) $\frac{n+1}{2n}$ (*b*) $\frac{2n}{n+1}$ (*c*) $\frac{n+1}{n}$ (*d*) $\frac{n}{n+1}$

Q.3. If $x^2 - 5x + 1 = 0$, find the value of $x + \frac{1}{x}$. **1**

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6

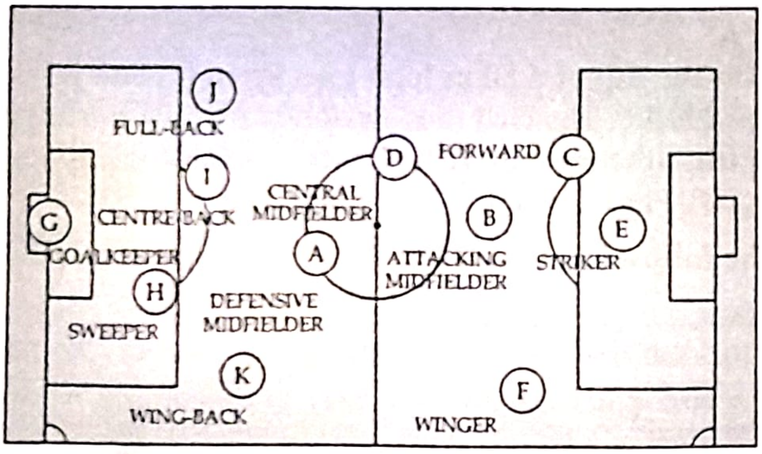
Q.4. HCF and LCM of two numbers is 9 and 459 respectively. If one of the numbers is 27, find the other number. **1**

(*a*) 18 (*b*) 36 (*c*) 153 (*d*) 9

Q.5. In an A.P. if $S_{21} = 1250$, $S_{20} = 1200$, find a_{21}. **1**

(*a*) 2450 (*b*) 50 (*c*) 40 (*d*) 60

Q.37. Tharunya was thrilled to know that the football tournament is fixed with a monthly timeframe from 20th July to 20th August 2024 and for the first time in the FIFA Women's World Cup's history, two nations host in 10 venues. Her father felt that the game can be better understood if the position of players is represented as points on a coordinate plane.



Based on the above information, answer the following questions:

(i) At an instance, the mid fielders and forward formed a parallelogram. Find the position of the central midfielder (D) if the position of other players who formed the parallelogram are :- A(1,2), B(4,3) and C(6,6). **1**

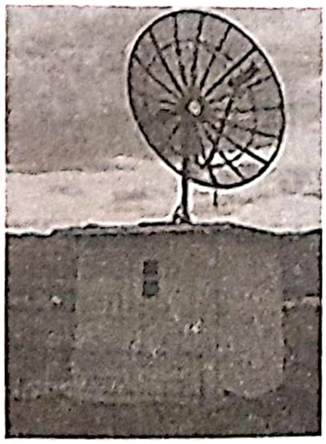
(ii) Check if the Goal Keeper G(-3,5), Sweeper H(3,1) and Wing-back K(0,3) fall on a same straight line. **2**

Or

Check if the Full-back J(5, –3) and centre-back I(–4, 6) are equidistant from forward C(0, 1) and if C is the mid-point of IJ.

(iii) If Defensive midfielder A(1, 4), Attacking midfielder B(2, –3) and Striker E(a, b) lie on the same straight line and B is equidistant from A and E, find the position of E. **1**

Q.38. This picture shows the antenna which is made with silver wire in the form of a circle with diameter 40 cm. The wire is also used in making 8 diameters which divide the circle into 16 equal sectors.



(i) Calculate the angle made each sector. **1**

(ii) Find the total length of the silver wire required. **1**

(iii) What is the length of one arc make by these 8 diameters? **2**

Or

(iii) Calculate the area of one sector.

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5 Sample Question Paper

For Solutions

Time allowed : 3 hours *Maximum marks : 80*

General Instructions:

1. *This question paper contains 38 questions.*
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3. *In **Section A**, Questions no. **1-18** are multiple choice questions (MCQs) and **Q. 19** and **Q. 20** are Assertion-Reason based questions of 1 mark each.*
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9. *Draw neat and clean figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.*
10. *Use of calculators is not allowed.*

SECTION-A

Section A consists of 20 questions of 1 mark each.

Q.1. If $xy = 180$ and HCF$(x, y) = 3$, then find the LCM(x, y). 1

(*a*) 540 (*b*) 60 (*c*) 20 (*d*) 50

Q.2. Value of α for which the following system of linear equations has an infinitive number of solutions:

$\alpha x + 3y = \alpha - 3;\ 12x + \alpha y = \alpha$ 1

(*a*) −6 (*b*) 6 (*c*) 3 (*d*) −3

Q.3. The graph of a polynomial $p(x)$ passes through the points (−5, 0), (0, −40), (8, 0) and (5, −30). Which among the following is a factor of $p(x)$? 1

(*a*) $(x - 5)$ (*b*) $(x - 8)$ (*c*) $(x + 30)$ (*d*) $(x + 40)$

Q.4. Graphically, the pair of equations given by 1

$6x - 3y + 10 = 0$ $2x - y + 9 = 0$

represents two lines which are

(*a*) intersecting at exactly one point (*b*) parallel
(*c*) coincident (*d*) intersecting at exactly two points

Q.5. Two APs have the same common difference. The first term of one of these is −1 and that of the other is −8. Then the difference between their 4th terms is 1

(*a*) −1 (*b*) −8 (*c*) 7 (*d*) −9

Q.6. Which term of the progression 4, 9, 14, 19, ... is 109?

(*a*) 22 (*b*) 21 (*c*) 20 (*d*) 19

Q.7. Solve for x: $x^2 + \frac{1}{2}x - 1 = 0$ 1

(*a*) 4, 1 (*b*) 4, −1 (*c*) $\frac{-1 \pm \sqrt{17}}{4}$ (*d*) $\frac{1 \pm \sqrt{17}}{4}$

Q.8. Find the value of 'k' for which the quadratic equation $x^2 - 2x + k = 0$ has no real roots. 1

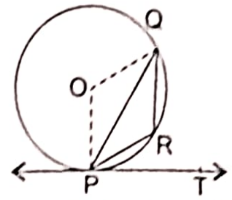
(*a*) $k > 1$ (*b*) $k > -1$

(*c*) $k < 1$ (*d*) $k < -1$

Q.9. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50°$, the find $\angle AOB$. 1

(*a*) 90° (*b*) 100° (*c*) 80° (*d*) 110°

Q.10. In the given figure, PQ is a chord of a circle with centre O. PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$. 1



(*a*) 150° (*b*) 60°

(*c*) 90° (*d*) 120°

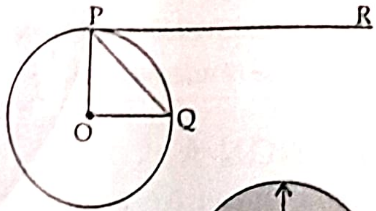
Q.11. A chord of a circle of radius 10 cm subtends a right angle at its centre. Calculate the length of the chord (in cm). 1

(*a*) 10 cm (*b*) 5 cm (*c*) $5\sqrt{2}$ cm (*d*) $10\sqrt{2}$ cm

Q.12. If $4 \cot \theta - 5 = 0$, then the value of $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is 1

(*a*) $\frac{5}{3}$ (*b*) $\frac{5}{6}$ (*c*) 0 (*d*) $\frac{1}{6}$

Q.13. If O is centre of a circle and Chord PQ makes an angle 50° with the tangent PR at the point of contact P, find the angle made by the chord at the centre. 1



(*a*) 130° (*b*) 100°

(*c*) 50° (*d*) 30°

Q.14. Shown here is a solid made by joining a right circular cylinder and a hemisphere of equal radius (r cm). The total surface area of the solid is equal to the surface area of a sphere with twice the radius of this solid. 1

Which of the following gives the height of the cylinder in the above solid?

(*a*) $6r$ cm (*b*) $6.5r$ cm

(*c*) $7r$ cm (*d*) $7.5r$ cm

Q.15. For the following distribution:

Class	0 - 5	5 - 10	10 -15	15 - 20	20 - 25
Frequency	10	15	12	20	9

the upper limit of the modal class is 1

(*a*) 10 (*b*) 15 (*c*) 20 (*d*) 25

Q.16. A box contains 70 cards numbered from 1 to 70. If one card is drawn at random from the box, find the probability that it bears a perfect square number. 1

(*a*) $\frac{9}{30}$ (*b*) $\frac{4}{35}$ (*c*) $\frac{1}{7}$ (*d*) $\frac{1}{10}$

Q.17. $\sin A + \cos B = 1$, $A = 30°$ and B is an acute angle, then find the value of B. 1

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°

Q.18. $(\sec A + \tan A)(1 - \sin A) =$ 1

(*a*) sec A (*b*) sin A (*c*) cosec A (*d*) cos A

DIRECTION: *In the questions numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option–*

(*a*) *Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).*

(*b*) *Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).*

(*c*) *Assertion (A) is true but Reason (R) is false.*

(*d*) *Assertion (A) is false but Reason (R) is true.*

Q.19. **Assertion (A):** Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone. 1

Reason (R): Top is obtained by fixing the plane surfaces of the hemisphere and cone together.

Q.20. **Assertion (A):** The probability of winning a game is 0.4, then the probability of losing it, is 0.6. 1

Reason (R): P(E) + P(not E) = 1

SECTION-B

Section B consists of 5 questions of 2 marks each.

Q.21. In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject. 2

Q.22. Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7).

Or, Find a point on the y-axis which is equidistant from the points A(4, 8) and B(–6, 6). Also find the distance AP.

Q.23. For what values of k will the following pair of linear equations have infinitely many solutions? 2

$$kx + 3y - (k - 3) = 0; \quad 12x + ky - k = 0$$

Q.24. If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, $0° < A + B \leq 90°$ and $A > B$, then find the measures of angles A and B. 2

Or, If $3 \sin\theta + 5 \cos\theta = 5$, then prove that $5 \sin\theta - 3 \cos\theta = \pm 3$.

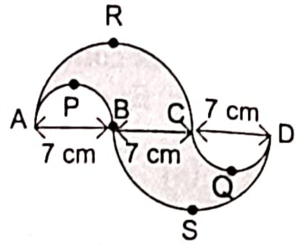
Q.25. The angles of a triangle are in A.P., the least being half the greatest. Find the angles. 2

SECTION-C

Section C consists of 6 questions of 3 marks each.

Q.26. Prove that $3 + \sqrt{2}$ is an irrational number. 3

Q.27. In the given fig., APB and CQD are semi-circles of diameter 7 cm each, while ARC and BSD are semi-circles of diameter 14 cm each. Find the perimeter of the shaded region. **3**



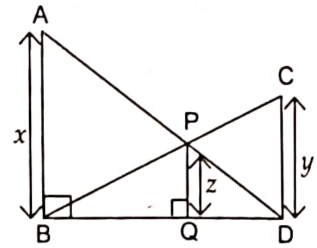
Q.28. Find the value of p for which the quadratic equation $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also find these roots. **3**

Or, A motor boat whose speed is 20 km/h in still water, takes 1 hour more to go 48 km upstream than to return down stream to the same spot. Find the speed of the stream.

Q.29. On reversing the digit of a two digit number, number obtained is 9 less than three times the original number. If difference of these two numbers is 45, find the original number. **3**

Q.30. In the given fig., $\angle ABD = \angle CDB = \angle PQB = 90^\circ$. If AB = x units, CD = y units and PQ = z units. **3**

Prove that: $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ or $x^{-1} + y^{-1} = z^{-1}$.



Q.31. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting **3**

(*i*) a face card or a black card (*ii*) neither an ace nor a king

(*iii*) a jack and a black card

Or, Five cards—the ten, jack, queen, king and ace of diamonds, are well shuffled with their faces downwards. One card is then picked up at random.

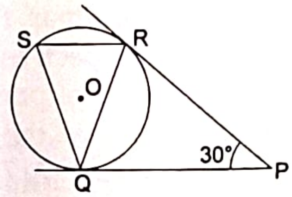
(*a*) What is the probability that the drawn card is the queen?

(*b*) If the queen is drawn and put aside, and a second card is drawn, find the probability that the second card is (*i*) an ace (*ii*) a queen.

SECTION-D

Section D consists of 4 questions of 5 marks each.

Q.32. In the given fig., tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$. **5**



Q.33. The marks obtained by 80 students of Class X in a mock test of Mathematics are given below in the table: **5**

Marks	*Number of students*
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43
60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

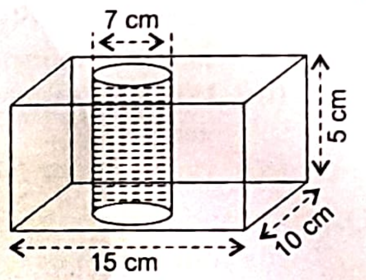
Find the median and the mode of the data.

Or, Find the median wage of the workers from the following distribution table:

Wages	*No. of Workers*
More than 150	0
More than 140	10
More than 130	29
More than 120	60
More than 110	104
More than 100	134
More than 90	151
More than 80	160

Q.34. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent in cylindrical with radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹120 per m^2, find the amount shared by each school to set up the tents. 5

Or, In the given fig., from a cuboidal solid metallic block of dimensions 15 cm × 10 cm × 5 cm a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. [Use $\pi = \frac{22}{7}$] 5

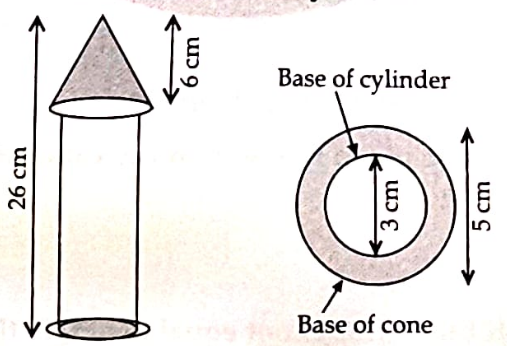


Q.35. An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at the instant. 5

SECTION-E

Case study based questions are compulsory.

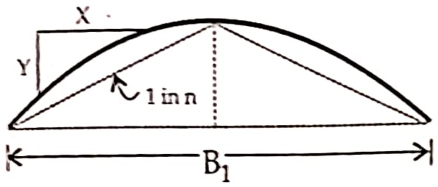
Q.36. A rocket is in the shape of a cone mounted on a cylinder.



(i) What is the area of the lower ring? **1**
(ii) What is the slant height of upper cone? **1**
(iii) What would be the area of the base of rocket with radius 3 cm? **2**
Or, **(iii)** How much metal sheet will be needed to cover the cylindrical portion? **2**

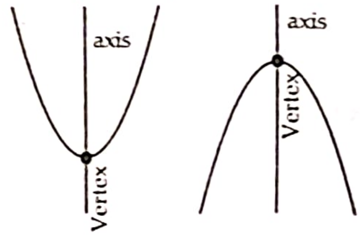
Q.37. Applications of Parabolas-Highway Overpasses/Underpasses

A highway underpass is parabolic in shape.



a. Parabolic Camber $y = 2x^2/nw$

Parabola. A parabola is the graph that results from $p(x) = ax^2 + bx + c$. Parabolas are symmetric about a vertical line known as the Axis of Symmetry. The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the Vertex.



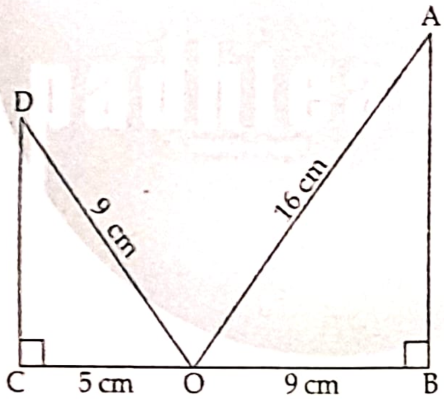
(i) If the highway overpass is represented by $x^2 - 2x - 8$. Then find its all zeroes. 1

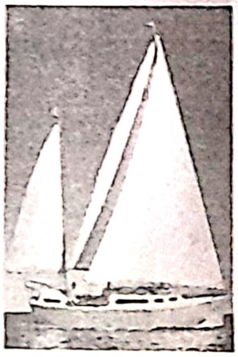
(ii) The highway overpass is represented graphically.
Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial 1

(iii) Write the representation of Highway Underpass whose zero is 6 and sum of the zeroes is 0. 2

Or, (iii) Write the number of zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have.

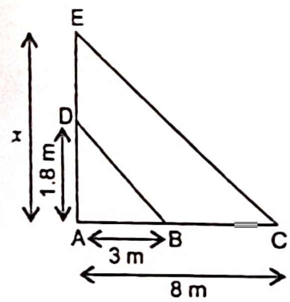
Q.38. On the basis of the following figures and associating them with triangle theorems answer the following questions:





(i) If the given triangles are similar. Find $\angle P$. 1

(ii) With reference to the given figure, find the value of x. 1



Or, (ii) If $\Delta ABC \sim \Delta PQR$ but ΔABC is not equal to ΔPQR, then AB.QR is equal to which of the two sides? 2

(iii) If in two triangles, ΔABC and ΔDEF, $\frac{AB}{DF} = \frac{BC}{FE} = \frac{CA}{ED}$ is 2

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